Spectrum of waves in randomly modulated multilayers

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The spectrum and damping of waves in partially randomized multilayer structures are calculated. A method of calculation which was proposed and demonstrated earlier for the model of a superlattice with a harmonic dependence of its material parameters along its axis in the initial state, is extended here to the case of a multilayer structure (i.e., a superlattice with sharp interfaces). One- and three-dimensional inhomogeneities are considered, and the correlation function of the superlattice is derived. The spectrum and damping of waves in the superlattice described by this correlation function are found in the weak-coupling approximation in the vicinities of all the odd Brillouin zone boundaries. [S0163-1829(99)14501-6]

I. INTRODUCTION

Investigations of the spectrum of waves in partially randomized superlattices (multilayer structures) have been carried out very intensively in recent years. Several approaches to this problem now exist.

The modeling of the randomization by altering the order of successive layers of two different materials is in wide use now. It is assumed that only the periodicity in the arrangement of the layers corresponding to the ideal superlattice is destroyed when the system is randomized. A number of important and interesting results have been derived with the help of this model in studies of electron dynamics,[1,2] or the propagation of elastic[3] and spin[4] waves. In several papers the study of wave propagation in a superlattice was conducted in the framework of a method which consists in the numerical modeling of the random deviations of the interfaces from their initial periodic arrangement.[5,6] A model based on a doubly-periodic dependence of a physical parameter along the superlattice axis has been used in another approach.[7,8]

One more approach to the description of partially randomized superlattices was proposed recently in Refs. 9,10. This approach is based on the well-known radio-physics model of the random modulation of the frequency of a periodic radio signal.[11,12] In Ref. 9 a brief outline of this approach is given for the case of a superlattice whose period is modulated by a one-dimensional random function of a coordinate. In Ref. 10 a detailed description of the approach, and its extension to the cases of two- and three -dimensional random modulations, are presented. The correlation function of the superlattice is found analytically for each type of random modulation. The spectrum and damping of the waves are calculated for the model with a harmonic dependence of material parameters along the axis of the initial superlattice. In the present paper this approach is extended to multilayer systems in which the dependence of material parameters in the initial state has the form of rectangular space pulses. For definiteness we consider here spin waves, but the main results are presented in a form that is also valid in some approximation for elastic and electromagnetic waves as well.

II. METHOD OF CALCULATION: CORRELATION FUNCTION

As in Ref. 10 we consider the consequences of inhomogeneities of material parameters for wave propagation for the example of spin waves in a ferromagnet in which only the value of the magnetic anisotropy β depends on $\mathbf{x}$. Such an anisotropy can be represented in the form

$$\beta(\mathbf{x})=\beta[1+\gamma\rho(\mathbf{x})], \tag{1}$$

where β is the average value of the anisotropy, $\gamma=\Delta\beta/\beta$ is its relative rms fluctuation, and $\rho(\mathbf{x})$ is a centralized ($\langle\rho\rangle=0$) and normalized ($\langle\rho^2\rangle=1$) function of coordinates.

Choosing a magnetic field $\mathbf{H}$ and the anisotropy axis to be directed along the z axis we obtain the following equation for the circular projection of the magnetization $m^+=M_x+iM_y$:

$$\nabla^2 m^+ +[\nu-\varepsilon\rho(\mathbf{x})]m^+=0. \tag{2}$$

In writing Eq. (2) we have introduced the notations

$$\nu=(\omega-\omega_0)/\alpha gM, \quad \varepsilon=\gamma\beta/\alpha, \tag{3}$$

where $\omega_0=g(H+\beta M)$. In the scalar approximation both the spectrum of elastic and electromagnetic waves are also described by this equation with redefinitions of the parameters. For elastic waves we have $\nu=(\omega/v)^2$, $\varepsilon=\nu\gamma_u$, where γ_u is the rms fluctuation of the density of the material and v is the wave velocity. For an electromagnetic wave we have $\nu=\varepsilon_e(\omega/c)^2$, $\varepsilon=\nu\gamma_e$, where ε_e is the average value of the dielectric permeability, γ_e is its rms deviation, and c is the speed of light.

By carrying out the Fourier transformation of Eq. (2), we obtain the equation satisfied by the transform $m_{\mathbf{k}}$,

$$(\nu-k^2)m_{\mathbf{k}}=\varepsilon\int m_{\mathbf{k}_1}\rho_{\mathbf{k}-\mathbf{k}_1}d\mathbf{k}_1. \tag{4}$$

The eigenfrequencies of the waves described by Eq. (4) are determined by the poles of the Fourier transform of the average of the corresponding Green function. In this paper we have restricted ourselves to considering only the first nonvanishing contribution in ε to the mass operator of the Green function (the Bourret approximation),[13] and we obtain the general equation for the dispersion law of the averaged waves in the form

$$\nu - k^2 = \varepsilon^2 \int \frac{S(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1}{\nu - k_1^2}, \tag{5}$$

where $S(\mathbf{k})$ is the spectral density of the random function $\rho(\mathbf{x})$, which is connected with the correlation function $K(\mathbf{r})$ by a Fourier transformation

$$K(\mathbf{r}) \equiv \langle \rho(\mathbf{x})\rho(\mathbf{x}+\mathbf{r})\rangle = \int S(\mathbf{k})e^{i\mathbf{kr}}d\mathbf{k}. \tag{6}$$

In the process of deriving of the correlation function we follow the method that was suggested in Ref. 10. The randomization is taken into account by introducing a random modulation u of the superlattice period. In the general case this modulation can be a function of all three coordinates x, y, and z:

$$\rho(\mathbf{x}) = \frac{4}{\pi}\sum_{m=0}^{\infty} \frac{(-1)^m}{p}\cos p[q(z-u(\mathbf{x}))+\psi], \tag{7}$$

where $p=2m+1$. In the absence of disorder $\rho(\mathbf{x})$ has the form of rectangular spatial pulses. The stochastic properties of the function $\rho(\mathbf{x})$ have to be derived from the stochastic properties of the function $u(\mathbf{x})$ which characterizes, in the main, the inhomogeneity of the positions and structure of the interfaces. Following the procedure that was used in Ref. 10 we obtain a general expression for the correlation function in the form

$$K(\mathbf{r}) = \frac{8}{\pi^2}\sum_{m=0}^{\infty} \frac{1}{p^2}\cos\, pqr_z \exp\left[-\frac{p^2}{2}Q(\mathbf{r})\right], \tag{8}$$

where $Q(\mathbf{r})$ is the structure function of the random displacements $u(\mathbf{x})$, which is related by Eq. (I.22) of Ref. 14 with the spectral density $S_\phi(\mathbf{k})$ of the homogeneous random function $\phi(\mathbf{x}) = \nabla u(\mathbf{x})$:

$$Q(\mathbf{r}) = 2q^2 \int \frac{d\mathbf{k}}{k^2} S_\phi(\mathbf{k})(1-\cos\,\mathbf{kr}). \tag{9}$$

The correlation properties of the function $\phi(\mathbf{x})$ can be modeled by some standard correlation function $K_\phi(\mathbf{r})$. One of the main results of Ref. 10 is that the structure function $Q(\mathbf{r})$ and, consequently, the correlation function of the superlattice $K(\mathbf{r})$, does not depend on the form of the modeling function $K_\phi(\mathbf{r})$, for the limiting cases of short-wavelength and long-wavelength inhomogeneities [see Eqs. (I.23) – (I.32)]. This statement is also valid for the multilayer type of a superlattice, but the determination of these limiting cases now depends on the number of a harmonic in the series (8). We obtain an approximate expression for $K(\mathbf{r})$ in the form

$$K(\mathbf{r}) = \frac{8}{\pi^2}\sum_{m=0}^{\infty} \frac{1}{p^2}\cos\, pqr_z\Phi_p, \tag{10}$$

where Φ_p for the one-dimensional case has the form

$$\Phi_p = \begin{cases} \exp(-p^2k_{c1}^2r_z^2/2), & p \gg p_0, \\ \exp(-p^2k_{c2}r_z), & p \ll p_0. \end{cases} \tag{11}$$

Here $k_{c1}=\sigma q$, $k_{c2}=(\sigma q)^2/k_\parallel$, $p_0=k_\parallel/\sigma q$, σ is the rms fluctuation of the one-dimensional random function $\phi(z)$, and $k_\parallel$ is its correlation wave number.

For three-dimensional inhomogeneities

$$\Phi_p = \begin{cases} \exp(-p^2k_{c3}^2r^2/2), & p \gg p_0, \\ -(p/p_0)^2[1-2/\eta+(1+2/\eta)e^{-\eta}] & p \ll p_0. \end{cases} \tag{12}$$

Here $k_{c3}=\sigma q/\sqrt{3}$, $p_0=k_0/\sigma q$, $\eta=k_0 r$, σ is the rms fluctuation of the three-dimensional random function $\phi(\mathbf{r})$, and k_0 is its correlation wave number.

III. SPECTRUM AND DAMPING OF WAVES

Performing Fourier transformations of Eq. (10) taking Eqs. (11) and (12) into account, substituting the expressions for $S(\mathbf{k})$ obtained in Eq. (5), and performing the integration, we obtain equations for the spectrum of waves in the extended zone scheme. For one-dimensional inhomogeneities this equation has the form

$$\nu - k^2 = \frac{\Lambda^2}{4}\left\{\sum_{|p|\ll p_0} F_p^{(1)}(\nu,k) + \sum_{|p|\gg p_0} F_p^{(2)}(\nu,k)\right\}. \tag{13}$$

The function $F_p^{(1)}$ and $F_p^{(2)}$ in this equation are

$$F_p^{(1)} = \frac{\sqrt{\nu_1} - ip^2k_{c2}}{p^2\sqrt{\nu_1}} \frac{1}{(\sqrt{\nu_1}-ip^2k_{c2})^2-(pq-k_z)^2}, \tag{14}$$

$$F_p^{(2)} = \frac{1}{p^3k_{c1}\sqrt{2\nu_1}}\left[D(u)+D(v)+i\frac{\sqrt{\pi}}{2}(e^{-u^2}+e^{-v^2})\right], \tag{15}$$

where $D(x)=e^{-x^2}\int_0^x e^{t^2}dt$ is Dawson's integral, whose arguments u and v are given by

$$u = [\sqrt{\nu_1} - |k_z - pq|]/(\sqrt{2}pk_{c1}),$$

$$v = [\sqrt{\nu_1} + |k_z - pq|]/(\sqrt{2}pk_{c1}). \tag{16}$$

In the two-wave approximation we can describe the spectrum in the vicinity of the nth Brillouin-zone boundary $k_{rn} = nq/2$ by using only the term of the series with $p=n$. The resulting equation has the complex solution $\nu = \nu' + i\xi$, which under the condition $nk_{c2}/k_r \ll 1$ for $n \ll p_0$ has the form

$$\nu^\pm = \nu_{rn} \pm \frac{1}{2}\Delta\nu_n + in^3G_2/2, \tag{17}$$

where

$$\Delta\nu_n = \sqrt{(\Lambda/n)^2-(n^3G_2)^2} \tag{18}$$

is the width of a gap, and $G_2=k_{c2}q$ is the damping parameter. If the inequality $\Lambda/n > n^3G_2$ is satisfied, the degeneracy is removed and a gap $\Delta\nu_n$ appears in the spectrum.

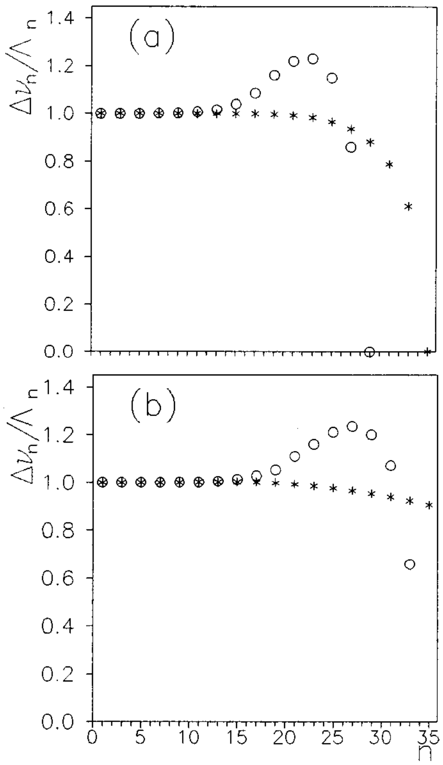

FIG. 1. The dependence of the normalized width of the gap $\Delta\nu_n/\Lambda_n$ on the zone number n for one-dimensional (a) and three-dimensional (b) inhomogeneities. Stars correspond to Eq. (14) for (a) and to Eq. (23) for (b); circles correspond to Eq. (15) for (a) and to Eq. (25) for (b).

For $n \gg p_0$ in the limiting case of small damping

$$\Delta\nu_n = (\Lambda/n)(1+1/b)^{1/2}, \tag{19}$$

$$\xi_\pm \approx \frac{1}{2}(\pi/2)^{1/2} G_1 n^2 (b-1)\exp\left[-\frac{1}{2}(b+1)\right], \tag{20}$$

where $b=(\Lambda/2n^3G_1)^2 \gg 1$, and $G_1 = qk_{c1}$. In the opposite limiting case when the gap becomes narrow we obtain

$$\Delta\nu_n \approx [\sqrt{\pi}(\Lambda/n)^2 - 2(n^2G_1)^2]^{1/2}, \tag{21}$$

$$\xi_\pm = n^2 G_1/\sqrt{2}. \tag{22}$$

The condition for the gap to be opened is now given by the inequality $\Lambda/n > (2/\sqrt{\pi})^{1/2} n^2 G_1$.

We calculate the dependence of the gap on the zone number n for both $n \ll p_0$ and $n \gg p_0$ by numerical methods [Fig. 1(a)]. The circles in Fig. 1(a) correspond to the case $n \gg p_0 < 1$. The form of this curve does not depend on $k_\parallel$; that is why for the given relation $\sigma q^2/\Lambda$ it has the same form for all values of $p_0 = k_\parallel/\sigma q < 1$. The stars in Fig. 1(a) correspond to the case $p_0 \gg 30$. In contrast to the preceding case, the form of the curve now depends on p_0. The opening of new gaps for $n > 27$ in this case is determined by the decrease of $G_2 = \sigma q^2/p_0$.

For three-dimensional inhomogeneities the equation for the spectrum has the same general form as Eq. (13), but $F_p^{(1)}$ and $F_p^{(2)}$ are defined by different expressions:

$$F_p^{(1)} = \left(\frac{1}{p^2} - \frac{1}{p_0^2}\right)\frac{1}{\nu - (\mathbf{k}-p\mathbf{q})^2} + \frac{1}{2k_0 p_0^2|\mathbf{k}-p\mathbf{q}|} \times\left[\frac{1}{v_1 - i} - \frac{1}{u_1 - i} + 2i\left(\ln\frac{u_1-i}{u_1} - \ln\frac{v_1-i}{v_1}\right)\right], \tag{23}$$

where

$$u_1 = (\sqrt{\nu} - |\mathbf{k}-p\mathbf{q}|)/k_0, \quad v_1 = (\sqrt{\nu} + |\mathbf{k}-p\mathbf{q}|)/k_0, \tag{24}$$

$$F_p^{(2)} = \frac{1}{\sqrt{2}k_{c3}p^3}\frac{1}{|\mathbf{k}-p\mathbf{q}|} \times\left[D(u_2) - D(v_2) + i\frac{\sqrt{\pi}}{2}(e^{-u_2^2} - e^{-v_2^2})\right], \tag{25}$$

with

$$u_2 = (\sqrt{\nu} - |\mathbf{k}-p\mathbf{q}|)/\sqrt{2}pk_{c3},$$

$$v_2 = (\sqrt{\nu} + |\mathbf{k}-p\mathbf{q}|)/\sqrt{2}pk_{c3}, \tag{26}$$

Just as in the case of the one-dimensional homogeneities treated above, for the three-dimensional inhomogeneities we can describe the spectrum in the vicinity of $k_{rn} = nq/2$ in the two-wave approximation by using only the term of the series with $p = n$. For $n \ll p_0$ we obtain in the limiting case $\eta_n = k_0 qn/\Lambda_n \ll 1$

$$\Delta\nu_n \approx \Lambda_n\left[1 + \frac{2}{3}\left(\frac{n}{p_0}\right)^2 \eta_n^2\right], \tag{27}$$

$$\xi_\pm = \Lambda_n\left(\frac{n}{p_0}\right)^2 \eta_n^3. \tag{28}$$

For the opposite limiting case $\eta_n \gg 1$ we have

$$\Delta\nu_n \approx \Lambda_n\left[1 - \frac{1}{2}\left(\frac{n}{p_0}\right)^2\left(1 - \frac{\pi}{2\eta_n}\right)\right], \tag{29}$$

$$\xi_\pm = \frac{\Lambda_n}{4\eta_n}\left(\frac{n}{p_0}\right)^2 \ln 2\eta_n. \tag{30}$$

For $n \gg p_0$ numerical calculations demonstrate that the solutions corresponding to Eq. (25) differ little from the solutions corresponding to Eq. (15) investigated above. An analytical analysis of the limiting cases $G_{n3} \ll \Lambda_n$ and $G_{n3} \sim \Lambda_n$, where $G_{n3} = n^2 G_3$ gives the same Eqs. (19), (20) and (21), (22), respectively, which have been obtained for Eq. (15), with the natural change of the damping parameter $G_1 = k_{c1}q$ to $G_3 = k_{c3}q$.

The dependence of $\Delta\nu_n$ on the zone number n is shown in Fig. 1(b). As in Fig. 1(a) the circles correspond to the case $p_0 < 1$, and the stars correspond to the case $p_0 \gg 30$. Comparing the results for one- and three-dimensional inhomogeneities one can see that for the case $p_0 < 1$, corresponding to smooth inhomogeneities with Gaussian correlations, new gaps corresponding to $n=29,\ 31$, and 33, open in the three-dimensional case in accordance with Eq. (21), where the damping parameter G_1 has been replaced by the smaller parameter G_3. Even greater differences between the one- and three-dimensional cases are found for the short-wavelength inhomogeneities (we assume that $k_0=k_\parallel$).

Both Figs. 1(a) and 1(b) have an illustrative nature. A system for which about 30 Brillouin zones with open gaps could be investigated is far from reality. But for a real system with only several open gaps the dependence of $\Delta\nu_n/\Lambda_n$ on n will be the same as in Figs. 1(a) and 1(b), only the points will be plotted very sparsely.

IV. CONCLUSIONS

The approach to the investigation of the wave spectrum of partially randomized superlattices which was suggested in Ref. 10 has been extended here to the case of superlattices with sharp interfaces, i.e., multilayer structures. As in Ref. 10, the spectrum and damping of the wave is investigated here in the Bourret approximation. For the harmonic superlattice this approximation permits investigating only the first Brillouin zone, because the spectrum of the zones with $n \neq 1$ is determined by the next terms of the series. In contrast to this, the Bourret approximation for the multilayer structure gives the possibility of investigating the spectrum and damping in the vicinity of the boundary of any odd Brillouin zone. Just as for superlattices with initial harmonic dependences of their material parameters, for superlattices with sharp interfaces different results are obtained for short-wavelength and smooth inhomogeneities. But the demarcation line between smooth and short-wavelength inhomogeneities depends now on the zone number n. The inhomogeneities characterized by the intensity σ and the correlation wave number $k_\parallel$ (for the one-dimensional case) or k_0 (for three-dimensional case) are the short-wavelength ones for the Brillouin zones with $n < k_\parallel/\sigma q$ and the smooth ones for the zones with $n > k_\parallel/\sigma q$. It was found that the damping parameter G_n is proportional to n^3 for the short-wavelength inhomogeneities, and to n^2 for the smooth ones. Correspondingly, the conditions for the closing of the gaps depend differently on the zone number n for the short-wavelength and smooth inhomogeneities. There are significant differences in the dependences of the gap width on n for the one- and three-dimensional inhomogeneities, especially for the short-wavelength ones. The appearance of the random deformation of the interfaces along with their random displacements from the initial positions leads to a decrease of the damping and to the opening of new gaps in comparison with the one-dimensional case where only random displacements of interfaces occur. In all cases, with increasing disorder the successive closing of the gaps in the spectrum takes place beginning with large values of n down to $n=1$.

Experimental investigations of spin-wave spectra are restricted for the present to the vicinity of the first Brillouin-zone boundary.[15] It would be of interest to carry out experiments covering several Brillouin zones to investigate the regularities described by the equations of this paper.

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