Strange attractor in resonant tunneling

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We consider the process of resonant electron tunneling through a double-barrier potential, taking into account nonlinear dynamical effects generated by charge accumulation in the interbarrier space. We use the perturbation approach of Davydov and Ermakov, which was developed for investigating intrinsic bistability in resonant tunneling. For incoming electron flow, which is modulated slowly in time, we show that the resulting nonlinear dynamics can become chaotic, with the chaos described (because of the open nature of the system) by a strange attractor. We determine the conditions for the existence of this strange attractor and estimate characteristic experimental parameters for its observation. [S0163-1829(98)08531-2]

I. INTRODUCTION

The rapid progress of nanotechnology fabrication has stimulated growing interest in electron transport through quantum low-dimensional structures in the ballistic regime.[1–9] One of the most frequently studied devices is the double-barrier resonant tunneling structure (DBRTS), which consists of two potential barriers surrounding a potential well. It is well known that the transmission of electrons through a DBRTS is effective only if certain resonance conditions are satisfied (see, for example, Refs. 1–9 and references therein).

Apart from the inherently nonlinear nature of the current-voltage (I-V) characteristics caused by the resonant tunneling process itself, additional nonlinear effects have been the focus of much current interest. For instance, a proper accounting of the dynamical charge accumulation within the potential well leads to an electrostatic feedback mechanism that shifts the resonance energy. Under some conditions, this can result in the appearance of nonlinear effects such as current instability, intrinsic bistability, self-oscillations,[10–19] Hamiltonian chaos,[20] dissipative chaos,[21] and others.

Experimentally, hysteresis (bistability) in the voltage dependence of the electron current through the DBRTS was observed for the first time by Goldman *et al.*[10–12] In these papers a simple theoretical explanation of the effect was given. Independently and simultaneously, Davydov and Ermakov[13] predicted the existence of bistability in resonant tunneling and developed a consistent theory of the phenomenon, relating it as well to previous discussions of similar phenomena in biological systems and in molecular electronics.[19,22–25] While their pioneering work is well known in these latter contexts, it is unfortunately practically unknown to experts in resonant tunneling in semiconductor microstructures.

Because the hysteresis and bistability are important nonlinear characteristics of the DBRTS, we shall mention some of their features in more detail. One of the main peculiarities of hysteresis and bistability in the I-V characteristic of the DBRTS is its independence of the external circuit in the system; indeed, for this reason the term "intrinsic bistability" was suggested in Ref. 10. Despite the discussion in Ref. 14 of the role of the external circuit in the appearance of a hysteresis, in the experiment of Goldman *et al.*,[10–12] the principal cause of intrinsic bistability in DBRTS is quite obvious now. Besides, this effect was also observed in the computer simulations.[15] Recently, several experimental and theoretical papers studying intrinsic bistability in a system of "three barriers plus two wells" have appeared (see Ref. 26 and references therein). Control of the carrier dynamics in a DBRTS, or in the three-barrier system, can be achieved using optical techniques based on photoluminescence.[26–28]

When the external bias applied to a DBRTS is larger than some critical value, nonlinear oscillations of the output current can appear even for stationary input current. Such self-oscillations were investigated in theoretical papers,[16,19] and were observed independently in computer simulations.[17] A possible connection between these self-oscillations and the well-known modulation instability of the nonlinear Schrödinger equation was discussed by Malomed and Azbel.[18]

In their computer experiment,[20] Jona-Lasino *et al.* considered the periodic transmission of a wave packet through a potential involving a DBRTS with high transparent barriers and with two infinite walls bounding the system on the both sides of the DBRTS. The system was thus closed and Hamiltonian, and one could study the "dynamics" by studying the energy level spectrum. The results of Ref. 20 established that incorporating the effects of charge accumulation within the DBRTS (using the Hartree approximation) led to chaotic behavior for the envelope of the wave packet for energies exceeding a critical value. In Ref. 21, the authors investigated numerically a realistic model of a doped quantum well heterostructure. Nonlinearity is introduced in the model through the effective potential due to the density of electrons in the well. Dephasing and energy relaxation are introduced through a density matrix approach. The resultant dynamics shows the periodic-doubling routes to chaos, in the form of strange attractor. It was mentioned by the authors of Ref. 21 that all materials parameters used are well within the present capabilities of present quantum well technology, and the driving field amplitudes and frequencies are easily obtainable with a free electron laser.

In the present paper, we consider a related but distinct time-dependent problem: the transition to dynamical chaos when a time-modulated incoming electron current passes through a DBRTS. In contrast to the studies of Refs. 20 and 21, we consider an open system, so that the electrons ''pass through'' the DBRTS only once. Thus, our consideration complements the closed system theory developed in Ref. 21. Employing the approach of Ref. 13, we show that the dynamical charge accumulation in the potential well leads to a self-consistent electric field that modifies the resonant tunneling. As a result, an effective shift of the resonance energy appears, which depends on the amplitude of the current transmitted through the DBRTS. Under certain conditions, the resulting nonlinearity, together with the time-periodic modulation of the incoming electron current, leads to *dissipative* (because the system is open) chaotic dynamics involving a strange attractor. When the width of a resonance is small compared to the value of the resonant energy, and also if the time-periodic modulation of the incoming electron flow consists of a sequence of short pulses, then the strange attractor that appears in this system is the well-known Ikeda attractor.[29,30] We explore the prospects for the experimental observation of this dynamical chaos and estimate that the conditions for chaos are close to the conditions for the realization of intrinsic bistability in the DBRTS.[10–12]

The paper is organized as follows. In Sec. II, we present the main equations describing the nonlinear dynamics of the DBRTS when charge accumulation effects are included. In Sec. III, we study the conditions for the transition to chaos. In Sec. IV, we discuss the necessary conditions for the experimental observation of chaos in the DBRTS. To avoid excessive technical detail in the main text while maintaining completeness, we present the derivations of some formulas in two short Appendixes.

II. BASIC EQUATIONS

Consider a current of electrons moving through a one-dimensional (1D) system consisting of two identical potential barriers of height V_0 and width a separated by a distance b. We shall refer to the region between the barriers as the ''potential well'' and take as our spatial origin the left edge of the left barrier. The potential can then be written in the following form:

$$V(x)=\begin{cases} 0 & \text{if } x<0, \quad a<x<a+b, \quad x>2a+b \\ V_0 & \text{if } 0<x<a, \quad a+b<x<2a+b. \end{cases} \tag{1}$$

We assume that an electron current with a slowly time-modulated intensity is injected from the left. The characteristic modulation frequency is much smaller than $E_r/\hbar$, where E_r is the energy of the resonant tunneling. We wish to study the time dependence of the outgoing current, accounting for charge accumulation within the potential well and the consequent formation of an electrostatic feedback mechanism.

We use the approach of Davydov and Ermakov,[13,19] which was previously developed in Ref. 13 for the investigation of transient processes in the intrinsic bistability of the DBRTS. In this section, we present the main results of Ref. 13 in the form that can be applied for potential barriers with an arbitrary transparency. This generalization allows us to investigate several physically interesting cases, and to determine the conditions of validity of the approach.

We shall treat the electron-electron interaction within the Hartree approximation, so that the many-body electron wave function can be represented as a product of single electron wave functions.[17] For the single electron wave functions, in the regions outside the DBRTS, we assume the form,

$$\psi_{in}(x,t)=[D_0(t)e^{ikx}+R(t)e^{-ikx}]e^{-i\omega_0 t}, \tag{2a}$$

$$\psi_{out}(x,t)=D(t)e^{i(kx-\omega_0 t)}, \tag{2b}$$

where E is the incoming electron energy, $k=(2m^*E/\hbar^2)^{1/2}$, and $\omega_0=E/\hbar=\hbar k^2/2m^*$, with m^* being the effective electron mass. We assume that the amplitudes of the incoming $D_0(t)$, the reflected $R(t)$, and the outgoing $D(t)$ waves are all slowly varying in time, so that an adiabatic approximation is valid. Within the Hartree approximation, the Coulomb interaction between the incoming electron wave and electrons accumulated in the region of the potential well ($a<x<a+b$) will simply add an additional potential term to the single particle Schrödinger equation. This additional potential term, V_c, in the homogeneous approximation which is considered below, is given by,

$$V_c=-e\varphi, \tag{3a}$$

where φ is the self-consistent electrostatic potential generated by all electrons in the potential well and e is the electron's charge. The total electric charge accumulated in the well is $Q=-Ne\rho$, where N is the number of electrons in the well, and ρ is defined in terms of the electron wave function ψ as

$$\rho(t)=\frac{1}{b}\int_a^{a+b}|\psi(x,t)|^2dx. \tag{3b}$$

Using Gauss's law, one can find the electric field $\mathcal{E}$ produced by the charge Q,

$$\mathcal{E}=-\frac{4\pi}{\epsilon}\frac{Q}{S}, \tag{3c}$$

where ϵ is the dielectric constant of inter-barrier medium and S is the area of the surface that is crossed by the electric flux. Using the homogeneous approximation for the field $\varphi=-\mathcal{E}b$, we find the final expression for V_c to be

$$V_c(t)=\sigma\rho(t), \tag{4a}$$

with ρ given by Eq. (3b) and

$$\sigma=\frac{4\pi}{\epsilon}e^2n_sb, \tag{4b}$$

where n_s in the electron density per unit area and all other quantities have been defined above. The coefficient σ in Eq. (4b) can be expressed through the area capacity C_s of the DBRTS as $\sigma = e^2 n_s / C_s$. This form of V_c was also used in the numerical experiment.[17] In what follows we shall consider only the case $V_c \ll E$. The potential in Eq. (4a) is a nonlinear and nonlocal functional of $\psi(x,t)$ and is a source of nonlinear effects, including intrinsic bistability and self-oscillations.[13,16,17]

We now consider the behavior of the wave function in the DBRTS. Using an adiabatic approximation, one can derive the wave functions inside the barriers by solving the stationary Schrödinger equation,[13]

$$\psi_{bj}(x,t) = [\alpha_j(t) e^{\gamma x} + \beta_j(t) e^{-\gamma x}] e^{-i\omega_0 t} \quad (j=1,2), \tag{5a}$$

where

$$\gamma = [2m^*(V_0 - E)/\hbar^2]^{1/2}. \tag{5b}$$

Here the complex amplitudes α_j and β_j are slowly varying functions of t.

Because the DBRTS itself has zero potential in the interbarrier region, the total potential energy inside the well is $V_c(t)$. Since the wave function depends (slowly) on time, $V_c(t)$ is also a slowly varying function of time. Thus, the wave function inside the potential well must be found as a solution of the nonstationary Schrödinger equation.[13] The problem of finding of the solution of the Schrödinger equation (either stationary or nonstationary) for nonlinear and nonlocal potential in Eq. (4a) is rather complicated. However, if the electrostatic interaction of electrons is weak enough ($V_c \ll E$) and $V_c(t)$ varies in time slowly, the wave function inside the potential well can be represented in the form,[13]

$$\psi_w(x,t) = [A(t) e^{iqx} + B(t) e^{-iqx}] e^{-i\omega_0 t}, \tag{6}$$

where the wave vector q will be determined below, and the complex amplitudes A and B are slowly varying functions of time, with characteristic frequency $\omega \ll \omega_0 = E/\hbar$. We will show below that ω is the maximum of Ω and ν, where Ω is the characteristic frequency of time-modulated incoming electron flow, and ν is the inverse width of the resonant level. Substituting Eq. (6) in the nonstationary Schrödinger equation with the potential $V_c(t)$, we find for the wave vector q,

$$q \approx k - \Delta k, \quad \Delta k = \Delta k_c + \Delta k_t \quad (\Delta k/k \ll 1), \tag{7a}$$

$$\Delta k_c = \frac{m^*}{k\hbar^2} V_c, \tag{7b}$$

$$\Delta k_t = -i \frac{m^*}{k\hbar} (\dot{A}A^{-1} + \dot{B}B^{-1}), \tag{7c}$$

$$V_c = \sigma \left\{ |A|^2 + |B|^2 + \left[AB^* \frac{e^{2iqa}}{2iqb} (e^{2iqb} - 1) + \text{c.c.} \right] \right\}. \tag{8}$$

The term Δk in Eq. (7a) takes into account both the influence of the nonlinearity and the dependence on time. The term Δk_c arises from the Coulomb interaction of electrons, and the term Δk_t arises from the slow time modulation of the wave function due to the dependence of V_c on t.

By direct substitution, one can show that the wave function (6) is an approximate solution of the nonstationary Schrödinger equation, valid up to the terms of the order of ω/ω_0 for $kb \simeq 1$, which corresponds to the conditions of the resonant tunneling.

One should notice that the final expression for the Coulomb energy V_c in Eq. (8) includes the wave vector q, which is a consequence of the nonlocal structure of our self-consistent approach to the Coulomb interaction. Thus, Eqs. (7) and (8) define the wave vector q only implicitly. However, as we will later show, in the most interesting case of near resonant tunneling, one can find an approximate explicit solution for q.

The correction Δk in Eq. (7a) can be expressed through the amplitude of the outgoing wave D, for $x > 2a+b$. To this end, we must find relations between the amplitudes A, B, and D. Matching the wave functions and their derivatives at the points $x = 2a+b$ and $a+b$, we obtain,

$$A = \frac{1}{2} D \left[\left(\frac{k+q}{q} \right) \cosh \gamma a - i \left(\frac{k}{\gamma} - \frac{\gamma}{q} \right) \sinh \gamma a \right] \times \exp[ik(2a+b) - iq(a+b)], \tag{9}$$

$$B = -\frac{1}{2} D \left[\left(\frac{k-q}{q} \right) \cosh \gamma a + i \left(\frac{k}{\gamma} + \frac{\gamma}{q} \right) \sinh \gamma a \right] \times \exp[ik(2a+b) + iq(a+b)]. \tag{10}$$

Substituting Eqs. (9) and (10) into the expressions for Δk_c in Eqs. (7b) and (8), we have

$$\Delta k_c = \kappa |D|^2, \tag{11}$$

$$\kappa = \kappa_0 \left\{ \cosh^2 \gamma a + \frac{1}{2} \left(\frac{k^2}{\gamma^2} + \frac{\gamma^2}{k^2} \right) \sinh^2 \gamma a + \left(\frac{k}{\gamma} + \frac{\gamma}{k} \right) \frac{1}{2kb} (1 - \cos 2kb) \cosh \gamma a \sinh \gamma a - \frac{1}{2} \left(\frac{k^2}{\gamma^2} - \frac{\gamma^2}{k^2} \right) \frac{\sin 2kb}{2kb} \sinh^2 \gamma a + O(\Delta k/k) \right\} \quad \left(\kappa_0 \equiv \frac{m^* \sigma}{k\hbar^2} \right),$$

where the constant σ is defined in Eq. (4b). The logarithmic time derivatives $\dot{A}A^{-1}$ and $\dot{B}B^{-1}$ in the expression for Δk_t in Eq. (7c) can be expressed in terms of the slowly time-dependent variable $\dot{D}D^{-1}$. These relations can be derived in general from Eqs. (9) and (10) (see Appendix A). The simplest form of these relations occurs in two interesting cases:

I. The case of high and wide barriers,

$$\gamma/k \gg 1, \quad \gamma a \gg 1, \tag{12}$$

corresponding to the barriers of *low* transparency (i.e., low transmission).

II. The case of high and relatively narrow barriers

$$\gamma/k \gg 1, \quad \gamma a \ll 1, \quad \gamma^2 a/k \gg 1.$$

This case corresponds to the ‘‘moderate transparency’’of the barriers. The corresponding inequalities can be written in the form,

$$1 \ll (\gamma a)^{-1} \ll \gamma/k. \tag{13}$$

These two cases will be considered below, and we shall refer to them as I and II.

In cases I and II, we have (see Appendix A),

$$\frac{\dot{A}}{A} \approx \frac{\dot{B}}{B} \approx \frac{\dot{D}}{D} + O(\Delta k \omega (a+b)), \tag{14}$$

where ω is the characteristic frequency of variation of the functions A, B, and D_0. Because we deal with a DBRTS whose characteristic size is of the order of the de Broglie wavelength, $k(a+b) \sim 1$, and, up to terms of order $\Delta k/k$, we have $\dot{A}A^{-1} \approx \dot{B}B^{-1} \approx \dot{D}D^{-1}$. Thus, the expression for Δk_t in Eq. (7c) takes the form,

$$\Delta k_t = -i \frac{2m^*}{k\hbar} \frac{\dot{D}}{D}. \tag{15}$$

The expressions (11) and (15) define the correction Δk in (7) depending on the complex amplitude D of the wave function of outgoing electrons. Hence, we next analyze how this correction Δk influences the process of resonant tunneling. Matching the wave functions and their spatial derivatives at the points $x=0$ and $x=a$, and taking into account Eqs. (9) and (10), one can derive the exact dependence between the complex incoming amplitude D_0 and the outgoing amplitude D,

$$\begin{aligned} D_0 = \frac{1}{4} D e^{ik(2a+b)} \Bigg\{ & \Bigg[\left(\frac{k+q}{k} \right) \cosh\gamma a - i \left(\frac{q}{\gamma} - \frac{\gamma}{k} \right) \sinh\gamma a \Bigg] \\ & \times \Bigg[\left(\frac{k+q}{q} \right) \cosh\gamma a - i \left(\frac{k}{\gamma} - \frac{\gamma}{q} \right) \sinh\gamma a \Bigg] e^{-iqb} \\ & - \Bigg[\left(\frac{k-q}{k} \right) \cosh\gamma a + i \left(\frac{q}{\gamma} + \frac{\gamma}{k} \right) \sinh\gamma a \Bigg] \\ & \times \Bigg[\left(\frac{k-q}{q} \right) \cosh\gamma a + i \left(\frac{k}{\gamma} + \frac{\gamma}{q} \right) \sinh\gamma a \Bigg] e^{iqb} \Bigg\}. \end{aligned} \tag{16}$$

The expression (16) has a complicated structure, so to interpret it, let us first consider the case of tunneling through the DBRTS without taking into account the Coulomb interaction $(\Delta k \equiv 0)$. Setting $q \to k$ in Eq. (16), we have the standard expression,[13,19,25]

$$D = \frac{D_0 \exp[-2ik(a+b)]}{(2G^2-1) - 2iSG}, \tag{17}$$

where

$$G(k) = \cosh\gamma a \cos kb - \frac{k^2-\gamma^2}{2k\gamma} \sinh\gamma a \sin kb, \tag{18}$$

$$S(k) = \cosh\gamma a \sin kb + \frac{k^2-\gamma^2}{2k\gamma} \sinh\gamma a \cos kb. \tag{19}$$

The conditions for the resonant tunneling $|D/D_0|^2 = 1$ is satisfied for the wave vector k_r, which is determined from the equation,

$$G(k_r) = 0. \tag{20}$$

Equation (20) is equivalent to

$$\cot k_r b = \frac{k_r^2 - \gamma^2}{2k_r\gamma} \tanh\gamma a. \tag{21}$$

In the limiting case I, the condition of the resonant tunneling in Eq. (21) takes a simpler form,

$$k_r b \approx \pi n - 2k_r/\gamma, \tag{22}$$

where n is an integer. In the case II, the condition for the resonant tunneling also has the form of Eq. (22) with the substitution: $\gamma \to \gamma^2 a$.

We now consider the modification of the resonant tunneling conditions when the Coulomb interaction is taken into account. Using Eq. (16), one can show (see Appendix B) that to first order in $\Delta k/k$ at $kb \sim 1$, D and D_0 are related by:

$$D = \frac{D_0 \exp[-2ik(a+b)]}{-1 + 2G(G-iS) + (\Delta k/k)[ikb + 2kbS(G-iS) + (G-iS)Q \sin kb]}, \tag{23}$$

where

$$Q(k) = \left(\frac{k}{\gamma} + \frac{\gamma}{k} \right) \sinh\gamma a, \tag{24}$$

and the expressions for $G(k)$ and $S(k)$ are given by Eqs. (18) and (19).

Consider wave vectors in the vicinity of the resonance, k_r (20), and introduce a small detuning parameter away from the resonance $\xi = k - k_r$, $|\xi|/k_r \ll 1$. Expanding $G(k)$ and $S(k)$ in Eqs. (18) and (19) in a power series up to the terms of first order in ξ, we have,

$$\begin{gathered} G(k_r + \xi) \approx -\xi b S(k_r), \\ S(k_r + \xi) \approx S(k_r). \end{gathered} \tag{25}$$

Using Eq. (25), we can expand Eq. (23), which to first order in ξ, takes the form,

$$D=D_0\exp[-2i(k_r+\xi)(a+b)]\times\left\{-1+iL\left[\xi-\Delta k_r\left(1-\frac{b}{L}+\frac{(l/L)^{1/2}}{k_rb}\sin k_rb\right)\right]\right\}^{-1}, \tag{26}$$

where

$$L=2bS^2(k_r),\quad l=Q^2(k_r)b/2, \tag{27}$$

and the expression for Δk_r is given by Eqs. (7), (11), and (15) with the substitution $q\to k_r$. In deriving Eqs. (26) and (27), we have neglected small terms proportional to $\xi\Delta k$ and $\Delta k/k$.

The parameter L in Eq. (27) has a simple physical meaning. It defines the half-width of the resonance in k space when the Coulomb interaction is neglected. The half-width $\delta E_{1/2}$ of the resonant level in the well is $\delta E_{1/2}=2\hbar\nu$, where,

$$\nu=\frac{\hbar k_r}{2m^*L}. \tag{28}$$

Indeed, using the standard definition (see, e.g., Ref. 31), and using Eq. (26), the half-width $\xi_{1/2}$ of the resonant level in the k space can be found from the equation: $|D/D_0|^2=[1+L^2\xi^2]^{-1}=\frac{1}{2}$.

The solution of this equation is: $\xi_{1/2}=L^{-1}$, and $\delta E_{1/2}\equiv E(k_r+\xi_{1/2})-E(k_r)\approx(\hbar^2k_r/m^*)\xi_{1/2}=\hbar^2k_r/m^*L$.

Notice that despite the small value of Δk_r in Eq. (26), the value of L in Eq. (27) can be rather large, and their product can be of order $\Delta k_rL\sim1$. In particular, for the limiting cases I and II, we have the following expressions for L,

$$L\approx\frac{\gamma^2b}{8k_r^2}\exp(2\gamma a)\quad(\text{case I}), \tag{29}$$

$$L\approx\frac{\gamma^4ba^2}{2k_r^2}\quad(\text{case II}). \tag{30}$$

Using the variable ν defined by Eq. (28), the correction Δk_t containing the time-derivatives can be represented in a simple form

$$\Delta k_t=-\frac{i}{\nu L}\frac{\dot{D}}{D}. \tag{31}$$

The expression for the correction Δk_c is given by Eq. (11) using the substitution $k\to k_r$. For cases I and II, the expression for κ in Eq. (11) can be simplified:

$$\kappa\approx\kappa_0L/b. \tag{32}$$

Using Eqs. (11), (26), and (31), we can derive the following differential equation for the complex amplitude D of the transmitted wave,

$$\frac{dD}{d\tau}=-D+iL\xi D-iL\overline{\kappa}|D|^2D+D_0(\tau)F_0(k_r,\xi), \tag{33}$$

where

$$\overline{\kappa}=\kappa U(k_r),\quad \tau=\overline{\nu}t,\quad \overline{\nu}=\nu/U(k_r),$$

$$U(k_r)=1+\frac{(l/L)^{1/2}}{k_rb}\sin(k_rb), \tag{34}$$

$$F_0(k_r,\xi)=-\exp[-2i(k_r+\xi)(a+b)].$$

For cases I and II, we have $(l/L)^{1/2}\approx(-1)^{n+1}$, $U(k_r)\to1$, $\overline{\nu}\to\nu$, and $\overline{\kappa}\to\kappa$. Thus, the characteristic time scale for the problem is the ''quarter-width'' of the resonance level ν in Eq. (28). Notice, that as shown in the experiment described in Ref. 27, the value ν^{-1} is the characteristic time scale that determines the dynamics of charge accumulation in the DBRTS.

The Eq. (33) was first derived in Ref. 13 in considerations of the intrinsic bistability in the DBRTS. In what follows, we shall consider the case in which the incoming wave $D_0(\tau)$ is slowly modulated in time in a periodic fashion. In preparation for this analysis, we shall first consider the small parameters of the problem and the relations among them.

The small parameters discussed thus far in our approach are $\Delta k/k_r\ll1$ (for the wave number), $\nu/\omega_0\ll1$ (for the frequency). To relate these parameters, we use the following estimates:

$$\Delta k_c\simeq\frac{m^*V_c}{k_r\hbar^2}=\frac{V_c}{2\hbar\nu L}, \tag{35}$$

so that

$$\frac{\Delta k_c}{k_r}\simeq\frac{\kappa}{k_r}=\left(\frac{V_c}{\hbar\nu}\right)\frac{1}{2k_rL}. \tag{36}$$

If $V_c\sim\hbar\nu$, then to satisfy the condition $\Delta k_r/k_r\ll1$, we require $k_rL\gg1$. This implies that the resonance is narrow. From Eq. (33), the dimensionless parameter of nonlinearity is $\overline{\kappa}L$. From Eq. (35) we have,

$$\overline{\kappa}L\sim\Delta k_cL=\left(\frac{V_c}{2\hbar\nu}\right)=\left(\frac{V_c}{2E}\right)\left(\frac{\hbar\nu}{E}\right)^{-1}. \tag{37}$$

As follows from Eq. (37), the regime of strong nonlinearity ($\overline{\kappa}L\sim1$) is $V_c\sim\hbar\nu$. This condition has simple physical interpretation — the charging energy is of the same order as a characteristic width of the resonant level [see Eq. (28)]. Note that we can achieve this regime even if both V_c and ν are small—$V_c/E\ll1$ and $\hbar\nu/E\ll1$—since it is their ratio that determines the strength of the nonlinearity and by Eq. (37) this ratio can be of the order one.

III. CHAOTIC DYNAMICS

We assume that the amplitude of the incoming electron current, $D_0(t)$, consists of a sequence of pulses periodic in time, with a pulse duration T_0, an amplitude A_0, and a time interval between the pulses T. Then, for $\overline{\nu}T_0\ll1$ we have,

$$D_0(t)=\frac{A_0T_0}{T}\sum_{n=-\infty}^{\infty}\exp\left(\frac{2\pi n}{T}\right)=A_0\Theta_0\sum_{n=-\infty}^{\infty}\delta(\tau-n\tilde{T}), \tag{38}$$

where $\Theta_0=\overline{\nu}T_0$ and $\tilde{T}=\overline{\nu}T$. We introduce the dimensionless variables,

$$z=(\overline{\kappa L})^{1/2}D \quad \varepsilon=A_0\Theta_0(\overline{\kappa L})^{1/2}F_0(k_r,\xi). \tag{39}$$

Using these variables, Eq. (33) can be written,

$$\frac{dz}{d\tau}=-z+iL\xi z-i|z|^2z+\varepsilon\sum_{n=-\infty}^{\infty}\delta(\tau-n\tilde{T}). \tag{40}$$

The differential Eq. (40) can be reduced to a discrete map,

$$z_{n+1}=(z_n+\varepsilon)p\ \exp[-i\tilde{T}(\lambda|z_n+\varepsilon|^2-L\xi)], \tag{41}$$

where

$$z_n\equiv z(n\tilde{T}-0), \quad p=e^{-\tilde{T}}, \quad \lambda=\frac{1-\exp(-2\tilde{T})}{2\tilde{T}}. \tag{42}$$

The discrete map in Eq. (41) is equivalent to the Ikeda map,[29,30] which is well known from studies of dynamical chaos and bistabilities in optical systems, and for which strange attractors dominate the dynamics for a large range of parameters. As introduced in Refs. 29 and 30, the Ikeda map is

$$Y_{n+1}=A+BY_n\exp[i(|Y_n|^2-\delta_0)]. \tag{43}$$

Within the following substitutions

$$A=(\lambda\tilde{T})^{1/2}\varepsilon=\left[\frac{1-\exp(-2\tilde{T})}{2}\right]^{1/2}\varepsilon,$$

$$B=p=\exp(-\tilde{T}), \quad \delta_0=L\xi\tilde{T}, \tag{44}$$

$$Y_n=(\lambda\tilde{T})^{1/2}(z_n^{\dagger}+\varepsilon),$$

we can establish the equivalence of the two maps given in Eqs. (41) and (43). Hence, the system of the DBRTS with periodically modulated incoming current and self-consistent electron-electron interactions should also exhibit regions of dissipative chaotic dynamics controlled by a strange attractor.

To estimate the region of parameters for chaotic behavior directly in terms of the ''natural'' variables of Eq. (41), we can apply a simple ''phase stretching'' method.[32,33] This criterion gives the rough conditions for the appearance of a local instability in a nonlinear system. Applying this criterion to the map in Eq. (41) determines the (approximate) condition for the transition to chaos to be,[34]

$$K=2|\varepsilon z_0|\tilde{T}\lambda\gtrsim 1. \tag{45}$$

Since $\lambda\tilde{T}=[1-\exp(-2\tilde{T})]/2\leqslant 1/2$, we have $K\sim|\varepsilon z_0|$. To estimate K, we use $|z_0|\sim(\overline{\kappa L})^{1/2}$, $\varepsilon=A_0\Theta_0F_0(\overline{\kappa L})^{1/2}\sim A_0\Theta_0(\overline{\kappa L})^{1/2}$ [see Eq. (39)]. Thus, $K\sim A_0\Theta_0\kappa L$, where $\theta_0=\overline{\nu}T_0$, and A_0 is the dimensionless amplitude of the modulation of the incoming electron current $(0\leqslant|A_0|\leqslant 1)$. It follows from Eq. (45) that the conditions for chaos are $\kappa L\gtrsim(A_0\Theta_0)^{-1}$.

In applications to the Ikeda attractor, the criterion in Eq. (45) is known to agree within an order of magnitude with the results of numerical experiments.[30,35–37] Indeed, in the variables in Eq. (44), the criterion in Eq. (45) take the form $K\approx 2A|Y_0|$. In the numerical calculations,[35–37] the strange attractor was observed, for example, at $B\approx 0.5$, $A\approx 1.56$, $|Y_0|\approx 1-5$. Thus, the rough condition for the appearance of the strange attractor in the slowly-modulated DBRTS is,

$$A_0\Theta_0\kappa L\gtrsim 1. \tag{46}$$

The criterion in Eq. (45) has a simple physical interpretation: namely, the variation of the phase $\Delta\varphi$ in (41) during one ''kick'' must be sufficiently large: $\Delta\varphi\sim 2|\varepsilon z_0|\tilde{T}\lambda\gtrsim 1$, where $z_n=|z_n|\exp(-i\varphi_n)$.

IV. DISCUSSION

Our analysis has shown that the resonant tunneling of electrons through a DBRTS can exhibit a regime of chaotic behavior when the Coulomb interaction is taken into account in the Hartree approximation and the incoming electron current is modulated with a frequency, $\Omega=2\pi/T_0$ [see Eq. (38)], of the order of the resonance level width, ν in Eq. (28). The rough criterion for the transition to chaos has the form in Eq. (46). As the amplitude of modulation satisfies $A_0\leqslant 1$, and $\theta_0\equiv\overline{\nu}T_0<1$, it then follows from Eq. (46) that the main condition for the transition to chaos is $\kappa L\gtrsim 1$, where κ is defined in Eq. (38) [see also Eq. (11)], and determines the shift of the resonant level due to the Coulomb interaction, and L^{-1} is the half-width of the resonant level in k-space.

It is important to stress that although our system is completely Hamiltonian, the dynamical chaos is effectively *dissipative* and is controlled by a strange attractor of the Ikeda type. Effective dissipation appears in this Hamiltonian system because it is open. In the process of quantum tunneling, electrons are partially transmitted through the DBRTS and partially reflected from the DBRTS but in either case are lost ''forever'' from the region of the DBRTS.

The slow modulation of the incoming electron current could be realized by the preliminary transformation of this flow using a modulator for which the potential barrier has slowly varying height.[38,39]

We now examine estimates of the characteristic values of parameters necessary to observe a transition to chaos in the experiment. For simplicity, consider case I: $a\gamma\gg 1$ and $\gamma/k_r\gg 1$. Then $k_rb\approx\pi n$ (n is an integer). The method used above requires the following conditions to be satisfied: $\Delta k/k\sim\kappa/k_r\ll 1$. Let us fix the value $\mu\equiv\kappa/k_r\ll 1$. Using Eq. (32) we obtain,

$$\mu\approx\frac{\kappa_0L}{bk_r}\approx\frac{\kappa_0L}{n\pi}, \tag{47}$$

where κ_0 is defined in Eq. (11). Using Eqs. (32) and (47), one can present the dimensionless parameter of nonlinearity κL through the parameter μ,

$$\kappa L\approx(\kappa_0L)\frac{L}{b}=\frac{(\pi\mu n)^2}{\kappa_0b}. \tag{48}$$

Using the explicit expression for κ_0 in Eq. (11), we have the following expression for κ_0b,

$$\kappa_0 b=\frac{m^*e^2b^3n_s}{\pi n\hbar^2}\frac{4\pi}{\epsilon}. \tag{49}$$

Substituting Eq. (49) in Eq. (48), we have for the parameter of nonlinearity,

$$\kappa L=\frac{\mu^2}{n_s}\left(\frac{\pi n}{b}\right)^3\frac{\hbar^2}{m^*e^2}\frac{\epsilon}{4\pi}. \tag{50}$$

If the dielectric constant of the heterostructure is $\epsilon\approx 13$, then for the first resonant level $(n=1)$ and for $b\lesssim 10^{-7}$ cm and $n_s\lesssim 10^{10}$ cm^{-2},[18] we find $\kappa Lb\lesssim 1$, even for $\mu\leqslant 10^{-2}$. Thus, for the nanostructures with typical area electron densities,[27] the necessary condition for the transition to chaos: $\kappa L\gtrsim 1$ is quite reasonable. These conditions can be achieved in the framework of the perturbation theory described in this paper $(\mu\ll 1)$. The characteristic frequency of modulation of the incoming electron current can be of the order $\Omega\sim 10$ GHz.

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APPENDIX A: THE RELATION BETWEEN $\dot{A}A^{-1}$ AND $\dot{D}D^{-1}$

In this appendix we derive the connection between $\dot{A}A^{-1}$ and $\dot{D}D^{-1}$. First, we introduce $f(k,q)\equiv B/D$, where the explicit form of $f(k,q)$ follows from Eq. (10). From the definition of f, we have the simple relation among the logarithmic derivatives

$$\dot{B}B^{-1}=\dot{D}D^{-1}+\dot{f}f^{-1}. \tag{A1}$$

Now we find the expression for $\dot{f}f^{-1}$. Using the expansions

$$q\approx k-\Delta k,\quad q^{-1}\approx k^{-1}+k^{-2}\Delta k, \tag{A2}$$

which are valid for $\Delta k/k\ll 1$, we obtain the expression for $f(k,q)$,

$$f(k,q)\approx f_0+\Delta k f_1,$$

$$f_0=-\frac{i}{2}\left(\frac{k}{\gamma}+\frac{\gamma}{k}\right)\sinh\gamma a\times\exp[ik(3a+2b)-i\Delta k(a+b)], \tag{A3}$$

$$f_1=-\frac{1}{2}\left(\frac{1}{k}\cosh\gamma a+i\frac{\gamma}{k^2}\sinh\gamma a\right)\times\exp[ik(3a+2b)-i\Delta k(a+b)].$$

Differentiating Eq. (A3) we get,

$$\dot{f}=\dot{f}_0+\dot{f}_1\Delta k+\Delta\dot{k}f_1,$$

$$\dot{f}_0=-i\Delta\dot{k}(a+b)f_0, \tag{A4}$$

$$\dot{f}_1=-i\Delta\dot{k}(a+b)f_1.$$

Using Eqs. (A3) and (A4) we have

$$\frac{\dot{f}}{f}=-i\Delta\dot{k}(a+b)+\frac{\Delta\dot{k}f_1}{f}\approx -i\Delta\dot{k}(a+b)+\Delta\dot{k}\frac{f_1}{f_0}. \tag{A5}$$

Let the characteristic frequency of variation of A and B be ω. Then, $\Delta\dot{k}\sim\omega\Delta k$, and it follows from Eqs. (A1) and (A5) that,

$$\frac{1}{\omega}\frac{\dot{B}}{B}=\frac{1}{\omega}\frac{\dot{D}}{D}+O(-i\Delta k(a+b)+\Delta k(f_1/f_0)). \tag{A6}$$

The expression for f_1/f_0 can be found using the definitions for f_1 and f_0 in Eq. (A3),

$$k\left|\frac{f_1}{f_0}\right|=\frac{\gamma k}{k^2+\gamma^2}\left[\coth^2\gamma a+\frac{\gamma^2}{k^2}\right]^{1/2} =\begin{cases}1, & \text{if }\ \gamma/k\gg 1 \quad\text{and}\quad \gamma a\gg 1\\ 1, & \text{if }\ \gamma/k\gg 1,\quad \gamma a\ll 1,\quad\text{but}\quad \gamma/k\gg 1/(\gamma a).\end{cases} \tag{A7}$$

Combining Eqs. (A6) and (A7), we derive Eq. (14). The same procedure can be used for deriving the relation between $\dot{A}A^{-1}$ and $\dot{D}D^{-1}$.

APPENDIX B: AN APPROXIMATE RELATION BETWEEN D_0 AND D

In this appendix we derive the approximate relation between D_0 and D valid to first order in $\Delta k/k$ and for $kb\sim 1$. We begin from the exact dependence of D_0 on D [Eq. (16) in the text] in the form

$$D_0=De^{2ik(a+b)}[M(k,q)e^{-2ikb+i\Delta kb}-N(k,q)e^{-i\Delta kb}], \tag{B1}$$

where

$$M(k,q)=\left[\left(\frac{k+q}{2k}\right)\cosh\gamma a-\frac{i}{2}\left(\frac{q}{\gamma}-\frac{\gamma}{k}\right)\sinh\gamma a\right]\times\left[\left(\frac{k+q}{2q}\right)\cosh\gamma a-\frac{i}{2}\left(\frac{k}{\gamma}-\frac{\gamma}{q}\right)\sinh\gamma a\right], \tag{B2}$$

$$N(k,q)=\left[\left(\frac{k-q}{2k}\right)\cosh\gamma a+\frac{i}{2}\left(\frac{q}{\gamma}+\frac{\gamma}{k}\right)\sinh\gamma a\right]\times\left[\left(\frac{k-q}{2q}\right)\cosh\gamma a+\frac{i}{2}\left(\frac{k}{\gamma}+\frac{\gamma}{q}\right)\sinh\gamma a\right]. \tag{B3}$$

When $\Delta k/k\ll 1$ and $kb\sim 1$, we have $\Delta kb\ll 1$. This allows us to expand the exponents in Eq. (B1): $\exp(\pm i\Delta kb)\approx 1\pm i\Delta kb$, and then, using Eq. (A2), to expand $M(k,q)$ in Eq.

(B2) and $N(k,q)$ in Eq. (B3) in a power series of $\Delta k/k$. As a result, we derive from Eq. (B1),

$$D_0 \approx De^{2ik(a+b)}\sum_{l=0}^{3}\left\{[M_l e^{-2ikb}-N_l]\left(\frac{\Delta k}{k}\right)^l + i[M_l e^{-2ikb}+N_l]\left(\frac{\Delta k}{k}\right)^{l+1}(kb)\right\}, \tag{B4}$$

where

$$M_0=\left[\cosh\gamma a - i\left(\frac{k^2-\gamma^2}{2\gamma k}\right)\sinh\gamma a\right]^2,$$

$$N_0=-\left(\frac{k^2+\gamma^2}{2\gamma k}\right)^2\sinh^2\gamma a,$$

$$M_1=\left[i\cosh\gamma a+\left(\frac{k^2-\gamma^2}{2\gamma k}\right)\sinh\gamma a\right]\times\left(\frac{k^2+\gamma^2}{2k\gamma}\right)\sinh\gamma a,$$

$$M_2=\frac{1}{4}\left[-\cosh^2\gamma a-\sinh^2\gamma a+2i\left(\frac{k^2-\gamma^2}{2k\gamma}\right)\cosh\gamma a\sinh\gamma a\right],$$

$$N_1=M_1, \quad N_2=-M_2. \tag{B5}$$

We consider the expansion (B4) only to the terms of zeroth and of first order in $\Delta k/k$. One can check that,

$$M_0e^{-2ikb}-N_0=2G^2-1-2iSG,$$

$$i\Delta kb[M_0e^{-2ikb}+N_0]=2\Delta kb(GS-iS^2)+i\Delta kb, \tag{B6}$$

$$M_1[e^{-2ikb}-1]=(G-iS)\left(\frac{k}{\gamma}+\frac{\gamma}{k}\right)\sinh\gamma a\sin kb,$$

where the expressions for G and S are defined in Eqs. (18) and (19), respectively. Thus, from the expansion (B4), which includes terms up to order $\sim\Delta k/k$, and using Eq. (B6), we derive Eq. (23).

To justify the neglect of the second and third order terms in $\Delta k/k$, it is necessary that $|M_2|,|N_2|\lesssim|M_1|,|N_1|$. In general, the expressions for $|M_2/M_1|$ and $|N_2/N_1|$ have complicated forms. Hence we present here only the expressions for the limiting cases I and II, which are sufficient for our present purposes,

$$\left|\frac{M_2}{M_1}\right|\sim\left|\frac{N_2}{N_1}\right|=\begin{cases}2k/\gamma\ll 1, & \text{case I}\\ k/(\gamma^2 a)\ll 1, & \text{case II.}\end{cases} \tag{B7}$$

It follows from Eq. (B7) that the procedure of neglecting of higher-order terms in Eq. (B4) is self-consistent.

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