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Crossing resonance of two wave fields in disordered media

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The interaction of two wave fields of different nature in a disordered medium with an arbitrary relation between the mean value P and rms fluctuation λ of the coupling parameter is studied. A significant reconstruction of the eigenfrequencies, damping parameters, and susceptibilities of the system occurs when the situation changes from the model of a homogeneous medium ($P\neq 0$, $\lambda=0$) to the model of the disorder-induced crossing resonance ($P=0$, $\lambda\neq 0$). The concept of two effective media in the same material, which was introduced for the latter model in earlier work, is extended here to the case $P\neq 0$.
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I. INTRODUCTION

The term ''crossing resonance'' describes a wide class of phenomena occurring when the dispersion curves of two wave fields of different physical nature intersect each other. An interaction between these fields removes the degeneracy, giving rise to a new compound excitation. Such crossing resonances as magnetoelastic resonance,[1,2] polaritons,[3] electron-nuclear magnetic resonance,[4] and others play an important role in solid-state physics, and have been studied in detail in ordered materials.

Recently, interest in these resonance interactions between wave fields in disordered materials has arisen. The influence of inhomogeneities of parameters of one of the interacting fields has been considered in a number of papers (see, for example, Refs. 5–7 for polaritons in a medium where electromagnetic parameters are random). This leads to a more complicated dispersion law and some other interesting effects in the vicinity of the crossing resonance. But the most significant changes occur in the physics of the crossing resonances if it is not the parameters of each of the interacting fields that are considered as random quantities but the coupling between them. Beginning with Ref. 8, the crossing resonance of two wave fields in a medium with an inhomogeneous coupling parameter between the fields was investigated in several papers.[9–14] All of these studies were conducted within the framework of the disorder-induced crossing resonance (DICR) model, which was introduced in Ref. 8. Within this model the coupling parameter between the wave fields was assumed to be a random zero-mean function of coordinates, so that the interaction occurs only due to spatial fluctuations of this parameter. The model is a special case of the more general situation where both the mean value and fluctuations of the coupling parameter exist in a material. It is a convenient model that describes the influence of the disorder on crossing resonances in the most prominent way, and is also related to some real situations. One can mention, for example, amorphous ferromagnets with a zero-mean magnetostriction, or polaritons arising due to the coupling between electromagnetic waves and vibrations that would be dipole inactive in the absence of the disorder.

The physical nature of the interacting wave fields as well as the nature of the coupling parameters are different in different media. Despite these differences, however, the main effects caused by inhomogeneities of the coupling parameter have quite a general nature, which is independent of a particular realization of the resonance situation. In order to emphasize the general features of the phenomenon, it was considered in Refs. 8, 13, and 14 in the most general form, which would be valid for the crossing resonance of any nature. In Ref. 8 the reconstruction of dispersion laws as well as the accompanying decay of the average waves caused by fluctuations of the coupling parameter were investigated. A considerable qualitative difference between DICR and crossing resonances in homogeneous media was found. Both coupled wave fields in a homogeneous medium are coherent and have a joint dispersion law that consists of two branches; depending on the value of the damping in the system, the gap between these branches at the resonance point can be opened up or closed. In contrast to this, the mixed excitations in the DICR model consist of the coherent part of one of the wave fields and scattered waves of the other field. In this case each averaged wave field is characterized by its own dispersion law. The situation is possible, for instance, when the dispersion law of the one averaged wave field has a gap at the resonance point, whereas the dispersion curve of the other field is continuous. In Ref. 13 the energy dynamics of the compound states which arise in a system of two wave fields coupled by a random interaction have been studied. Randomness of the interaction causes energy flow from the coherent wave to scattered states of the second participating wave field. If disorder-induced crossing resonance occurs, it has been found that there is also a current from the scattered waves to the coherent component. In Ref. 14 a susceptibility matrix in the vicinity of DICR has been studied. The diagonal components of the susceptibility matrix demonstrate resonance features caused by the interaction of the averaged wave of one nature with fluctuation waves of another nature, while the off-diagonal components are equal to zero. It has been shown that the average properties of the system can be considered within the concept of two effective media that can be introduced in the same real material in order to describe properties of the fields. These two media have different characteristics and are independent in the sense that each of the averaged wave fields propagates through its own effective medium without any interactions.

In addition to these properties, which are general for any disorder-induced crossing resonance, there are properties and phenomena that are specific for particular systems. The specific features of the disorder-induced magnetoelastic resonance in ferromagnets with zero-mean magnetostriction,[9–11] and disorder-induced polaritons in disordered dielectrics,[12] have been examined. It was shown that the main characteristics of the stochastic magnetoelastic interaction can be measured by experimental studies of either the modified dispersion law of acoustic waves[9] or the elastic analogs of the Faraday and Cotton-Moutton effects.[10] Polaritons were shown[12] to be good candidates for the experimental observation of effects of the inhomogeneity of the coupling parameter in a medium with dipole inactive phonons by optical methods.

All of these studies[8–14] were carried out in the framework of the DICR model. The mean value P of the coupling parameter was considered to be equal to zero, and it was assumed that the interaction between wave fields occurs only due to fluctuations of this parameter. The parameter, which characterizes the strength of the interaction in this case, is the rms fluctuation of the coupling parameter, λ. In real disordered media the average value of the coupling parameter is not, in most cases, equal to zero, and it may be in any kind of relationship with its rms value λ. The main objective of the present paper is to consider this general situation. Our main goal is to study how dispersion laws of the coupled excitations change in going from the case of the pure DICR model $(P=0,\lambda\neq 0)$ to the case of a homogeneous medium $(P\neq 0,\lambda=0)$. For the sake of definiteness we consider the magnetoelastic crossing resonance, but the main results obtained hold qualitatively for crossing resonances of any nature.

II. EQUATIONS OF THE COUPLED AVERAGED WAVES

We shall analyze the crossing resonance of two wave fields for the example of magnetoelastic resonance in inhomogeneous ferromagnets.

Excitations in a magnetoelastic medium are governed by the system of Landau-Lifshitz equations for the magnetization and the equations for the elastic displacements,

$$\dot{\mathbf{M}}=-g\left[\mathbf{M}\times\left(-\frac{\partial\mathcal{H}}{\partial\mathbf{M}}+\frac{\partial}{\partial\mathbf{x}}\frac{\partial\mathcal{H}}{\partial(\partial\mathbf{M}/\partial\mathbf{x})}\right)\right],$$
$$\mu\ddot{u}_i=\frac{\partial}{\partial x_i}\frac{\partial\mathcal{H}}{\partial u_{ij}},\tag{1}$$

where $\mathbf{M}$ is the magnetization, $\vec{u}$ is the elastic displacement vector, $u_{ij}=1/2(\partial u_i/\partial x_j+\partial u_j/\partial x_i)$ is the elastic strain tensor, g is the gyromagnetic ratio, and μ is the density of the medium.

We assume that our system is an elastically isotropic ferromagnet with a single magnetic symmetry axis, so that the corresponding magnetoelastic potential energy $\mathcal{H}$ takes the form

$$\mathcal{H}=\frac{1}{2}\alpha(\nabla\mathbf{M})^2-\frac{1}{2}\beta(\mathbf{Mn})^2-\mathbf{HM}+\frac{1}{2}d_1u_{ii}^2+\frac{1}{2}d_2(u_{ij}u_{ij}+u_{ij}u_{ji})+\frac{1}{2}B(\mathbf{x})M_iM_ju_{ij}.\tag{2}$$

Here α is the exchange parameter, β and $\mathbf{n}$ are the magnitude and direction of the magnetic anisotropy axis, respectively, d_1 and d_2 are the elastic Lamé constants, $\mathbf{H}$ is the magnetic field, and $B(\mathbf{x})$ is the magnetoelastic parameter.

In the general case the parameter B coupling the subsystems can be represented as the sum of a nonrandom and a random component,

$$B(\mathbf{x})=B_0+\Delta B\rho(\mathbf{x}),\tag{3}$$

where B_0 is the mean value and ΔB is the rms fluctuation of the magnetoelastic parameter, and $\rho(\mathbf{x})$ is a centered $[\langle\rho(\mathbf{x})\rangle=0]$ and normalized $[\langle\rho^2(\mathbf{x})\rangle=1]$ random function. The stochastic properties of $\rho(\mathbf{x})$ are characterized by the normalized correlation function

$$K(\mathbf{r})=\langle\rho(\mathbf{x})\rho(\mathbf{x}+\mathbf{r})\rangle.\tag{4}$$

Let an external dc magnetic field and the anisotropy axis be directed along the z axis of the coordinate system. The equilibrium direction of the magnetization, then, also coincides with the z axis. We consider the excitation of the medium by bulk forces f_a and f_b, with the first of them affecting the elastic subsystem and the second one influencing the magnetic subsystem. We assume that these forces are perpendicular to the z axis. Therefore, only their x and y components have nonzero values. Linearizing the system (1) with respect to the small deviations $\mathbf{m}(\mathbf{x},\mathbf{t})$ from the equilibrium magnetization $\mathbf{M}_0$, using the scalar approximation for the elastic waves ($v_t=v_l=v$, where v_t and v_l are the speeds of the transverse and longitudinal elastic waves, respectively), and neglecting the terms describing both the nonresonant interaction between the elastic and the left-polarized spin waves and the terms describing the interaction between the spin waves and the longitudinal elastic waves (u_z), we obtain the following integral equations for the Fourier transforms of the circular components $m=m_x+im_y$ and $u=u_x+iu_y$:

$$[(\omega-i\Gamma_u)^2-\omega_u^2(\mathbf{k})]u(\mathbf{k})+\frac{iMk_z}{2\mu}\left[B_0m(\mathbf{k})+\Delta B\int m(\mathbf{k}_1)\rho(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1\right]=\Omega_u^2f_k,\tag{5}$$

$$[\omega-\omega_s(\mathbf{k})-i\Gamma_s]m(\mathbf{k})-\frac{i\omega_M M}{2}\Big[B_0 k_z u(\mathbf{k})+\Delta B\int k_{1z}u(\mathbf{k}_1)\rho(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1\Big]=\omega_M h_k .$$

In these equations ω_u and ω_s are the initial dispersion laws of the elastic and spin waves, respectively,

$$\omega_u=vk,\quad \omega_s=\omega_0+\alpha\omega_M k^2, \tag{6}$$

where $\omega_0=g(H+\beta M)$ and $\omega_M=gM$; we added the parameters Γ_u and Γ_s in order to model the initial dampings of the corresponding waves.

We shall examine these equations in the vicinity of the crossing resonance point (ω_r,k_r), where ω_r and k_r are determined by the equations

$$\omega_u(k_r)=\omega_s(k_r)=\omega_r\equiv vk_r . \tag{7}$$

Introducing the dimensionless variables ϕ and ψ and the dimensionless forces Φ and Ψ by

$$\phi_k=(2\mu\omega_r\omega_M)^{1/2}\frac{u_k}{M},\quad \psi_k=\frac{m_k}{M},$$
$$\Phi_k=\left(\frac{\mu\omega_M}{2\omega_r}\right)^{1/2}\Omega_u f_k,\quad \Psi_k=\frac{h_k}{M}, \tag{8}$$

we obtain the following system of equations:

$$\frac{1}{2\omega_r}[(\omega-i\Gamma_u)^2-\omega_u^2(\mathbf{k})]\phi(\mathbf{k})+\frac{i}{2}P\frac{k_z}{k_r}\psi(\mathbf{k})+\frac{i}{2}\lambda\frac{k_z}{k_r}\int\psi(\mathbf{k}_1)\rho(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1=\Omega_u\Phi_k ,$$
$$[\omega-\omega_s(\mathbf{k})-i\Gamma_s]\psi(\mathbf{k})-\frac{i}{2}P\frac{k_z}{k_r}\phi(\mathbf{k})-\frac{i\lambda}{2k_r}\int k_{1z}\phi(\mathbf{k}_1)\rho(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1=\omega_M\Psi_k . \tag{9}$$

The mean value P and the rms fluctuation λ of the coupling parameter are determined by the expressions

$$P=B_0\left(\frac{\omega_M}{2\mu\omega_r}\right)^{1/2}Mk_r,\quad \lambda=\Delta B\left(\frac{\omega_M}{2\mu\omega_r}\right)^{1/2}Mk_r . \tag{10}$$

Both of them have the dimensions of frequency.

In order to deduce the averaged Green function of Eqs. (9), it is convenient to introduce matrix notations:

$$G_0^{-1}=\begin{pmatrix}\frac{1}{2\omega_r}[(\omega-i\Gamma_u)^2-\omega_u^2(\mathbf{k})] & i\frac{Pk_z}{2k_r}\\ -i\frac{Pk_z}{2k_r} & \omega-\omega_s(\mathbf{k})-i\Gamma_s\end{pmatrix},$$

$$R=\begin{pmatrix}0 & -\frac{ik_z}{2k_r}\int\rho(\mathbf{k}-\mathbf{k}_1)\cdots d\mathbf{k}_1\\ \frac{i}{2k_r}\int k_{1z}\rho(\mathbf{k}-\mathbf{k}_1)\cdots d\mathbf{k}_1 & 0\end{pmatrix},$$

$$f=\begin{pmatrix}\phi_{\mathbf{k}}\\ \psi_{\mathbf{k}}\end{pmatrix},\quad F=\begin{pmatrix}\Omega_u\Phi_{\mathbf{k}}\\ \omega_M\Psi_{\mathbf{k}}\end{pmatrix}.$$

Here G_0 is the matrix Green function, which describes only a uniform coupling between $\phi_{\mathbf{k}}$ and $\psi_{\mathbf{k}}$; R is the matrix of integral operators which takes into account the nonuniform coupling; and f and F are the vectors of the variables and forces of the system under consideration, respectively.

Using these notations we can rewrite the system of Eqs. (9) in the form of a matrix equation

$$G_0^{-1}f=\lambda Rf+F. \tag{11}$$

The averaged Green matrix of this equation can be found by any of the standard methods that have been developed for the usual scalar equation. Specifically, in the approximation that is analogous to the Bourret approximation for the usual equation we obtain

$$\langle f\rangle=\langle G\rangle F=\frac{1}{G_0^{-1}-\lambda^2\langle RG_0R\rangle}F. \tag{12}$$

The system of equations for the averaged variables $\langle\phi\rangle$ and $\langle\psi\rangle$ in this approximation has the form

$$\left\{\frac{1}{2\omega_r}[(\omega-i\Gamma_u)^2-\omega_u^2(k)]-Q_u(k)\right\}\langle\phi\rangle+\frac{iP_k e(k)}{2}\langle\psi\rangle =\Omega_u\Phi_k, \tag{13}$$

$$\{\omega-i\Gamma_s-\omega_s(k)-Q_s(k)\}\langle\psi\rangle-\frac{iP_k e(k)}{2}\langle\phi\rangle=\omega_M\Psi_k,$$

where

$$Q_u(\mathbf{k})=\frac{\lambda_k^2}{4}\int\frac{(\omega-i\Gamma_u)^2-\omega_u^2(\mathbf{k}_1)}{2\omega_r\Delta_{\mathbf{k}_1}}S(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1,$$

$$Q_s(\mathbf{k})=\frac{\lambda_k^2}{2k_z^2}\int\frac{k_{1z}^2[\omega-i\Gamma_s-\omega_s(\mathbf{k}_1)]}{\Delta_{\mathbf{k}_1}}S(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1,$$

$$e(k)=1+\frac{\lambda_k^2}{4k_z^2}\int\frac{k_{1z}^2}{\Delta_{k_1}}S(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1, \tag{14}$$

$$\Delta_k=\frac{1}{2\omega_r}[(\omega-i\Gamma_u)^2-\omega_u^2(\mathbf{k})][\omega-i\Gamma_s-\omega_s(\mathbf{k})]-\frac{P^2}{4},$$

$$P_k=\frac{k_z}{k_r}P,\quad \lambda_k=\frac{k_z}{k_r}\lambda.$$

Here $S(\mathbf{k})$ is the spectral density of the random function $\rho(\mathbf{x})$. It is the Fourier transform of the correlation function $K(\mathbf{r})$.

In what follows we consider waves propagating in the direction of the equilibrium magnetization $\mathbf{M}_0$, assuming $k_z=k$. We also consider P and λ as small quantities of the same order of magnitude, and neglect their product of the third order $P\lambda^2$ and, accordingly, the term $P^2/4$ in the denominators of the integrands. The first term in the curly brackets in the first of Eqs. (13) can be simplified in the vicinity of the crossing resonance by the use of the approximation

$$\omega-i\Gamma_u+\omega_u(k)\approx 2\omega_r. \tag{15}$$

Using all of these approximations we obtain a simpler form of Eqs. (13):

$$[\omega-\omega_u(k)-i\Gamma_u-Q_u]\langle\phi\rangle+\frac{iP_k}{2}\langle\psi\rangle=\Omega_u\Phi_k, \tag{16}$$

$$[\omega-\omega_s(k)-i\Gamma_s-Q_s]\langle\psi\rangle-\frac{iP_k}{2}\langle\phi\rangle=\omega_M\Psi_k,$$

where Q_u and Q_s play the role of the mass operators of the averaged Green matrix; within the approximations made they are determined by the following expressions:

$$Q_u=\frac{\lambda_k^2}{4}\int\frac{S(\mathbf{k}-\mathbf{k}_1)dk_1}{\omega-\omega_s(k_1)-i\Gamma_s}, \tag{17}$$

$$Q_s=\frac{\lambda_k^2\omega_r}{2k^2}\int\frac{k_{1z}^2S(\mathbf{k}-\mathbf{k}_1)dk_1}{(\omega-i\Gamma_u)^2-\omega_u^2(k_1)}.$$

Let us choose the standard exponential form of the correlation function to characterize the inhomogeneities:

$$K(r)=e^{-k_c r},\quad S(k)=\frac{1}{\pi^2}\frac{k_c}{(k^2+k_c^2)^2}. \tag{18}$$

Here k_c is the correlation wave number ($k_c\approx r_c^{-1}$, where r_c is the correlation radius of the inhomogeneities). The integrals (17) with the spectral density given by Eq. (18) have been calculated in Ref. 9. They have cumbersome forms (especially for Q_s), but we use here their simplified forms which have been obtained in Ref. 14 for small values of Γ_u, Γ_s, and vk_c in comparison with ω_r:

$$Q_u=\frac{\lambda_k^2}{4}\frac{1}{\omega-\omega_s(k)-i\Gamma_s^*}, \tag{19}$$

$$Q_s=\frac{\lambda_k^2}{4}\frac{1}{\omega-\omega_u(k)-i\Gamma_u^*}.$$

The effective relaxation parameters Γ_s^* and Γ_u^* are sums of the initial damping constants and the relaxations due to scattering

$$\Gamma_s^*\approx\Gamma_s+2k_c\sqrt{\alpha\omega_M(\omega-\omega_0)}\approx\Gamma_s+v_s k_c, \tag{20}$$

$$\Gamma_u^*\approx\Gamma_u+vk_c,$$

where $v_s=2\alpha\omega_M k_r$ is the velocity of the spin waves at the crossing resonance point. The expression for the addition to Γ_s is valid only for $\omega>\omega_0$. Both additions are the products of k_c and the velocity of the corresponding wave. Therefore, the addition to Γ_u is significantly larger than the addition to Γ_s.

The system of equations (16) with the mass operators Q_u and Q_s in the forms given by Eq. (19) is analyzed in the next section of this paper. It is worth emphasizing that in the approximations chosen, the system of equations for a crossing resonance of any nature in a medium with an inhomogeneous coupling parameter has a form analogous to Eqs. (16).

III. EIGENFREQUENCIES OF THE AVERAGED WAVES

The general equations (16) describe the averaged waves in the vicinity of the crossing resonance for $\Phi_k=\Psi_k=0$. The complex dispersion law of the waves is determined by the equation

$$D(\tilde{\omega},k)=0, \tag{21}$$

where D is the determinant of the system (16). Here $\tilde{\omega}$ is the complex frequency

$$\tilde{\omega}=\omega+i\xi, \tag{22}$$

where ω and ξ are the eigenfrequency and damping parameter of the averaged waves, respectively.

Using Eqs. (19) for the mass operators, we obtain the complex dispersion law in the form

$$D\equiv\left[\tilde{\omega}-\omega_u-i\Gamma_u-\frac{\lambda_k^2/4}{\tilde{\omega}-\omega_s-i\Gamma_s^*}\right]\left[\tilde{\omega}-\omega_s-i\Gamma_s-\frac{\lambda_k^2/4}{\tilde{\omega}-\omega_u-i\Gamma_u^*}\right]-\frac{P_k^2}{4}=0. \tag{23}$$

It is a fourth-order equation for the complex frequency $\tilde{\omega}$. This equation has been studied earlier in two limiting cases: $\lambda=0,P\neq 0$ and $P=0,\lambda\neq 0$. Here we recall briefly the results of these investigations. In an ordered medium ($\lambda=0$, $P\neq 0$), Eq. (23) becomes a quadratic equation, and the behavior of the resulting two branches at the crossing resonance point depends on the relation between the parameters P, Γ_u, and Γ_s. If $P>\Gamma\equiv|\Gamma_u-\Gamma_s|$, a gap $\Delta\omega$ between the branches appears at the crossing resonance point [Fig. 1(a)], and the damping parameters for both branches $\omega_\pm$ of the spectrum become equal to each other:

$$\omega_\pm=\omega_r\pm\frac{1}{2}\Delta\omega,\quad \xi_\pm=\frac{1}{2}(\Gamma_u+\Gamma_s), \tag{24}$$

where

$$\Delta\omega=\sqrt{P^2-\Gamma^2}. \tag{25}$$

In the opposite case $P<\Gamma$ the branches cross each other at the resonance point, while the damping parameters differ:

$$\omega_\pm=\omega_r,\quad \xi_\pm=\frac{1}{2}(\Gamma_u+\Gamma_s\pm\Delta\xi), \tag{26}$$

where

$$\Delta\xi=\sqrt{\Gamma^2-P^2}. \tag{27}$$

When the ratio Γ/P increases, the damping parameters $\xi_\pm$ tend to their initial values Γ_u and Γ_s.

In a medium with an inhomogeneous coupling parameter in the case where $P=0$ (the model of DICR), Eq. (23) splits into two different quadratic equations:

$$D_u\equiv(\tilde{\omega}-\omega_u-i\Gamma_u)(\tilde{\omega}-\omega_s-i\Gamma_s^*)-\frac{\lambda_k^2}{4}=0,$$
$$D_s\equiv(\tilde{\omega}-\omega_s-i\Gamma_s)(\tilde{\omega}-\omega_u-i\Gamma_u^*)-\frac{\lambda_k^2}{4}=0. \tag{28}$$

This difference is caused by the fact that D_u and D_s contain different pairs of the relaxation parameters, Γ_u,Γ_s^* and Γ_s,Γ_u^*, respectively. The first of these equations describes the averaged elastic waves, the second one the averaged spin waves. The dispersion law for the averaged elastic waves is modified by the interaction with the scattered spin waves, and the dispersion law for the averaged spin waves is modified by the scattered elastic waves. The shape of the dispersion curves following from Eqs. (28) can also be considerably different for the elastic and spin waves. In particular, the conditions for a gap between the different branches to appear at the crossing point have the following forms for the elastic and spin waves:

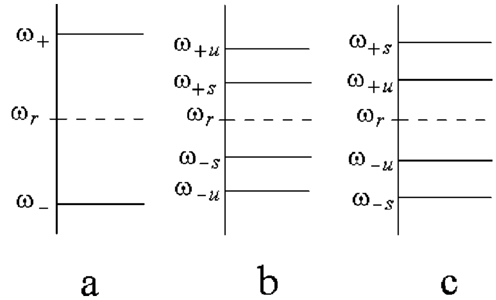


FIG. 1. Spectrum at the crossing resonance point for both the homogeneous medium (a) and for the DICR model (b) and (c).

$$\lambda^2>(\Gamma_s^*-\Gamma_u)^2,\quad \lambda^2>(\Gamma_s-\Gamma_u^*)^2, \tag{29}$$

respectively. If both of these inequalities are satisfied, we have for the elastic waves at the crossing point

$$\omega_{\pm u}=\omega_r\pm\frac{1}{2}\Delta\omega_u,\quad \xi_{\pm u}=\frac{1}{2}(\Gamma_u+\Gamma_s^*), \tag{30}$$

where

$$\Delta\omega_u=\sqrt{\lambda^2-(\Gamma_s^*-\Gamma_u)^2}, \tag{31}$$

and for the spin waves

$$\omega_{\pm s}=\omega_r\pm\frac{1}{2}\Delta\omega_s,\quad \xi_{\pm s}=\frac{1}{2}(\Gamma_u^*+\Gamma_s), \tag{32}$$

where

$$\Delta\omega_s=\sqrt{\lambda^2-(\Gamma_s-\Gamma_u^*)^2}. \tag{33}$$

One can see that the gap in the spin wave spectrum $\Delta\omega_s$ and the gap in the elastic wave spectrum $\Delta\omega_u$ are not equal to each other in the general case. If the initial damping parameters Γ_u and Γ_s are equal to zero, or $\Gamma_u=\Gamma_s$, the difference between the gaps is determined by the difference between the additions $\Delta\Gamma_u=v k_c$ and $\Delta\Gamma_s=v_s k_c$. Therefore $\Delta\omega_u>\Delta\omega_s$ in such cases [Fig. 1(b)]. If there is a difference between the initial damping constants, the relation between $\Delta\omega_s$ and $\Delta\omega_s$ can be more complex. For the case $\Gamma_s>\Gamma_u$ we have $\Delta\omega_s>\Delta\omega_u$ [Fig. 1(c)] for small values of k_c until the inequality

$$\Delta\Gamma_u<2\Gamma+\Delta\Gamma_s \tag{34}$$

is satisfied, and $\Delta\omega_s<\Delta\omega_u$ for larger values of k_c (Fig. 2).

If the sign of one or the other of the inequalities (29) changes, the corresponding branch becomes continuous at the resonance point. For example, we have for the spin waves in this case

$$\omega_{\pm s}=\omega_r,\quad \xi_{\pm s}=\frac{1}{2}(\Gamma_u^*+\Gamma_s\pm\Delta\xi_s), \tag{35}$$

where

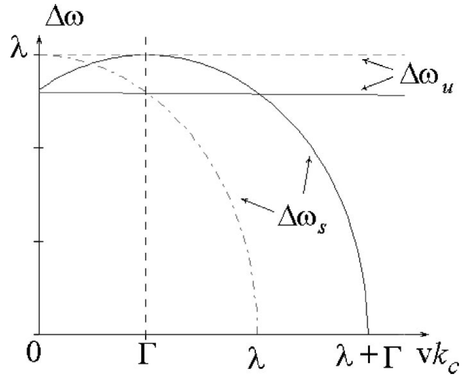


FIG. 2. Dependences of the gaps $\Delta\omega_1$ and $\Delta\omega_2$ in the spectrum on k_c for the DICR model. The solid curves correspond to $\Gamma=0.5\lambda$, the dashed ones correspond to $\Gamma=0$.

$$\Delta\xi_s=\sqrt{(\Gamma_s-\Gamma_u^*)^2-\lambda^2}. \tag{36}$$

It has been shown in Ref. 14 that the average properties of a system of two wave fields of different nature coupled by a coupling parameter with a zero mean value can be considered within the concept of two effective media that should be introduced in the same material. These two media have different relaxation characteristics (Γ_u and Γ_s^* for one of the media and Γ_u^* and Γ_s for the other), and are independent in the sense that each of the averaged wave fields propagates through its own effective medium without interacting with the partner wave field.

Let us study the changes in both the spectrum and damping determined by Eq. (23) at the point of the crossing resonance $k=k_r$ as P and λ change subject to the condition

$$P^2+\lambda^2=C^2, \tag{37}$$

where C is a constant with the dimensions of frequency ($C \ll \omega_r$). When λ decreases from C to zero, the four frequencies of the independent magnetic and elastic oscillations [Figs. 1(b) or 1(c)] must transform into the two frequencies of the coupled magnetoelastic oscillations [Fig. 1(a)]. It might be assumed that this transformation proceeds as follows. The frequencies ω_{+s} and ω_{+u} of the independent magnetic and elastic oscillations merge together into the frequency ω_+ of the coupled magnetoelastic oscillations and, correspondingly, the frequencies ω_{-s} and ω_{-u} merge into the frequency ω_- at $\lambda=0$. But the real picture is not so simple.

The inhomogeneities of the coupling parameter are characterized by two main quantities: λ and k_c. To distinguish effects due to each of them we investigate Eq. (23) for several values of k_c. For $k_c=0$ (i.e., $\Gamma_u^*=\Gamma_u, \Gamma_s^*=\Gamma_s$) a simple analytic solution of Eq. (23) can be obtained. In reality the case $k_c=0$ has no physical meaning because the condition of ergodicity is not satisfied for the random function $\rho(\mathbf{x})$ in this case. But it can be considered as a zero approximation for the case of long-wavelength inhomogeneities. In this case Eq. (23) has four complex solutions,

$$\omega_{\pm1}=\omega_r\pm\frac{1}{2}\sqrt{\mu_1^2-\Gamma^2}+i\,\frac{\Gamma_a+\Gamma_s}{2}, \tag{38}$$

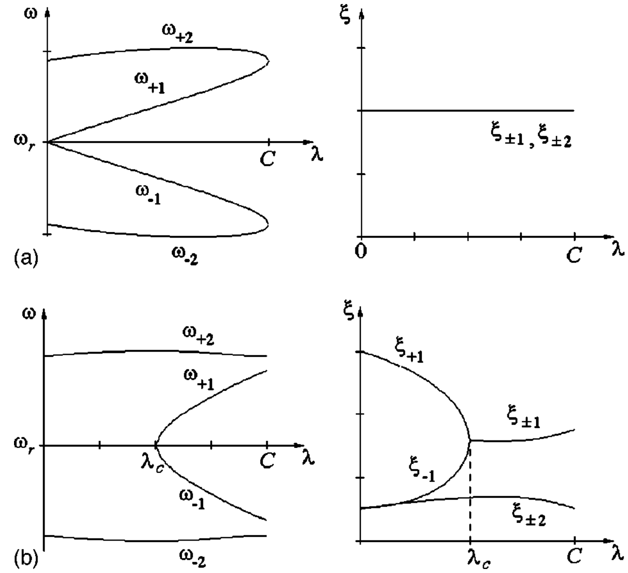


FIG. 3. The dependence on λ of the reconstruction of both the eigenfrequencies ω_i and damping parameters ξ_i at the crossing resonance point $k=k_r$ for $\Gamma=0$ ($\Gamma_u/\omega_r=\Gamma_s/\omega_r=0.01$) for the cases $k_c=0$ (a) and $k_c/k_r=0.15$ (b).

$$\omega_{\pm2}=\omega_r\pm\frac{1}{2}\sqrt{\mu_2^2-\Gamma^2}+i\,\frac{\Gamma_a+\Gamma_s}{2},$$

where

$$\mu_{1,2}=\frac{1}{2}(\sqrt{P^2+4\lambda^2}\mp P) \tag{39}$$

or, if the condition (37) is satisfied,

$$\mu_{1,2}=\frac{1}{2}(\sqrt{C^2+3\lambda^2}\mp\sqrt{C^2-\lambda^2}). \tag{40}$$

For the cases where $k_c\neq0$, numerical solutions of Eq. (23) have been obtained.

The dependences of the spectrum on λ are different for the cases where the initial damping constants Γ_s and Γ_u are equal to each other and when $\Gamma_s\neq\Gamma_u$. Let us begin with the simpler case $\Gamma_s=\Gamma_u=\Gamma_0, \Gamma=\Gamma_s-\Gamma_u=0$ [Figs. 3(a) and 3(b)]. These figures show the reconstruction of both the spectrum and damping for $k_c=0$ [Fig. 3(a)] and $k_c\neq0$ [Fig. 3(b)]. From the latter picture one can see that with the decrease of λ the frequencies ω_{+1} and ω_{-1} which belong (at $\lambda=C$) to the magnetic oscillations merge together at some critical point $\lambda=\lambda_c$, while the frequencies ω_{+2} and ω_{-2} of the elastic (at $\lambda=C$) oscillations continuously transform into the frequencies ω_+ and ω_- of the magnetoelastic oscillations at $\lambda=0$. The critical point $\lambda=\lambda_c$ for the oscillations with the frequencies $\omega_{\pm1}$ is also the critical point for their damping parameters $\xi_{\pm1}$. But if the frequencies ω_{+1} and ω_{-1} became equal to each other for $\lambda<\lambda_c$, their damping parameters ξ_{+1} and ξ_{-1} are equal for $\lambda>\lambda_c$. The damping parameters $\xi_{\pm2}$ of the oscillations with the frequencies $\omega_{\pm2}$ change only slightly over the entire range of λ values. The critical point λ_c shifts to smaller values of λ when k_c de-

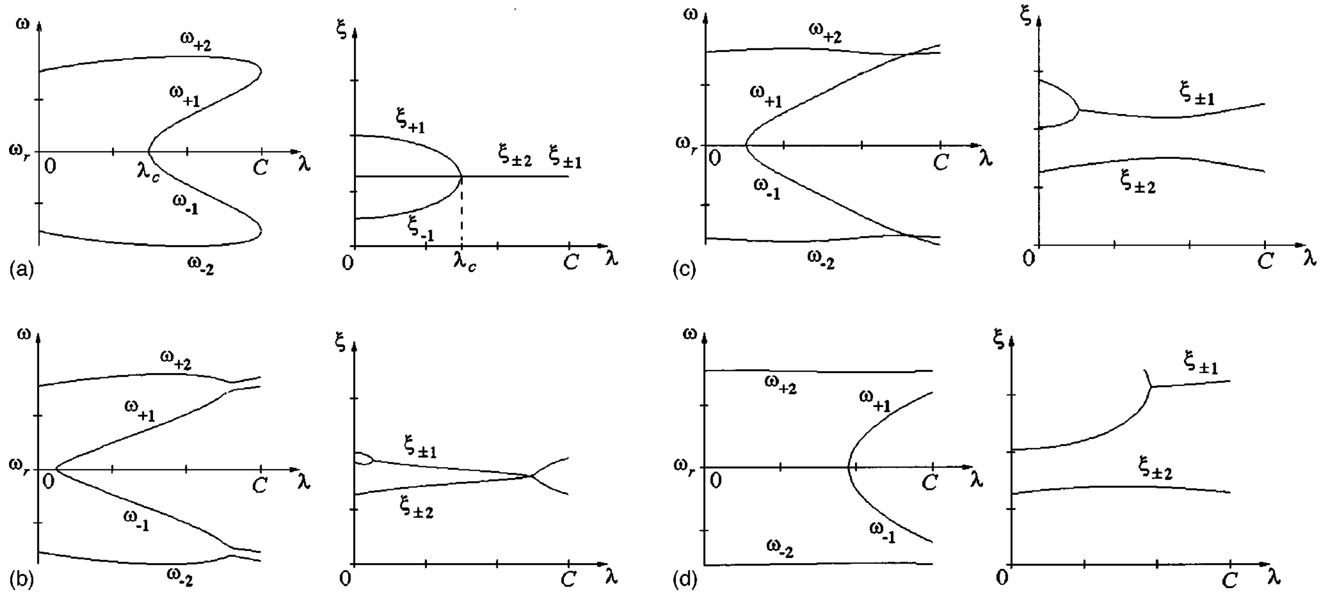


FIG. 4. The dependence on λ of the reconstruction of both the eigenfrequencies ω_i and damping parameters ξ_i at the crossing resonance point $k=k_r$ for $\Gamma_s>\Gamma_u$ ($\Gamma_s/\omega_r=0.2, \Gamma_u/\omega_r=0.05$) for the cases $k_c/k_r=0$ (a), $k_c/k_r=0.133$ (b), $k_c/k_r=0.233$ (c), and $k_c/k_r=0.4$ (d).

creases, and $\lambda_c=0$ when $k_c=0$ [Fig. 3(a)]. The difference between $\Delta\omega_1$ and $\Delta\omega_2$ at the point $\lambda=C$ also disappears at $k_c=0$.

Let us now consider the more general case $\Gamma_s>\Gamma_u$ [Figs. 4(a)–4(d)]. The analytic solutions (23) corresponding to $k_c=0$ are shown in Fig. 4(a). The critical point λ_c is determined from the condition for closing the gap between the solutions ω_{+1} and ω_{-1} (i.e., $\mu_1=\Gamma$):

$$\lambda_c^2=\Gamma(\Gamma+P) \tag{41}$$

or, if the condition $P^2+\lambda^2=C^2$ is satisfied:

$$\lambda_c^2=\frac{1}{2}\Gamma(\Gamma+\sqrt{4C^2-3\Gamma^2}). \tag{42}$$

In contrast to Fig. 3(a), λ_c is not equal to zero at $k_c=0$ now, because $\Gamma\neq 0$.

The critical point λ_c shifts to smaller values of λ when k_c increases [Fig. 4(b)], until λ_c reaches the zero point. Then λ_c begins to increase with the increase of k_c [Fig. 4(c)]. For values of λ that are close to C the frequencies $\omega_{\pm 1}$ and $\omega_{\pm 2}$ interact with each other [Fig. 4(b)] or cross each other [Fig. 4(c)], and the gap $\Delta\omega_1$ is larger than the gap $\Delta\omega_2$ at $\lambda=C$. With the further increase of k_c the gap $\Delta\omega_1$ becomes smaller than the gap $\Delta\omega_2$ for all values of λ [Fig. 4(d)]. Remarkable changes occur also in the damping of the corresponding oscillations. A finite correlation radius of the inhomogeneities ($k_c\neq 0$) destroys the symmetry in the damping dependences. For large enough k_c the damping parameters $\xi_{\pm 1}$ become larger than $\xi_{\pm 2}$ for the entire region of λ values.

The study of the amplitudes of the induced oscillations that will be carried out below helps us to answer the following question: to what type of oscillations—magnetic or elastic—do these four frequencies belong in the different cases?

IV. SUSCEPTIBILITIES OF THE SYSTEM

The system response to an external excitation can be described by the components of the susceptibility matrix $\hat{\chi}$:

$$\phi=\chi_u\Phi+\chi_{us}\Psi, \quad \psi=\chi_s\Psi+\chi_{su}\Phi. \tag{43}$$

The diagonal components $\chi_{u,s}$ of the matrix describe the direct excitation of oscillations by their own forces, while the off-diagonal components χ_{us} and χ_{su} are responsible for the indirect excitation of an oscillation by a force applied to its partner. The diagonal and off-diagonal susceptibilities are found to be

$$\chi_u\equiv\frac{\langle\phi\rangle}{\Phi}=\frac{\Omega}{D}\left[\omega-\omega_k^s-i\Gamma_s-\frac{\lambda_k^2/4}{\omega-\omega_k^u-i\Gamma_u^*}\right],$$

$$\chi_s\equiv\frac{\langle\psi\rangle}{\Psi}=\frac{\omega_M}{D}\left[\omega-\omega_k^u-i\Gamma_u-\frac{\lambda_k^2/4}{\omega-\omega_k^s-i\Gamma_s^*}\right], \tag{44}$$

$$\chi_{us}\equiv\frac{\langle\phi\rangle}{\Psi}=\frac{iP_k\omega_M}{2D}, \quad \chi_{su}\equiv\frac{\langle\psi\rangle}{\Phi}=\frac{iP_k\Omega}{2D},$$

where D is defined by Eq. (23). The imaginary parts of the susceptibilities which determine the energy absorbed by the corresponding wave field have been studied.

Let us consider the simplest situation where $\Gamma=0$ and $k_c=0$, which corresponds to the spectrum depicted in Fig. 4(a). Both the elastic $\chi_u''(\omega,\lambda)$ and magnetic $\chi_s''(\omega,\lambda)$ susceptibilities have the same form in this case (Fig. 5) if they are normalized to their maximum values at $\lambda=0$. One can see that the largest amplitudes of the excitations correspond to the frequencies $\omega_{\pm 2}$ for the entire region of λ values. However, there are two additional maxima at the frequencies $\omega_{\pm 1}$ which appear at intermediate values of λ (Fig. 6) even in the case of total symmetry between the magnetic and elastic systems.

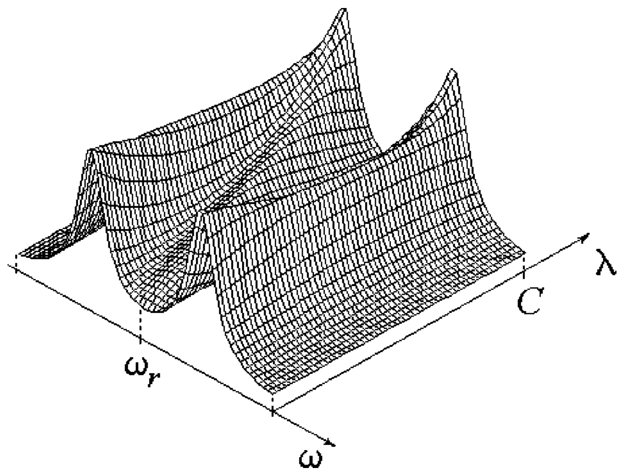


FIG. 5. The magnetic χ_s'' and elastic χ_u'' susceptibilities as functions of both ω and λ at the crossing resonance point for the case $\Gamma=0,k_c=0$. Both susceptibilities are identical if we use the normalizations $\chi_s''(\omega,\lambda)/\chi_{s\,\max}''(\omega,0)$ and $\chi_u''(\omega,\lambda)/\chi_{u\,\max}''(\omega,0)$.

When Γ or k_c is not equal to zero the symmetry between χ_u'' and χ_s'' disappears. In Figs. 7(a) and 7(b), the elastic (a) and magnetic (b) susceptibilities (normalized to their maximum values at $\lambda=0$) are shown for $k_c=0$ but $\Gamma\neq 0$. This corresponds to the spectrum in Fig. 4(a). One can see that intense elastic excitations appear not only at the frequencies $\omega_{\pm 2}$ but also at the frequencies $\omega_{\pm 1}$. The amplitudes of the latter excitations grow when λ increases, while the amplitudes of the excitations at the frequencies $\omega_{\pm 2}$ decrease. The magnetic susceptibility has a quite different dependence on λ: its maxima correspond to the frequencies $\omega_{\pm 2}$ in the entire region of λ values.

In Figs. 8(a) and 8(b), the elastic (a) and magnetic (b) susceptibilities are shown for $\Gamma=0$ but $k_c\neq 0$. This picture is the opposite in some ways to that in Fig. 7. Now the amplitudes of the magnetic excitations at the frequencies $\omega_{\pm 1}$ increase with the increase of λ. The largest amplitudes of the elastic excitations correspond to the frequencies $\omega_{\pm 2}$ in the entire region of λ values.

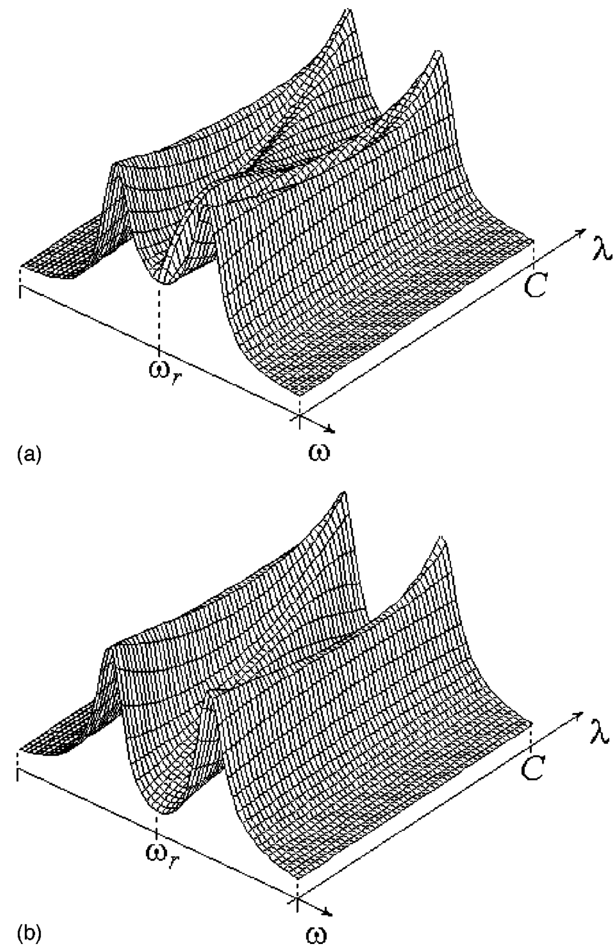


FIG. 7. The elastic (a) and magnetic (b) susceptibilities for $\Gamma\neq 0,k_c=0$. The normalization is as in Fig. 5.

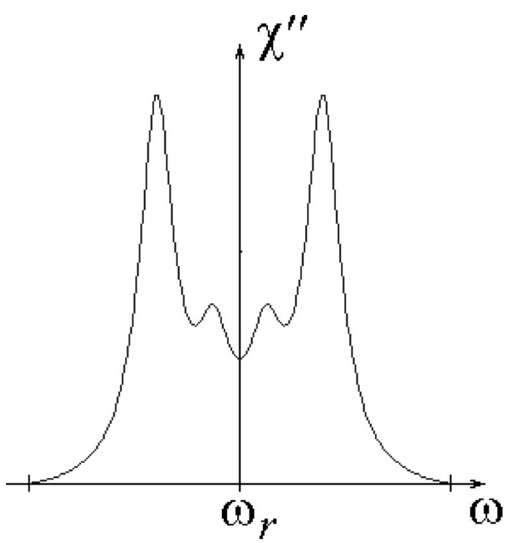
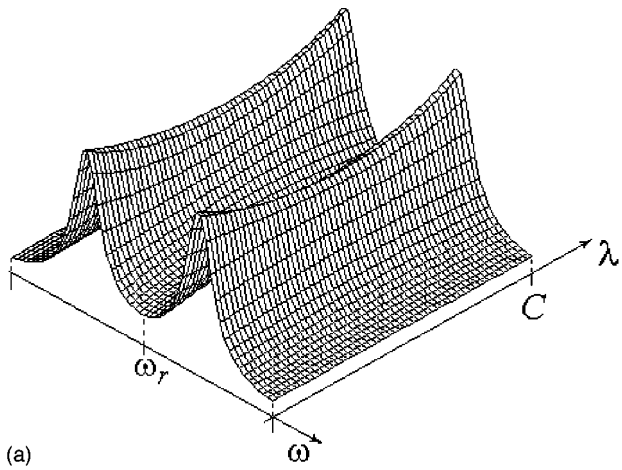


FIG. 6. The cross section of Fig. 5 at $\lambda=0.4C$.

V. CONCLUSION

In this paper we have studied the spectrum of two coupled waves with a random coupling parameter. The primary goal of the study was to examine how the general characteristics of the spectrum evolve from the case of a completely random coupling, with the mean value P of the coupling parameter being equal to zero, to the ordered situation where the rms deviation of the coupling parameter λ is equal to zero. In order to keep the total strength of the coupling unchanged, we subjected the characteristics of the coupling to the constraint $P^2+\lambda^2=C^2$, where C was kept constant. As a particular example, we considered the magnetoelastic resonance in ferromagnets, but the main results of the paper remain valid for any type of crossing resonance.

It is well known that in a homogeneous medium ($\lambda=0,P=C$), coupled magnetoelastic oscillations at the point of the crossing resonance are characterized by two eigenfrequencies $\omega_\pm$ separated by the gap $\Delta\omega=\omega_+-\omega_-$, which is proportional to P. In the opposite limit of the DICR model ($\lambda=C,P=0$), averaged spin and elastic waves are not coupled. The splitting of the dispersion curves of each of the averaged waves, however, appears due to the interaction of the averaged wave of one nature with fluctuation waves of the other nature. The averaged magnetic and elastic oscillations are characterized by the frequencies $\omega_{\pm s}$ and $\omega_{\pm u}$ correspondingly, and the values of the gaps $\Delta\omega_s=\omega_{+s}-\omega_{-s}$



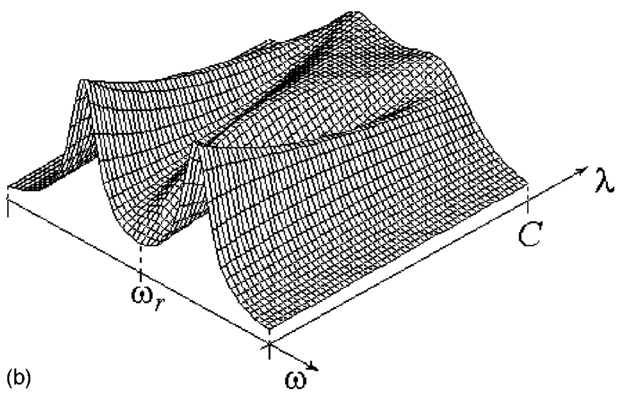


FIG. 8. The elastic (a) and magnetic (b) susceptibilities for $\Gamma=0,k_c\neq 0$. The normalization is as in Fig. 5.

and $\Delta\omega_u=\omega_{+u}-\omega_{-u}$ are, in this case, different.

One would expect that the transition from these four frequencies in the DICR model to the two frequencies in a homogeneous medium will be going as follows. The frequencies of magnetic oscillations ω_{+s} and elastic oscillations ω_{+u} approach each other with the increase P, and merge together at $\lambda=0$ $(P=C)$ into the frequency ω_+ of coupled magnetoelastic oscillations; simultaneously, the frequencies ω_{-s} and ω_{-u} also approach each other merging at $\lambda=0$ into the frequency ω_- of coupled magnetoelastic oscillations.

In reality, however, a nontrivial reconstruction of the spectrum takes place. The frequencies that correspond at $P=0$ to the oscillations of one physical nature (in our case to the elastic oscillations $\omega_{\pm u}$) continuously transform into the frequencies of the coupled magnetoelastic oscillations $\omega_\pm$ when P increases and λ decreases. At the same time the frequencies that correspond at $P=0$ to the magnetic oscillations $\omega_{\pm s}$ approach each other and merge together at some critical value $\lambda=\lambda_c$. The value of λ_c is determined by the parameters of the magnetic and elastic dampings.

When $\lambda\neq 0$ and $P\neq 0$ the four eigenfrequencies can be found in the system (for $\lambda<\lambda_c$ two of them coincide with each other). The distribution of amplitudes of the averaged magnetic and elastic oscillations between these frequencies changes in a complicated manner when the relation between λ and P changes. The study of the susceptibilities of the system shows that, depending on the value of the ratio λ/P, each of these frequencies can correspond to the oscillations with the main contribution from one or the other of the participating wave fields.

The properties of crossing resonances in random materials can be understood in terms of the concept of two effective media in the same material introduced in Ref. 14 for the DICR model $(P=0,\lambda\neq 0)$. In the general case of $P\neq 0,\lambda\neq 0$, the concept of two effective media can be developed as follows. ‘‘Initial’’ averaged spin waves propagate in the effective magnetic medium whose properties are modified by the interaction with scattered elastic waves. The properties of the averaged spin waves depend on the parameters Γ_s, Γ_u^*, and λ. The ‘‘initial’’ averaged elastic waves propagate in the effective elastic medium whose properties are modified by the interaction with scattered spin waves. The properties of the averaged elastic waves depend on the parameters Γ_u, Γ_s^*, and λ. In the general case there is a homogeneous coupling P between the effective magnetic and effective elastic media. The coupled averaged magnetoelastic waves propagate in these coupled effective media. In the case $P=0,\lambda\neq 0$ (DICR model), the effective magnetic and elastic media become independent. In the opposite case, when $\lambda\to 0,P\neq 0$, these coupled effective media transform into the regular magnetoelastic medium.

Possibilities for experimental observation of the disorder-induced crossing resonance were considered in Ref. 9 for the magnetoelastic resonance and in Ref. 12 for polaritons. It was found that special ferromagnet alloys with particularly long-wave inhomogeneities must be created for such experiments. At the same time the estimates for polaritons turned out to be more optimistic owing to much stronger interaction between electromagnetic waves and phonons in ionic crystals. Allowing for a nonzero mean value of the interaction relaxes the conditions for the observation of the crossing resonances in disordered materials. The attenuation of waves increases with the increase of λ. Because of this it is most difficult to satisfy the conditions of the observation of the open gap in the spectrum and two maxima in the high-frequency susceptibility in the case $P=0$, corresponding to the DICR model. If these conditions are satisfied for this case the observation of the changes of the frequencies and susceptibilities that have been obtained in this paper is possible for all relations between λ and P.

The results presented in this paper were obtained with the use of the Bourret approximation.[15,16] This approximation gives reliable results when the effects due to disorder are small enough, and when the function $\rho(\mathbf{x})$ is close enough to the dichotomic random process. The first condition can be easily evaluated in any concrete situation. At the same time, the study of the permissible departures of $\rho(\mathbf{x})$ from the dichotomic random function, which do not change significantly the results of the present paper, is an independent research problem, which is outside the scope of this paper.

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