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Relativistic chaos for an electron in a standing microwave field

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Abstract. – We study the dynamics of an electron in a standing wave of a microwave field. For large field amplitude the system undergoes a transition to a chaotic regime which is shown to be entirely related with the transition to relativistic dynamics. A possible laboratory experiment is shortly discussed.

This paper discusses the chaotic dynamics of a relativistic electron in a standing wave of a high-frequency electromagnetic field. Although the relativistic chaos is not a new problem, it is still a subject of considerable interest [1]-[7]. The key feature of a relativistic system is that it has an intrinsic nonlinearity enabling a chaotic dynamics even in the case, when it is completely integrable in the nonrelativistic limit. Perhaps the simplest example of such a system is a linearly driven harmonic oscillator [7]. The system we discuss in this paper provides a similar example, where the relativistic regime is accompanied by a transition to chaos. Being so simple as the harmonic oscillator, this system has the advantage to be a "laboratory" system not requiring a sophisticated set-up. A possible experimental arrangement is described in the concluding paragraphs of the paper.

We consider an electron moving along the z-axis, which crosses a standing wave with the wave vector k parallel to the x-axis and a polarization of the electric component parallel to the y-axis. In the Coulomb gauge the vector potential $\mathbf{A}(\mathbf{r},t) = \mathbf{n}_y(Ec/\nu)\cos(kx)\cos(\nu t)$ corresponds to the given field and the Hamiltonian of the system has the form

$$H = \left(m^2c^4 + c^2\left[\mathbf{P} - \mathbf{n}_y\frac{eE}{\nu}\cos(kx)\cos(\nu t)\right]^2\right)^{1/2} . \qquad (1)$$

After rescaling $x' = kx$, $t' = \nu t$, $P' = P/mc$, $H' = H/mc^2$ the Hamiltonian (1) can be reduced to the following form:

$$H = \left(1 + [\mathbf{P} - \mathbf{n}_y\kappa\cos x\cos t]^2\right)^{1/2} , \quad \kappa = \frac{eE}{mc\nu} \qquad (2)$$

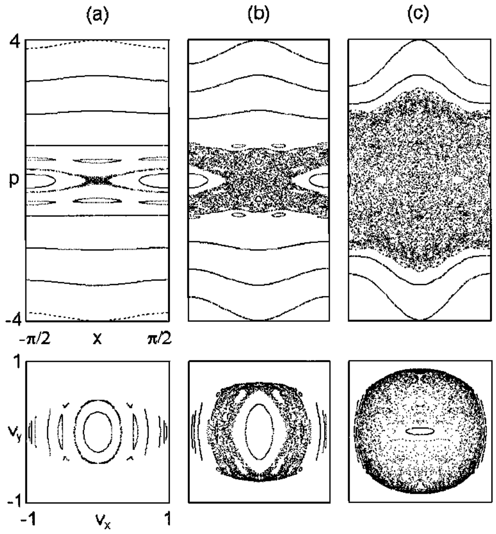

Fig. 1. – Stroboscopic map of the system (2) for $\kappa = 0.5$ (a), $\kappa = 1$ (b), and $\kappa = 2$ (c). The upper row is a cross-section by the plane (x, P_x), the lower row is a cross-section by the plane (v_x, v_y), $v_{x,y} = P_{x,y}/(1 + P^2_{x,y})$.

(primes are omitted). It is easy to see from eqs. (1), (2) that two components of the momentum, P_y and P_z, are integrals of the motion and, therefore, the system analyzed is effectively 1-dimensional. For the sake of simplicity let us assume $P_y = 0$ and $P_z = 0$, then

$$H = (1 + p^2 + \kappa^2 \cos^2 x \cos^2 t)^{1/2} \qquad (p \equiv P_x) \; . \tag{3}$$

The Hamiltonian (3) contains the single parameter κ which is actually a parameter of the transition to the chaotic regime. We proceed with a qualitative analysis of the system dynamics for different κ.

For $\kappa \ll 1$ the Hamiltonian (3) can be approximated by the Hamiltonian

$$H^{(1)} \approx (1 + p^2)^{1/2} + \frac{\kappa^2}{2(1 + p^2)^{1/2}} \cos^2 x \cos^2 t \tag{4}$$

and the dynamics of the system is almost regular. In fact, the condition of resonance with the field, which reads as $\dot{x} = p/(1 + p^2)^{1/2} = \pm 1$, cannot be satisfied for a finite value of p. Thus one can apply nonresonant perturbation theory. It leads to the effective Hamiltonian $H_{\text{eff}} = (1 + p^2)^{1/2} + (\kappa^2/4)(1 + p^2)^{-1/2} \cos^2 x$, and the dynamics of the system resembles that of a pendulum.

For a larger value of the parameter κ the next terms in the expansion of eq. (3) over κ^2 become important and the resonance condition $\dot{x} = p/(1 + p^2)^{1/2} = l/r$ (l, r are integers) can be satisfied for $|l/r| < 1$. Namely, the second term originates the nonlinear resonance $l = 1$, $r = 2$ at $p = \pm 1/\sqrt{3}$ with the width $\delta p \sim \kappa^2$, the third term causes the resonance with the width $\delta p \sim \kappa^3$, and so on. Employing the resonance overlap criteria one expects the transition to chaos at $\kappa \sim 1$ (see fig. 1). It should be noted that this value of κ requires a pretty large magnitude of the field (for instance, for $\lambda = 2\pi/k = 1$ cm the value $\kappa = 1$ corresponds to $E = 0.27 \cdot 10^6$ V/cm), which is actually on the border of modern MW-generators facilities.

Now we discuss a possible scheme of laboratory experiment (see fig. 2). As a critical value κ_{cr} of the transition to developed chaos it is convenient to choose the value of κ at

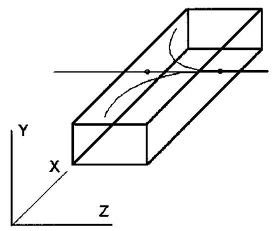

Fig. 2. – Schematic drawing of the proposed experiment. The electron beam passes through the holes in the wave guide located at a node of the standing wave. For the field magnitude larger than some critical value this trajectory becomes unstable and electrons scatter randomly.

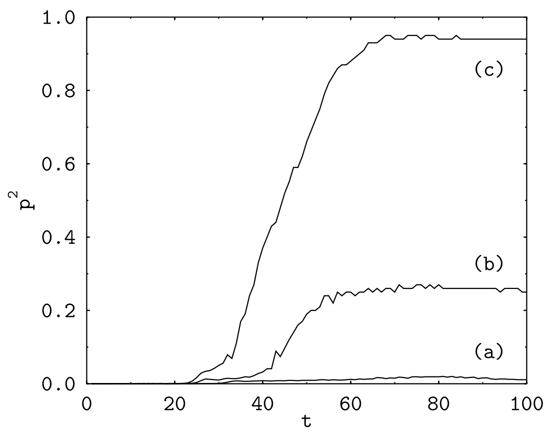

Fig. 3. – Chaotic diffusion of the momentum for the electron beam injected in the wave guide at a node of the standing wave: (a) $\kappa = 1.5$; (b) $\kappa = 1.8$; (c) $\kappa = 2$.

which the periodic point $p = 0$, $x = \pi/2$ loses its stability. This critical value can be found experimentally by measuring the intensity of the electron beam passing through the holes in the wave guide at a node ($x = \pi/2$) of the standing wave. Then the increase of κ over κ_{cr} will reflect itself in a decreasing of the intensity almost to zero. In fact, for $\kappa < \kappa_{\mathrm{cr}}$ the vicinity of the point $p = 0$, $x = \pi/2$ is occupied by stable trajectories (see fig. 1 (a), (b)). Because of this the initial (incoming) and final (outgoing) x-coordinates of an electron almost coincide and the electrons freely escape through the hole in the opposite wall of the wave guide. For $\kappa > \kappa_{\mathrm{cr}}$ the vicinity of the periodic point belongs to the chaotic component, thus the final position (and momentum) does not correlate with the initial one but takes a random value.

We simulated the dynamics of electrons passing through the wave guide at a nodal point. The initial distribution of the electrons in the beam was chosen Gaussian with a variance $\sigma_x = 0.1$ and $\sigma_p = 0.01$. The nonuniform amplitude of the field inside the wave guide was taken into account by substituting the parameter κ in the Hamiltonian (3) with $\kappa(t) = \kappa \sin(\pi v_z t/L_z)$ and we chose $L_z/v_z = 100$. Figure 3 shows the mean-squared momentum of the electrons in the beam as a function of time for different values of the field amplitude. It is seen that for $\kappa > 1.5$ there is a strong diffusion of the momentum. The chaotic diffusion in momentum space is accompanied by diffusion of the coordinate. For instance, for $\kappa = 2$ the electrons reaching the opposite wall of the wave guide spread over an interval of 50 wavelengths. Thus only a negligible fraction of the electron beam "has the chance" to leave the wave guide through the second hole.

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