Spontaneous dc Current Generation in a Resistively Shunted Semiconductor Superlattice Driven by a Terahertz Field

Kirill N. Alekseev,[1,2,3] Ethan H. Cannon,[1] Jonathan C. McKinney,[1] Feodor V. Kusmartsev,[2,4] and David K. Campbell[1]
[1]*Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801*
[2]*NORDITA, Bledgamsvej 17, DK-2100, Copenhagen 0, Denmark*
[3]*Theory of Nonlinear Processes Laboratory, Kirensky Institute of Physics, Krasnoyarsk 660036, Russia*
[4]*Department of Physics, Loughborough University, Loughborough LE11 3TU, United Kingdom*
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We study a resistively shunted semiconductor superlattice subject to a high-frequency electric field. Using a balance equation approach that incorporates the influence of the electric circuit, we determine numerically a range of amplitude and frequency of the ac field for which a dc bias and current are generated *spontaneously* and show that this region is likely accessible to current experiments. Our simulations reveal that the Bloch frequency corresponding to the spontaneous dc bias is approximately an integer multiple of the ac field frequency. [S0031-9007(98)05660-9]

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In 1928 Bloch demonstrated [1] that electrons in a periodic lattice potential with period a subject to a dc electric field $E_{\rm dc}$ undergo oscillations with characteristic frequency $\omega_B = eaE_{\rm dc}/\hbar$. In naturally occurring solids, observations of Bloch oscillations are precluded by the need for very high applied electric fields to reduce the Bloch oscillation period below the dephasing times arising from the ever-present scattering due to impurities and phonons. In 1970, Esaki and Tsu [2] recognized that due to their longer spatial periods, semiconductor superlattices (SSLs) can support Bloch oscillations in the terahertz (THz) domain—comparable to or faster than the corresponding scattering frequencies—even for modest field strengths ($\sim$1 kV/cm). Thus, not only should the Bloch oscillations be observable in SSLs, but SSLs could also serve as devices to produce THz radiation [2]. When, in addition to the static field $E_{\rm dc}$, an SSL is subjected to an ac field of frequency Ω, the static current-voltage characteristic can exhibit structures at $\omega_B = n\Omega$ (where n is an integer) [3,4]. In the presence of dissipation, multiphoton transitions involving different n can lead to a Bloch oscillator capable of producing radiation from almost zero to the THz frequencies. Recently, experiments involving novel electromagnetic radiation sources and coupling techniques have revealed additional effects of THz radiation fields on the dc conductivity of an SSL [5–8], including (i) THz multiphoton-assisted tunneling [5], (ii) ac-field-induced reduction of the dc current [6], (iii) *absolute* negative conductance [7], and (iv) resonant changes in conductivity [8].

In the present Letter, we examine a question that is roughly the converse of the Bloch oscillations in an SSL: namely, can a purely ac external field applied to an appropriately configured (but unbiased) SSL create a dc bias and corresponding dc current? In technical terms, this corresponds to a spontaneous breaking of spatial reflection symmetry. Using the methods of nonlinear dynamics, we show that the answer to this question is "yes" and find the condition for spontaneous dc current generation in the circuit consisting of the SSL shunted by an external resistance. We observe that the Bloch frequency corresponding to the spontaneously generated bias across the SSL is approximately an integer multiple of the frequency of the external ac field and explain the origin of this "phase locking."

Consider an SSL of spatial period a, length l, and area S that is homogeneously doped with N carriers per unit volume. The SSL is shunted by an external measuring device of resistance R and is exposed to an external ac electric field $E_{\rm ext}(t) = E_0 \cos\Omega t$. We use the standard tight-binding dispersion relation for electrons in a single miniband, so that $\varepsilon(p) = \Delta/2[1 - \cos(pa/\hbar)]$, with ε being the electron energy, p the quasimomentum along the SSL's axis, and Δ the miniband width. We describe the dynamics of electrons by the balance equations [4,9]

$$\begin{aligned} \dot{V} &= -eE_{\rm tot}(t)/m(\mathcal{E}) - \gamma_v V\,, \\ \dot{\mathcal{E}} &= -eE_{\rm tot}(t)V - \gamma_\varepsilon(\mathcal{E} - \mathcal{E}^{(0)})\,, \\ \dot{E}_{\rm sc} &= 4\pi eNV/\epsilon_0 - \alpha E_{\rm sc}\,, \end{aligned} \tag{1}$$

where $V(t)$ and $\mathcal{E}(t)$ are the electron velocity and energy averaged over the time-dependent distribution function satisfying the Boltzmann equation, and γ_v and γ_ε are phenomenological relaxation constants for the average velocity and energy, respectively. Elastic scattering by impurities, interface roughness, and structural disorder contribute to γ_v; inelastic phonon scattering dominates γ_ε. The parameter $\mathcal{E}^{(0)}$ describes the electron equilibrium energy [4], which is a function of the lattice temperature. Also appearing in (1) is the energy-dependent effective mass $m(\mathcal{E}) = m_0(1 - 2\mathcal{E}/\Delta)^{-1}$, where $m_0 \equiv \frac{2\hbar^2}{\Delta a^2}$. The first two balance equations, obtained in [4], have been well studied, while the third equation describing the self-consistent field—which is discussed below—has only recently been introduced [9].

The electric field acting on the electrons, $E_{\rm tot}(t) = E_{\rm sc}(t) + E_{\rm ext}(t)$, is the sum of the external ac field $E_{\rm ext}(t)$ and the self-consistent field $E_{\rm sc}(t)$, which incorporates the

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influence of the circuit and of the repulsive interaction with other electrons on a single electron's dynamics. The phenomenological constant α describes the relaxation of the self-consistent field [9]. The self-consistent field is related to the voltage U across the SSL by $E_{\rm sc} = U/l$. The current density through the SSL consists of two parts: the displacement current $j_{\rm disp} = \frac{\epsilon_0}{4\pi}\dot{E}_{\rm sc}$ (ϵ_0 is the average dielectric constant for the SSL) and the current of ballistic electrons $j = -eNV$, where V is the solution to Eqs. (1). Kirchoff's equation for the resistively shunted SSL, $[(\epsilon_0/4\pi)\dot{E}_{\rm sc} - eNV]S + (E_{\rm sc}l)/R = 0$, provides the circuit's contribution to the damping of the self-consistent field, $\alpha_{\rm cir} = (RC)^{-1}$, $C = \frac{\epsilon_0 S}{4\pi l}$; the total damping rate is the sum of the rates from the external circuit and internal mechanisms. The self-consistent field is responsible for the novel nonlinear effects of symmetry breaking and phase locking that we observe. Introducing the scaled variables $\mathcal{F}(t) = eaE_{\rm sc}(t)/\hbar$, $v = m_0 Va/\hbar$, and $w = (\mathcal{E} - \Delta/2)(\Delta/2)^{-1}$, so that the lower (upper) edge of the miniband corresponds to $w = -1$ ($w = +1$), we obtain the self-consistent set of equations

$$\begin{aligned} \dot{v} &= [\mathcal{F}(t) + \omega_s \cos\Omega t]w - \gamma_v v\,, \\ \dot{w} &= -[\mathcal{F}(t) + \omega_s \cos\Omega t]v - \gamma_\varepsilon(w - w^{(0)})\,, \\ \dot{\mathcal{F}} &= \omega_c^2 v - \alpha\mathcal{F}\,. \end{aligned} \tag{2}$$

Here $\omega_s = (eaE_0)/\hbar$ and $\omega_c = [(2\pi e^2 Na^2\Delta)/(\hbar^2\epsilon_0)]^{1/2}$, where ω_c is a characteristic frequency that can be interpreted as a miniband plasma frequency [9]. Throughout this paper we assume that initially the electrons in the SSL are at the bottom of miniband, $w(0) = -1$, with both current and voltage absent, $v(0) = \mathcal{F}(0) = 0$, and that the electrons relax to the bottom of the miniband, i.e., $w^{(0)} = -1$, where $w^{(0)} = (\mathcal{E}^{(0)} - \Delta/2)(\Delta/2)^{-1}$.

Since we are here interested primarily in stationary solutions of (2), we do not consider transient effects. The appearance of a dc current and a dc bias indicates the presence of the zeroth Fourier harmonic in the stationary solutions for $v(t)$ and $\mathcal{F}(t)$. Hence, we first establish how the existence of these zeroth harmonics relates to the symmetry properties of (2). Note that the appearance of these zeroth harmonics is accompanied by other (normally forbidden) even harmonics in the current, e.g., the second harmonic.

The following transformation leaves the system (2) invariant:

$$\begin{aligned} v \to -v, \qquad \mathcal{F} \to -\mathcal{F}, \qquad t \to t + T_{\rm sym}/2\,, \\ T_{\rm sym} \equiv (2n+1)T\,, \end{aligned} \tag{3}$$

where n is an integer and $T = 2\pi/\Omega$ is the period of the external ac field. Thus, for any solution of system (2) $\{v(t), w(t), \mathcal{F}(t)\}$, there exists a symmetry-related "conjugate" solution $\{-v(t + T_{\rm sym}/2), w(t + T_{\rm sym}/2), -\mathcal{F}(t + T_{\rm sym}/2)\}$. Physically, the transformation (3) demonstrates that both signs of electron velocity v and self-consistent field (voltage) $\mathcal{F}$ are equivalent in an SSL driven by an ac field without dc bias; in contrast, the lower and upper edges of the miniband are not equivalent; thus $w \not\to -w$.

When $\gamma_v = \gamma_\varepsilon = 0$, Eqs. (2) can be transformed [9] to the resistively shunted junction (RSJ) model familiar from the theory of Josephson junctions [10]. Since in this limiting case symmetry properties have been applied effectively to characterize and interpret solutions, we expect this also to be true in the present, more general case.

The form of the symmetry transformation suggests the use of a stroboscopic map of period $T/2$ to characterize the solutions of system (2): this amounts to considering v, w, $\mathcal{F}$ at discrete times $t_n = nT/2$, where n is an integer. We are especially interested in the projection of this stroboscopic phase portrait onto the v-$\mathcal{F}$ plane. Let $\phi(t)$ be one of the variables $v(t)$ or $\mathcal{F}(t)$ and denote by $\langle\phi\rangle$ the value of ϕ averaged over a large time interval. By analogy to the RSJ model [10,11], we have the following natural classification for attractors of (2) based on their symmetry properties.

(i) A *symmetric limit cycle* is a solution that obeys the equality $-\phi(t + T_{\rm sym}/2) = \phi(t)$, which implies that it is periodic with period $T_{\rm sym} = (2n+1)T$; as this period is $(2n+1)$ times the external driving period, this limit cycle is also known as a period $(2n+1)$ solution. The $T/2$-stroboscopic plot consists of $2(2n+1)$ points, and its projection on the v-$\mathcal{F}$ plane has inversion symmetry about the origin. More importantly, $\langle\phi\rangle = 0$, and therefore *dc current and bias are absent*. The simplest example of a symmetric limit cycle is a period-1 solution of the form $\phi(t) = A_\phi \cos\Omega t + B_\phi \sin\Omega t$, where A_ϕ and B_ϕ are constants.

(ii) For a *symmetry-broken limit cycle*, the $T/2$-stroboscopic phase portrait in the v-$\mathcal{F}$ plane lacks inversion symmetry about the origin. The simplest example is a period-1 solution of the form $\phi(t) = A_\phi \cos\Omega t + B_\phi \sin\Omega t + C_\phi/2$ with zero harmonic equal to $C_\phi/2$ (A_ϕ, B_ϕ, and C_ϕ are constants). This solution satisfies the equality $-\phi(t + T_{\rm sym}/2) = \phi(t) - C_\phi$. Actually, there are two symmetry-related conjugate limit cycles differing from each other only by the sign of the constant C_ϕ, and for fixed parameters, a trajectory selects one limit cycle depending on its initial conditions. For our case with fixed initial conditions, a trajectory can switch to a limit cycle with different symmetry only when the parameters are varied. This means that *for symmetry-broken limit cycles dc current and bias are nonzero* and their signs are fixed for fixed values of the system parameters.

(iii) For *symmetric chaos*, the solutions are aperiodic, and the $T/2$-stroboscopic phase portrait has a fractal structure with an infinite number of points but possesses inversion symmetry about the origin of coordinates in the v-$\mathcal{F}$ plane. Such solutions correspond to strange attractors with $\langle\phi\rangle = 0$, and *no dc current*.

(iv) For *asymmetric chaos*, the fractal pattern in the $T/2$ stroboscopic representation *lacks* this v-$\mathcal{F}$ inversion symmetry, and one has a symmetry-related conjugate pair

of strange attractors. For asymmetric chaos, $\langle \phi \rangle \neq 0$, and *dc current is spontaneously generated*.

We observe all four types of attractors in our model. The most unusual type of behavior—asymmetric chaos—is illustrated in Fig. 1, which shows a stroboscopic phase portrait of an attractor lacking inversion symmetry about the origin. The symmetry-related conjugate attractor exists but is *not* depicted in Fig. 1.

The above symmetry-based classification of attractors demonstrates that stationary solutions both with and without dc current can be realized in our model. To determine where in parameter space the different types of attractors exist, we have undertaken numerical simulation of system (2), holding the dissipation rates fixed but varying the external field strength and frequency. For modern SSLs, the velocity relaxation times range from tens of femtoseconds up to picoseconds. If we take $\gamma_v^{-1} = 0.35$ ps [8], then at $\Delta = 22$ meV, $a = 42$ nm, $N = 1.2 \times 10^{16}$ cm^{-3}, and $\epsilon_0 \approx 13$ (GaAs), we have $\gamma_v/\omega_c = 0.1$. As a rule, the relaxation time for energy is an order of magnitude longer than for velocity [12]; therefore we take $\gamma_\varepsilon/\omega_c = 0.01$. We choose $\alpha/\omega_c = 0.01$, which corresponds to a resistance of $R = 5.4$ kΩ and an SSL self-capacitance of $C = 0.64$ fF ($S = 10^{-7}$ cm^2, $l = 40 \times a = 1.68 \times 10^{-4}$ cm). For these parameters, the locations of broken-symmetry attractors are shown in the Ω–ω_s plane in Fig. 2. To distinguish genuine symmetry breaking from long transients and rounding errors, we take advantage of the observed phase locking and mark as symmetry broken those solutions having $|\langle \mathcal{F} \rangle| > \Omega/4$. There exists a small region of asymmetric chaos with $\langle v \rangle$ and $\langle \mathcal{F} \rangle \neq 0$. However, the majority of the symmetry-broken attractors are broken-symmetry limit cycles with higher even and odd harmonics as well as the zeroth harmonic [13]. We also observe some solutions with subharmonics, corresponding to period doubling.

The dependence of the spontaneously generated bias and dc current on the frequency of the external field is indicated in Fig. 3. Recall that $\langle \mathcal{F} \rangle$ is the *long-time average* of the *scaled* self-consistent field, i.e., it reflects the dc component of the spontaneously generated electric field in the SSL and is in fact equal to the "induced" Bloch frequency ($\omega_B \equiv eaE_{\rm dc}/\hbar = \langle \mathcal{F} \rangle$). We observe from Fig. 3 that $\langle \mathcal{F} \rangle$ is approximately equal to an integer multiple of Ω. This "mode locking" is the analog of the well-known zero-bias ac Josephson steps in a Josephson junction (compare in particular our Fig. 3 and Fig. 1 from [10]). But in contrast to the RSJ model, the mode locking in our case is only approximate—the exact value of $\langle \mathcal{F} \rangle$ depends weakly on the field strength, ω_s. We observe mode locking for periodic limit cycles as well as for asymmetric chaos. The efficiency of ac to dc field conversion, i.e., the ratio $\omega_B/\omega_s = |E_{\rm dc}/E_0|$, is often in the range 0.6–0.67, and the maximal efficiency we observed is $\approx$0.9. Therefore, a resistively shunted SSL could be an effective THz ac field to dc current converter.

From Eqs. (2) it follows that for *periodic* solutions $\langle v \rangle = \alpha \omega_c^{-2} \langle \mathcal{F} \rangle$. Therefore, *if the bias is phase locked* $\langle \mathcal{F} \rangle \approx n\Omega$ (n is an integer), *then the dc current is*

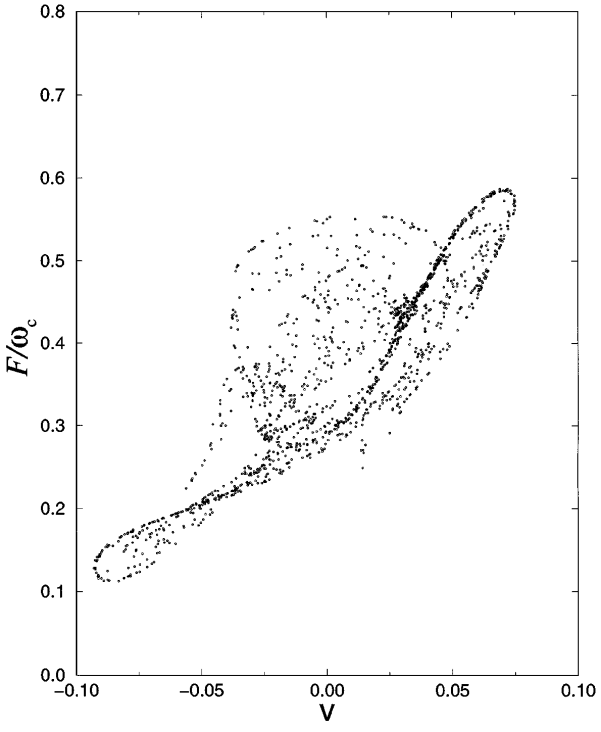

FIG. 1. Projection of the half-period stroboscopic phase portrait on the velocity–self-consistent field plane for asymmetric chaos at $\Omega/\omega_c = 0.47$, $\omega_s/\omega_c = 0.78$, $\gamma_v/\omega_c = 0.1$, $\gamma_\varepsilon/\omega_c = \alpha/\omega_c = 0.01$.

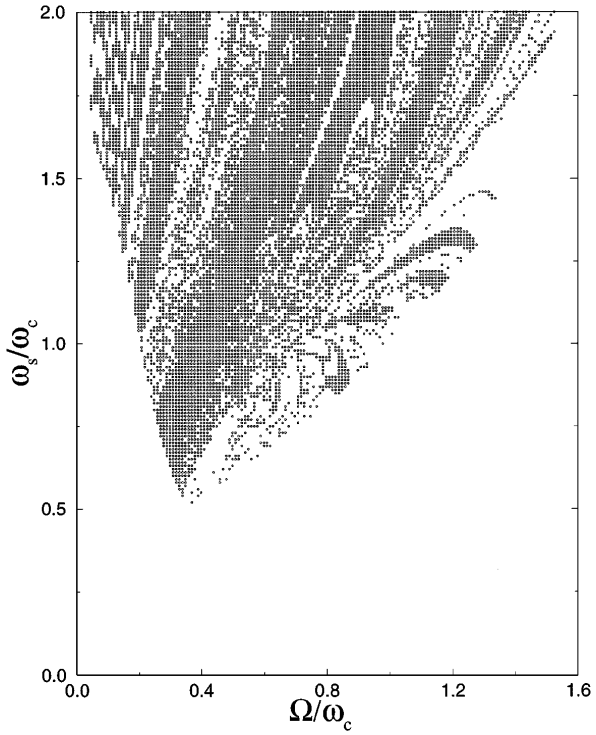

FIG. 2. A plot of region of symmetry-broken motion in the external field frequency(Ω)-external field strength(ω_s) plane. $\gamma_v/\omega_c = 0.1$, $\gamma_\varepsilon/\omega_c = \alpha/\omega_c = 0.01$.

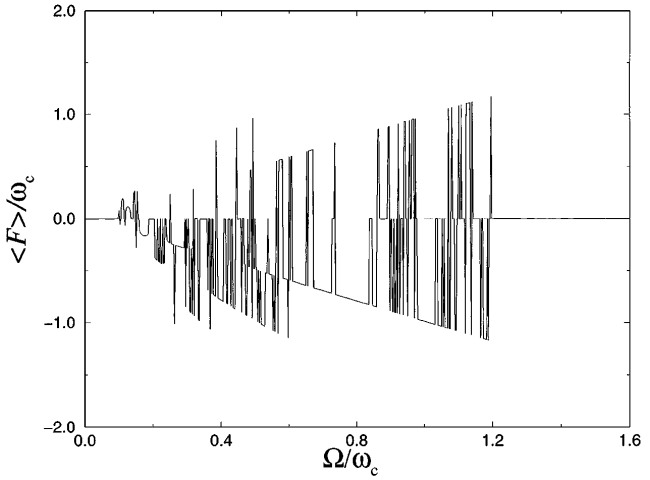

FIG. 3. Dependence of the spontaneously generated bias $\langle \mathcal{F} \rangle / \omega_c$ on the external field frequency Ω/ω_c at $\omega_s/\omega_c = 1.5$, $\gamma_v/\omega_c = 0.1$, $\gamma_\varepsilon/\omega_c = \alpha/\omega_c = 0.01$.

also proportional to $n\Omega$. Although this formula cannot rigorously be defined for aperiodic motion, we observed the same dependence for asymmetric chaos, at least during our simulation time of $17 \times 10^3\ \omega_c^{-1}$. To understand the origin of this phase locking, we recall that the wave functions of a time-periodic Hamiltonian with period T can be written as Floquet functions, where $\psi(x,t) = \exp(-\imath\epsilon t/\hbar) u_\psi(x,t)$, ϵ is the quasienergy, and u_ψ is periodic in time with period T [14]. Further, as shown by Krieger and Iafrate [15], when an electric field is applied to a perfect lattice, the Bloch wave functions evolve as Houston functions until tunneling to higher bands occurs. Therefore, in our single-band model *in the absence of scattering,* the electrons would evolve as Houston functions indefinitely. But Zak has demonstrated [14] that for a Houston function to be a Floquet state, the dc component of the electric field must satisfy the relation $eE_{\rm dc}a = 2\pi n\hbar/T$, for an integer n. In scaled units, this equation becomes precisely the phase-locking condition $\langle \mathcal{F} \rangle = n\Omega$; intuitively, this corresponds to resonant photon absorption between Stark ladder levels separated by an integer multiple of the photon energy. Our results demonstrate that this picture generally continues to be valid with the inclusion of scattering, when the Stark levels broaden and the phase-locking condition is only approximately satisfied. In our situation of an SSL without external bias, the self-consistent field leads to the spontaneous formation of a Stark ladder with suitable spacing.

We conclude with three comments. First, we estimate the size of the spontaneously generated current corresponding to present experimental parameters. For $\Omega = 0.9$ THz ($\Omega/\omega_c \approx 0.2$) and for the typical SSL parameters and dissipation constants mentioned above, we find that $I_{\rm dc} \equiv (eNS\Delta a/2\hbar)\langle v \rangle \simeq 28\ \mu$A ($\langle v \rangle \approx 0.002$) at field strength $E_0 \geq E_0^{(\rm cr)} \approx 4$ kV/cm ($\omega_s^{(\rm cr)}/\omega_c \approx 1$), which should be within the parameter range of current experiments [5–8]. Second, we stress that the physical situation we consider corresponds to transport in doped and/or current-injected SSLs, in which only electrons are present; if the carriers are photogenerated, so that for each electron a hole is also created, then excitonic binding energy effects, known to be important in other contexts (see, e.g., [16]), must be incorporated. How these will alter the situation is the matter of a separate major study. Third, we should distinguish our results from a recent study by Ignatov *et al.* [17], who pointed out that absolute negative conductance in the voltage-current characteristic of an SSL irradiated by an ac field of frequency Ω could cause an instability of the zero bias-zero current state, resulting in the switching to a new state with dc voltage per SSL period close to $\hbar\Omega/e$. These studies apply to a *biased* SSL operating in the high frequency regime, in which there can be no chaos. In contrast, our studies apply to an *unbiased* SSL at lower frequencies and lower amplitudes of the ac field and in a regime where chaotic behavior may occur.

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