Evidence of the $h_c \to K_S^0 K^+ \pi^- + \mathrm{c.c.}$ decay

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Based on $(2.712 \pm 0.014) \times 10^9$ $\psi(3686)$ events collected by the BESIII Collaboration, evidence of the hadronic decay $h_c \to K_S^0 K^+ \pi^- + \text{c.c.}$ is found with a significance of 4.3σ in the $\psi(3686) \to \pi^0 h_c$ process. The branching fraction of $h_c \to K_S^0 K^+ \pi^- + \text{c.c.}$ is measured to be $(7.3 \pm 1.8 \pm 0.8) \times 10^{-4}$, where the first and second uncertainties are statistical and systematic, respectively. Combining with the exclusive decay width of $\eta_c \to K\bar{K}\pi$, our result indicates inconsistencies with both pQCD and NRQCD predictions.

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I. INTRODUCTION

In the framework of the Standard Model, one fundamental assumption is that the strong interaction between quarks is mediated by colored gluons, as described by quantum chromodynamics (QCD) [1–3]. Since the derivative of the QCD coupling strength (α_s) with respect to the logarithm of the energy scale, i.e., the β function, is negative [1,2], α_s is divergent in the low-energy limit, bringing large uncertainties to the theoretical calculations in this nonperturbative region. Because of their large mass scales and nonrelativistic nature, charmonia are ideal probes to study and understand QCD from both perturbative and nonperturbative aspects. Based on perturbative QCD (pQCD) and nonrelativistic QCD (NRQCD), the branching fractions of several light hadron decay channels of the P-wave spin singlet charmonium $h_c(1P)$ are calculated [4]. For the channel $K\bar{K}\pi$, the predictions are $(1.4 \pm 0.9)\%$ (pQCD) and $(5.5 \pm 3.3)\%$ (NRQCD), respectively. An experimental measurement on this channel is helpful for testing the validity of these approaches.

While many hadronic decay modes of the S-wave charmonia including J/ψ and $\psi(3686)$ have been discovered and determined precisely, the knowledge of the hadronic decays of h_c is still sparse since its direct production via e^+e^- annihilation is forbidden. Until now, only a few hadronic decay modes have been observed [5,6] via the $\psi(3686) \to \pi^0 h_c$ process, with a sum of branching fractions less than 5% [7]. According to the experimental result on the branching fraction of the prominent electromagnetic transition $\mathcal{B}(h_c \to \gamma\eta_c) = (57.66^{+3.62}_{-3.50} \pm 0.58)\%$ [8], the fraction of gluonic annihilation width $\mathcal{B}(h_c \to 3g)$ is estimated to be about 42%, indicating that a significant fraction of h_c hadronic decay modes remain unknown. Furthermore, although the decay $h_c \to K_S^0 K^+ \pi^-$ has been predicted for a long time [4], it has not yet been discovered; only an upper limit of 6×10^{-4} has been determined [6,7].

A search for the hadronic decay mode $h_c \to K_S^0 K^+ \pi^-$ is performed based on the $\psi(3686) \to \pi^0 h_c$ process from the data sample with $(2.712 \pm 0.014) \times 10^9$ $\psi(3686)$ events [9] collected at center-of-mass energy $\sqrt{s} = 3.686$ GeV with the BESIII detector. Throughout this paper, charged conjugation is always implied unless otherwise specified.

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II. BESIII DETECTOR AND MONTE CARLO SIMULATION

The BESIII detector [10] records symmetric e^+e^- collisions provided by the BEPCII storage ring [11], which operates with a center-of-mass energy range from 2.00 to 4.95 GeV, with a peak luminosity of 1.1×10^{33} cm^{-2} s^{-1} achieved at 3.773 GeV. BESIII has collected large data samples in this energy region [12–14]. The cylindrical core of the BESIII detector covers 93% of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon-identification modules interleaved with steel. The charged-particle momentum resolution at 1 GeV$/c$ is 0.5%, and the resolution of the rate of energy loss, d$E/$dx, is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end-cap) region. The time resolution in the TOF barrel region is 68 ps, while that in the end-cap region is 110 ps. The end-cap TOF system was upgraded in 2015 using multigap resistive plate chamber technology, providing a time resolution of 60 ps, which benefits 80% of the data used in this analysis [15–17].

Simulated data samples produced with a Geant4-based [18] Monte Carlo (MC) simulation, which includes the geometric description of the BESIII detector and the detector response, are used to determine detection efficiencies and to estimate backgrounds. The simulation models the beam-energy spread and initial-state radiation in the e^+e^- annihilations with the generator KKMC [19]. The inclusive MC sample includes the production of the $\psi(3686)$ resonance, the ISR production of the J/ψ, and the continuum processes incorporated in KKMC [19]. All particle decays are modeled with EvtGen [20,21] using branching fractions either taken from the Particle Data Group [7], when available, or otherwise estimated with LUNDCHARM [22,23]. Final-state radiation from charged final-state particles is incorporated using PHOTOS [24]. Additionally, an exclusive MC sample of the signal process $\psi(3686) \to \pi^0 h_c$, $h_c \to K_S^0 K^+ \pi^-$ is generated for the selection criteria optimization and detection efficiency determination, where both $\psi(3686) \to \pi^0 h_c$ and $h_c \to K_S^0 K^+ \pi^-$ are simulated based on a uniform phase space distribution.

III. DATA ANALYSIS

A. Event selection

To select $\pi^0 K_S^0 K^+ \pi^-$ events, all the charged tracks are required to be within a polar angle (θ) range of $|\cos\theta| < 0.93$, where θ is measured with respect to the z axis, i.e., the symmetry axis of MDC. Exactly four charged tracks are required after the polar angle selection, and the total net charge is required to be zero.

To reconstruct K_S^0, all pairs of oppositely charged tracks are subjected to a secondary vertex fit, which constrains the two tracks to originate from a common vertex. The pair with invariant mass closest to the K_S^0 known mass is chosen, and the distance of the common vertex to the interaction point normalized by its uncertainty, $L/\Delta L$, is required to be greater than 2.

For the charged tracks not originating from K_S^0 decays, the distance of closest approach to the interaction point (IP) must be less than 10 cm along the z axis and less than 1 cm in the transverse plane. For each track, particle identification (PID) chi-square values $\chi^2_{\rm PID}(\mathcal{P}_\pm)$ under different particle hypotheses are computed utilizing the combined information of dE/dx and TOF, where $\mathcal{P} \in \{K, \pi\}$ refers to the particle types and the subscript indicates the positive or negative charge of the particle.

Photon candidates are reconstructed from electromagnetic clusters produced in the crystals of the EMC. Clusters with deposited energy larger than 25 MeV in the barrel region ($|\cos\theta| < 0.8$) or 50 MeV in the end-cap region ($0.86 < |\cos\theta| < 0.92$) are selected as photon candidates. The EMC timing of each photon counted from the event start time is required to be within [0, 700] ns to suppress noise and energy deposits unrelated to the event. The opening angle between the photon and the nearest charged track in angle is required to be larger than 10° to suppress the showers from charged tracks.

A pair of photons is accepted as a π^0 candidate if their invariant mass falls into the range $(0.12, 0.15)$ MeV$/c^2$. At least one π^0 is required. All combinations of final-state particles are subjected to a five-constraint kinematic fit, with the constraints provided by four-momentum conservation and the π^0 mass. PID of the two tracks and the selection of the best π^0 among multiple candidates are accomplished by minimizing $\chi^2 = \chi^2_{5C} + \chi^2_{\rm PID}(\mathcal{P}_+) + \chi^2_{\rm PID}(\mathcal{P}_-)$, where χ^2_{5C} is the fit quality of the 5C kinematic fit. The combination with the minimum χ^2 satisfying $\mathcal{P}_+\mathcal{P}_- = K^+\pi^-$ or $K^-\pi^+$ is accepted as a signal candidate. Additionally, $\chi^2_{5C} < 124$ is required to further suppress the non-$\pi^0 K_S^0 K^+ \pi^-$ background.

To suppress the background with one more or one less photon in the final state, a four-constraint kinematic fit, with the constraint provided by four-momentum conservation, is performed, and the chi-square values with one or three photons $\chi^2_{4C}(1\gamma)$ and $\chi^2_{4C}(3\gamma)$ are required to be larger than that of the signal candidate, where the test is done for all the photon candidates. Backgrounds π^0 produced from $K^*(892)^+ \to K^+\pi^0$ are found in the inclusive MC sample and removed by applying $|M(K^+\pi^0) - m(K^*(892)^+)| > 10\,{\rm MeV}/c^2$, where $M(K^+\pi^0)$ is the invariant mass of $K^+\pi^0$ and $m(K^*(892)^+)$ is the known mass of $K^*(892)^+$ [7]. All the selection criteria have been optimized by maximizing the figure of merit defined in Ref. [25], $\epsilon/(a/2+\sqrt{B})$, where $a=5$ is the desired one-tailed significance, ϵ is the selection efficiency, and B is the expected number of background events falling in the signal range from MC simulation. Here, the signal range is defined as $|M(K_S^0K^+\pi^-)-m(h_c)|<6\ \mathrm{MeV}/c^2$, where $M(K_SK^+\pi^-)$ is the invariant mass of $K_SK^+\pi^-$, $m(h_c)$ is the h_c known mass [7], and the interval is determined to be about $\pm2\sigma$ with respect to the resolution (3.16 MeV/c^2).

B. Fit to data

To determine the number of signal events N, an unbinned maximum-likelihood fit is performed on the $M(K_SK^+\pi^-)$ distribution, as shown in Fig. 1. In the fit, the h_c signal is modeled by the shape obtained from the signal MC sample convolved with a Gaussian function, which accounts for the difference in the mass resolution between MC simulation and data. The central value and standard deviation of the Gaussian function are fixed to those obtained by studying the control sample of $\psi(3686)\to\gamma\chi_{c1,c2},\chi_{c1,c2}\to K_S^0K^+\pi^-$. A peaking background component from $\psi(3686)\to\gamma\chi_{c2},\chi_{c2}\to K_S^0K^+\pi^-$ plus a fake photon, which forms a peak at the left side of the h_c signal, is included in the fit. The number of peaking background events is fixed based on the measured branching fraction [7]. Background from other channels distributes smoothly and is described by an ARGUS function [26], with the threshold parameter fixed to the kinematic threshold 3.551 GeV/c^2. The fit gives $N=205\pm50$ with a statistical significance 4.6σ, determined by the likelihood ratio of the fit with and without the h_c signal function [27].

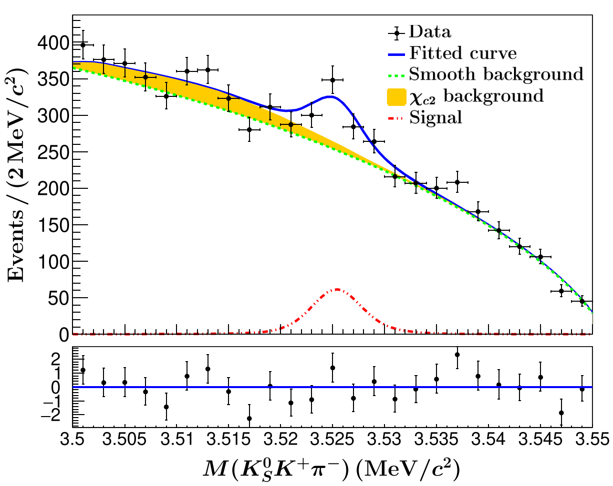

FIG. 1. Fit to the $M(K_S^0K^+\pi^-)$ distribution. The black dots with error bars are data, the blue solid curve is the total fit result, the red dashed-dotted curve is the h_c signal, the green dashed curve represents the smooth background, and the orange-filled shape represents the χ_{c2} peaking background.

The branching fraction is calculated with

$$\mathcal{B}(h_c\to K_S^0K^+\pi^-)\times\mathcal{B}(\psi(3686))=\frac{N}{\epsilon N_{\psi(3686)}\mathcal{B}(K_S^0)\mathcal{B}(\pi^0)},\qquad(1)$$

where $\epsilon=20.8\%$ is the selection efficiency, $N_{\psi(3686)}$ is the number of $\psi(3686)$ events [9], $\mathcal{B}(K_S^0)$ and $\mathcal{B}(\pi^0)$ are the branching fractions of $K_S^0\to\pi^+\pi^-$ and $\pi^0\to\gamma\gamma$, respectively, from the Particle Data Group [7], and $\mathcal{B}(\psi(3686))$ is the branching fraction of $\psi(3686)\to\pi^0h_c$ from Ref. [8]. The resulting values are $\mathcal{B}(h_c\to K_S^0K^+\pi^-)=(7.3\pm1.8)\times10^{-4}$ and $\mathcal{B}(h_c\to K_S^0K^+\pi^-)\times\mathcal{B}(\psi(3686))=(5.3\pm1.3)\times10^{-7}$ with statistical uncertainties only.

IV. SYSTEMATIC UNCERTAINTIES

The sources of systematic uncertainties in the branching fraction measurement are tracking and PID efficiencies, reconstruction of photons, π^0 and K_S^0, $K^+\pi^0$ mass ranges, mass resolution of the signal shape, the MC model, input values, and the fit method. All sources of systematic uncertainties, which are summarized in Table I, are treated as independent and summed in quadrature. The details of their estimation are discussed in the following.

Tracking efficiencies. The uncertainties of tracking efficiencies of the kaon and pion from the primary vertex are 1.0% per track [28] and are added linearly.

PID efficiency. The uncertainty of PID is studied with a $\psi(2S)\to K_S^0K^\pm\pi^\mp\pi^0$ control sample. The sample is selected by reconstructing one K_S^0 with a stricter requirement $L/\Delta L>20$ and one π^0 with the number of photons restricted to be two. The signal region is $0.12<M(\gamma\gamma)<0.14\ \mathrm{GeV}/c^2$ and $487.6<M(K_S^0)<507.6\ \mathrm{MeV}/c^2$, and the PID efficiency is defined as $N_{\mathrm{PID}}/N_{\mathrm{all}}$, where N_{PID} and N_{all} are the number of events in the signal region with

TABLE I. Summary of systematic uncertainties.

Source	Relative uncertainty (%)
Tracking	2.0
PID	2.1
Photon reconstruction	2.0
K_S^0 reconstruction	1.0
π^0 reconstruction	0.7
5C kinematic fit	0.9
MC model	4.9
$M(K^+\pi^0)$ mass window	⋯
Mass resolution	4.4
Fit method	3.4
Number of $\psi(3686)$	0.5
$\mathcal{B}(\psi(3686))$	7.3
Sum	11.1

and without PID. The difference between the efficiencies of MC and data is taken as the systematic uncertainty.

Photon reconstruction. The uncertainty of photon reconstruction, which is 1.0% for each photon, was determined with a control sample of $J/\psi \to \rho^0\pi^0, \rho^0 \to \pi^+\pi^-, \pi^0 \to \gamma\gamma$ [29].

K_S^0 *reconstruction*. The difference of K_S^0 reconstruction between MC simulation and data was studied based on the control samples of $J/\psi \to K^*(892)^\pm K^\mp$, $K^*(892)^\pm \to K_S^0\pi^\pm$, and $J/\psi \to \phi K_S^0 K^\pm \pi^\mp$ [30], and its related uncertainty is 1.0%.

π^0 *reconstruction*. The uncertainty of π^0 reconstruction is studied using $\psi(3686) \to K_S^0 K^+\pi^-\pi^0$ reconstructed with and without a π^0 mass constraint $(0.12, 0.15)$ GeV$/c^2$ and with the events in the h_c signal range excluded. The difference of π^0 reconstruction efficiency between data and MC simulation gives a relative uncertainty of 0.7%.

Kinematic fit. A correction of the helix parameters of the charged tracks in the MC samples is applied to improve the consistency between MC simulation and data. The uncertainty is estimated as half of the difference of efficiency with and without the correction, as suggested in Ref. [31], resulting in an uncertainty of 0.9%.

MC model. Because of the limited knowledge of intermediate states in the h_c decays, alternative MC samples are generated including the intermediate state $K^*(892)$ with the fractions of neutral and charged channels constrained by isospin symmetry. The result is compared with that of the nominal phase space result, and the difference is treated as the systematic uncertainty.

Mass resolution. An alternative control sample, $\psi(3686) \to \pi^0 J/\psi, J/\psi \to K_S^0 K^\pm\pi^\mp$, is used to determine the parameters of the mass resolution function, and the fit is repeated with the alternative mass resolution function. The difference of the fit yields to the nominal value gives a systematic uncertainty of 4.4%.

$K^+\pi^0$ *mass window*. To study the potential uncertainty arising from the $M(K^+\pi^0)$ mass window, a test introduced in Ref. [32] is performed by varying the range of the mass window around its nominal width. The test statistic ξ is defined as $\xi = \frac{|\mathcal{B}-\mathcal{B}'|}{\sqrt{|\sigma^2-\sigma'^2|}}$, where $\mathcal{B}$ (σ) and $\mathcal{B}'$ (σ') are the resulting branching fractions (uncertainties) of the nominal result and from varied mass windows, respectively. If the values of ξ show a trending behavior and exceed two, a systematic uncertainty is assigned. We find the values of ξ of the branching fraction are always less than two and show no trending behavior around the nominal mass window; therefore, the related uncertainty is negligible.

Fit method. The same test as described in the last paragraph is performed with respect to the fitting ranges, and the related systematic effect is proved to be negligible. For the background modeling, the shape for the smooth background is changed from an ARGUS function to a second-order Chebychev polynomial, and for the peaking background, the normalization scale is varied according to the uncertainty of the world average value of the branching fraction $\psi(3686) \to \gamma\chi_{c2}, \chi_{c2} \to K_S^0 K^+\pi^-$ [7]. The corresponding change of the result is taken as the systematic uncertainty.

Input values. The number of $\psi(3686)$ events is quoted from Ref. [9], and the related uncertainty is taken into account. For the input branching fractions, the uncertainties associated with $K_S^0 \to \pi^+\pi^-$ and $\pi^0 \to \gamma\gamma$ are negligible, and the uncertainty of $\psi(3686) \to \pi^0 h_c$ is assigned according to Ref. [8].

The signal significance is reestimated after considering alternative $M(K^+\pi^0)$ mass windows, mass resolution parameters, fitting ranges, and background shape. Among these tests, the lowest significance, 4.3σ, is taken as the final signal significance.

V. SUMMARY AND DISCUSSION

Based on $(2.712 \pm 0.014) \times 10^9$ $\psi(3686)$ events collected at $\sqrt{s} = 3.686$ GeV with the BESIII detector, evidence of the hadronic decay $h_c \to K_S^0 K^+\pi^-$ is found with a significance of 4.3σ after taking the systematic uncertainties into account. The product branching fraction $\mathcal{B}(\psi(3686) \to \pi^0 h_c) \times \mathcal{B}(h_c \to K_S^0 K^+\pi^- + \text{c.c.})$ is $(5.3 \pm 1.3 \pm 0.4) \times 10^{-7}$, and the branching fraction $\mathcal{B}(h_c \to K_S^0 K^+\pi^- + \text{c.c.})$ is determined to be $(7.3 \pm 1.8 \pm 0.8) \times 10^{-4}$, where the first and second uncertainties are statistical and systematic, respectively. Compared to the previous study by BESIII [6], in which an upper limit of $\mathcal{B}(\psi(3686) \to \pi^0 h_c) \times \mathcal{B}(h_c \to K_S^0 K^+\pi^- + \text{c.c.}) < 4.8 \times 10^{-7}, CL = 90\%$ is obtained based on 4.48×10^8 $\psi(3686)$ events, our result is consistent.

Assuming that isospin symmetry holds in the decay of $h_c \to K\bar{K}\pi$, the ratios between isospin multiplets are $\mathcal{B}(h_c \to K^0 K^-\pi^+) : \mathcal{B}(h_c \to \bar{K}^0 K^+\pi^-) : \mathcal{B}(h_c \to K^+K^-\pi^0) : \mathcal{B}(h_c \to K^0\bar{K}^0\pi^0) = 2:2:1:1$. Combining with $\mathcal{B}(K^0 \to K_S^0) = 0.5$, we get the branching fraction $\mathcal{B}(h_c \to K\bar{K}\pi) = 3 \times \mathcal{B}(h_c \to K_S^0 K^+\pi^- + \text{c.c.}) = (0.22 \pm 0.06)\%$, which is listed in Table II. Our result is consistent with the

TABLE II. Comparison between the theoretical predictions and the measurement of this work. Both statistical and systematic uncertainties are included.

Item	Value (%)	Source
$\mathcal{B}(h_c \to K\bar{K}\pi)$	1.4 ± 0.9	pQCD [4]
$\mathcal{B}(h_c \to K\bar{K}\pi)$	5.5 ± 3.3	NRQCD [4]
$\mathcal{B}(h_c \to K\bar{K}\pi)$	0.22 ± 0.06	This work
$\Gamma(h_c \to 3g)/\Gamma(\eta_c \to 2g)$	1.0 ± 0.1	pQCD [4]
$\Gamma(h_c \to 3g)/\Gamma(\eta_c \to 2g)$	8.3 ± 1.8	NRQCD [4]
$\Gamma(h_c \to 3g)/\Gamma(\eta_c \to 2g)$	0.93 ± 0.54	Refs. [7,33]
$\Gamma(h_c \to K\bar{K}\pi)/\Gamma(\eta_c \to K\bar{K}\pi)$	0.069 ± 0.062	This work

predictions based on pQCD and NRQCD, within 1.3 and 1.6 times the uncertainties, respectively.

Because the uncertainties of theoretical prediction are mainly caused by the input branching fractions of η_c, the comparison is rearranged by checking the following relation [34,35], which is used for calculating the exclusive branching fractions in Ref. [4],

$$\frac{\Gamma(h_c \to h)}{\Gamma(\eta_c \to h)} \approx \frac{\Gamma(h_c \to 3g)}{\Gamma(\eta_c \to 2g)}, \tag{2}$$

where h is any exclusive hadronic channel and $\Gamma(h_c \to 3g)$ and $\Gamma(\eta_c \to 2g)$ are the inclusive gluonic annihilation widths of h_c and η_c, respectively. While the theoretical uncertainties for the right-hand side of Eq. (2) are small, an estimation is also made as follows based on experimental results. Taking $\Gamma(h_c \to 3g) \approx \Gamma(h_c) \times (1 - \mathcal{B}(h_c \to \gamma\eta_c)) \approx 0.30 \pm 0.17$ MeV by neglecting all other radiative channels of h_c, and $\Gamma(\eta_c \to 2g) \approx \Gamma(\eta_c) \approx 32.0 \pm 0.7$ MeV by assuming a negligible radiative partial width of η_c, the ratio in the right-hand side of Eq. (2) is $\frac{\Gamma(h_c \to 3g)}{\Gamma(\eta_c \to 2g)} \approx (0.93 \pm 0.54)\%$, which is in good agreement with the pQCD prediction $(1.0 \pm 0.1)\%$ but much smaller than the NRQCD prediction of $(8.3 \pm 1.8)\%$ [4]. Using the result from this work and the result from a global fit of η_c branching fractions [33] as inputs, we determine the ratio of partial widths $\frac{\Gamma(h_c \to K\bar{K}\pi)}{\Gamma(\eta_c \to K\bar{K}\pi)} = (0.069 \pm 0.062)\%$, where the central value differs from the right-hand side of Eq. (2) by 1 order of magnitude.

As a consequence, the test with Eq. (2) using our result for $\Gamma(h_c \to K\bar{K}\pi)/\Gamma(\eta_c \to K\bar{K}\pi)$, which is more sensitive than that of $\mathcal{B}(h_c \to K\bar{K}\pi)$, indicates inconsistencies with both pQCD and NRQCD predictions. Given that the predictions of the theoretical models have large uncertainties and only leading-order formulas for both pQCD and NRQCD are implemented, to resolve the gap between theoretical predictions and experimental measurements, refined predictions are required with improved precision and involving high-order effects such as corrections of renormalization scale [36] or relativistic effects [37]. On the other hand, the precision of our measurement is still strongly limited by statistical uncertainty, preventing us from drawing a solid conclusion.

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