Hall-Like Effect Induced by Spin-Orbit Interaction

E. N. Bulgakov,[1] K. N. Pichugin,[1,2] A. F. Sadreev,[1,2] P. Středa,[3] and P. Šeba[3]
[1]*Kirensky Institute of Physics, 660036, Krasnoyarsk, Russia*
[2]*Åbo Akademi, Institutionen för Fysik, Department of Physics, SF-20500, Åbo, Finland*
[3]*Institute of Physics, Academy of Sciences of the Czech Republic, Cukrovarnická 10, 162 53 Praha, Czech Republic*
(Received 25 February 1999)

We study the effect of spin-orbit interaction on the electron-transport properties of a cross-junction structure. It results in spin polarization of left and right outgoing electron waves. Consequently, the incoming electron wave of a certain polarization induces a voltage drop perpendicular to the direct current flow between the source and drain of the four-terminal cross structure investigated. The resulting Hall-like resistance is estimated to be of the order of 10^{-3}–$10^{-2} h/e^2$ for technologically feasible structures. The effect becomes more pronounced in the vicinity of resonances where the Hall-like resistance changes its sign as a function of the Fermi energy.

PACS numbers: 73.20.Dx, 71.70.Ej, 72.10.–d

The spin-orbit interaction has a polarization effect on particle scattering processes [1]. It is well known that an unpolarized beam of nucleons scattered by a zero-spin nuclei becomes polarized. On the other side, the polarized incident beam results in azimuthal asymmetry of the scattering process. These effects were observed a long time ago in Stern-Gerlach experiments. Similar effects might be expected for electron scattering processes in microstructures which can be viewed as electron waveguides.

Influence of the spin-orbit interaction on the electron transport properties of mesoscopic systems has attracted attention of physicists since the early 1980's. At that time it was found that it is responsible for so-called antilocalization effect [2]. Later, spin-orbit interaction in devices of the Aharonov-Bohm geometry was systematically studied. In one-dimensional rings it affects the sign of the persistent currents [3,4] and leads to a topological spin phase [5]. These effects originate in a spin-orbit coupling term which is linear in momentum $\vec{p}$

$$\frac{1}{2m^2}\,\sigma \times \vec{\nabla} V(\vec{r}) \cdot \vec{p} \approx \frac{\hbar}{2} \sum_{\mu,\nu} \sigma_\mu \beta_{\mu,\nu} p_\nu \,, \qquad (1)$$

where σ_μ denotes Pauli matrices, $V(\vec{r})$ is a background potential, and $\beta_{\mu,\nu}$ represents a coupling strength. This term is responsible for spin-orbit splitting of electron states at $p \neq 0$. Just recently, splitting of Aharonov-Bohm oscillations caused by a strong spin-orbit coupling has been reported [6].

In semiconductor-based devices there are two main contributions to the spin-orbit coupling [7,8]. One of them arises due to the absence of an inversion center in the bulk $A_{\rm III}B_{\rm V}$ material, from which devices are usually fabricated, resulting in k-odd terms in the Hamiltonian of 3D electrons. The second contribution originates in a low spatial symmetry of the confining potential caused by asymmetry of the space charge distribution. Lifting of the spin degeneracy in zero magnetic field has already been experimentally verified for two-dimensional electron systems in different semiconductor structures [9–11]. In all cases the found spin-orbit coupling constant $\hbar^2\beta$ has been of the order of a few mV $\cdot$ nm.

Anomalous resistance due to asymmetry of elastic scattering processes induced by spin-orbit interaction might be expected for the cross-junction device sketched in Fig. 1. Transmission probabilities between perpendicular arms of the device should differ for spin-up and spin-down states of incident electrons. Consequently, a polarized incident electron beam may lead to a Hall-like effect in the absence of an external magnetic field.

We will assume the cross-junction device fabricated from a semiconductor heterostructure with a two-dimensional electron gas. The model Hamiltonian of such systems is usually assumed to be of the following form [5,12]:

$$H = \frac{p_x^2 + p_y^2}{2m^*} + \hbar\beta[\sigma_x p_y - \sigma_y p_x] + V(x,y)\,, \qquad (2)$$

where potential $V(x,y)$ represents hard-wall conditions at the device boundary, i.e., it is zero inside and infinite outside of the cross-junction area. The coupling strength

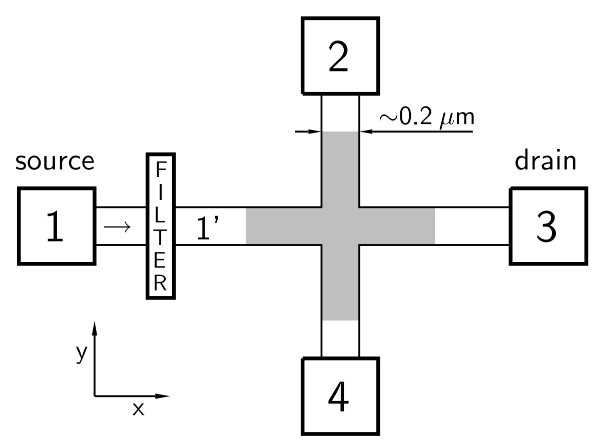

FIG. 1. The cross-junction device. Spin-orbit coupling is supposed to be nonzero in the shadowed area only.

 0031-9007/99/83(2)/376(4)$15.00

β represents an effective electric field along the $\hat{z}$ direction given by the form of the confining potential and absence of an inversion center.

The Hamiltonian H is invariant under time reversal represented by the operator $\hat{T} = i\sigma_y K$ with K being the operator of complex conjugation. The spin matrix $i\sigma_y$ acting upon the wave function of a state with well-defined value of the z component of the spin, s_z, changes the value of the z component of the spin to its opposite, $-s_z$ [1].

For the symmetrical cross structure described by the Hamiltonian H there is additional inversion symmetry related to transformation $x \rightarrow -x$ and $y \rightarrow -y$ represented by the operators $\hat{P}_x$ and $\hat{P}_y$, respectively. The Hamiltonian H, Eq. (2), commutes with operators $\sigma_x \hat{P}_x$ and $\sigma_y \hat{P}_y$, and transformed eigenfunctions

$$\begin{aligned} \psi'(x,y) &= \sigma_y P_y \psi(x,y)\,, \\ \psi'(x,y) &= \sigma_x P_x \psi(x,y) \end{aligned} \tag{3}$$

are thus eigenfunctions of the same Hamiltonian as well.

Current and voltage contacts are modeled by huge electron reservoirs with negligible spin-orbit interaction. To simplify scattering boundary condition we have placed ideal leads with vanishing spin-orbit coupling, $\beta \equiv 0$, between electron reservoirs and studied cross-junction, as sketched in Fig. 1. In these asymptotic regions electron wave functions can be expressed as a linear combination of eigenfunctions of the straight infinite lead at a given energy E. For each subband n they have the form of a plane wave, e.g., for leads connecting reservoirs 1 and 3 we have

$$\begin{aligned} \psi(x,y) &= \sqrt{\frac{1}{\pi w}}\, e^{\pm i k_n x} \sin \frac{\pi n y}{w}\, \chi(s_z)\,, \\ \chi(s_z) &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \end{aligned} \tag{4}$$

where w denotes the lead width and $k_n^2 = 2m^* E/\hbar^2 - \pi^2 n^2/w^2$. Each spin state $s_z = \pm\frac{1}{2}$, within a given subband n is forming its own quantum channel.

Electron transport properties of a quantum device allowing elastic scattering only are fully determined by transition probabilities $t_{i,j}(n, s_z \rightarrow m, s_z')$ representing electron transition of the wave k_n with spin s_z approaching crossing via ith lead into an outgoing channel state (k_m, s_z') within the lead j. The symmetry properties of the Hamiltonian H discussed above imply the following useful identities:

$$\begin{aligned} t_{1,2}(n, s_z \rightarrow m, s_z') &= t_{1,4}(n, -s_z \rightarrow m, -s_z')\,, \\ t_{1,2}(n, s_z \rightarrow m, -s_z) &= t_{1,2}(n, -s_z \rightarrow m, s_z)\,, \\ t_{1,3}(n, s_z \rightarrow m, s_z') &= t_{1,3}(n, -s_z \rightarrow m, -s_z') \end{aligned} \tag{5}$$

that remain valid forcyclic interchange of the lead numbering.

To obtain transition probabilities the following coupled equations for electron eigenfunctions have to be solved:

$$\begin{aligned} \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \varepsilon u_1 + i\alpha \frac{\partial u_2}{\partial y} - \alpha \frac{\partial u_2}{\partial x} &= 0\,, \\ \frac{\partial u_2^2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + \varepsilon u_2 + i\alpha \frac{\partial u_1}{\partial y} + \alpha \frac{\partial u_1}{\partial x} &= 0\,, \end{aligned} \tag{6}$$

together with scattering boundary conditions discussed above. Here we have introduced the following notations:

$$\psi \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \qquad \varepsilon = \frac{2m^* w^2 E}{\hbar^2}, \qquad \alpha = 2m^* \beta w\,.$$

The value $\alpha = 1$ has been chosen for numerical calculation. It represents an InAs structure ($m^* = 0.023 m_0$) of the lead width $w \approx 0.2\ \mu$m and with the spin-orbit coupling constant $\hbar^2 \beta \sim 6 \times 10^{-3}$ eV $\cdot$ nm [12]. Numerical results have been obtained by using a similar procedure as that already described by Ando [13].

To describe scattering asymmetry for a more general case of several subbands it is useful to introduce partial transmission coefficients $T_{i\uparrow,j\uparrow}$, $T_{i\uparrow,j\downarrow}$, $T_{i\downarrow,j\uparrow}$, and $T_{i\downarrow,j\downarrow}$ representing scattering of the fully polarized wave along the $\hat{z}$ direction ($s_z = \frac{1}{2}$ or $-\frac{1}{2}$) into outgoing channels of one particular spin orientation. They are given as the sum of transition probabilities, Eqs. (5), over relevant channels, well defined in asymptotic lead-regions. Obtained spin-depend coefficients representing electron transitions from the lead 1 into the left arm of the cross junction are shown in Fig. 2. Partial transmission coefficients representing right turn have the same energy dependence and are related to those describing left turn as follows:

$$\begin{aligned} T_{1\uparrow,2\uparrow} &= T_{1\downarrow,4\downarrow}, \qquad T_{1\downarrow,2\downarrow} = T_{1\uparrow,4\uparrow}\,, \\ T_{1\uparrow,2\downarrow} &= T_{1\downarrow,4\uparrow} = T_{1\downarrow,2\uparrow} = T_{1\uparrow,4\downarrow}\,. \end{aligned} \tag{7}$$

These identities are a direct consequence of the symmetry of transition probabilities, Eq. (5).

It is natural to suppose that reservoirs act as black bodies and that they are emitting and absorbing electrons independently on their spin orientation. It implies that the

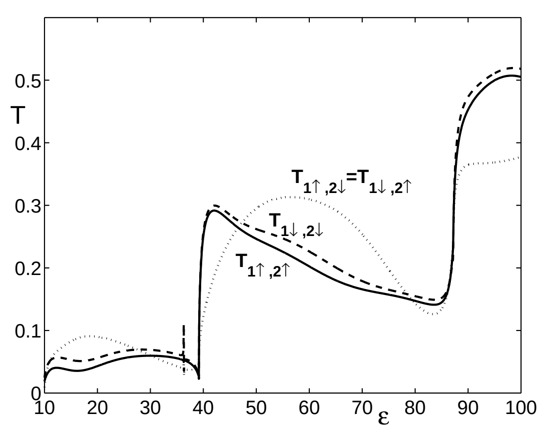

FIG. 2. Energy dependence of the partial transmission coefficients $T_{1\uparrow,2\uparrow}$, $T_{1\uparrow,2\downarrow}$, $T_{1\downarrow,2\uparrow}$, and $T_{1\downarrow,2\downarrow}$ describing transition of polarized electron wave incoming from the lead 1 into the spin-up and spin-down channels of the lead 2.

incoming wave should be considered as an unpolarized wave. Nevertheless, even in this case left and right outgoing waves can be partially polarized since the transmission into spin-up channels ($T_{1\uparrow,2\uparrow} + T_{1\downarrow,2\uparrow}$) differs from that into spin-down channels ($T_{1\uparrow,2\downarrow} + T_{1\downarrow,2\downarrow}$), as seen in Fig. 2.

Asymmetry of the scattering process also leads to the tendency of injected electrons to prefer a left or right turn at the crossing in the dependence on their spin orientation. To study this effect we have evaluated scattering coefficients $T^{(\uparrow)}$ and $T^{(\downarrow)}$ for the case of fully polarized incoming waves defined as follows:

$$\begin{aligned} T_L^{(\uparrow)} &\equiv T_{1\uparrow,2\uparrow} + T_{1\uparrow,2\downarrow} = T_{1\downarrow,4\uparrow} + T_{1\downarrow,4\downarrow} = T_R^{(\downarrow)}, \\ T_S^{(\uparrow)} &\equiv T_{1\uparrow,3\uparrow} + T_{1\uparrow,3\downarrow} = T_{1\downarrow,3\uparrow} + T_{1\downarrow,3\downarrow} = T_S^{(\downarrow)}, \\ T_R^{(\uparrow)} &\equiv T_{1\uparrow,4\uparrow} + T_{1\uparrow,4\downarrow} = T_{1\downarrow,2\uparrow} + T_{1\downarrow,2\downarrow} = T_L^{(\downarrow)}, \\ R^{(\uparrow)} &\equiv R_{i\uparrow,i\uparrow} + R_{i\uparrow,i\downarrow} = R_{i\downarrow,i\uparrow} + R_{i\downarrow,i\downarrow} = R^{(\downarrow)}. \end{aligned} \tag{8}$$

Their energy dependence is shown in Fig. 3. Other sets of identities can be obtained by cyclic interchange of the lead numbering.

The expected tendency of electrons with one particular spin orientation to prefer a left or right turn is evident. Exceptions have been found in the vicinity of subband edges at energies $\varepsilon_n = \pi^2 n^2$. A sharp peak in the transmission probabilities also appears at the energy of the second bound state ($\varepsilon_b \approx 36.72$) formed in cross structures [14]. It originates in a mixing of bound and transport states caused by spin-orbit interaction. It is a similar effect as that induced by radiation field [15].

Under particular conditions the discussed spin dependent scattering could lead to a Hall-like effect. Current flow J applied along $\hat{x}$ direction, i.e., from a source 1 to a drain 3, could not only induce a voltage drop between source and drain, $U_\parallel \equiv U_1 - U_3$, but there might also appear a voltage drop in the perpendicular direction, between voltage contacts 2 and 4, $U_\perp \equiv U_2 - U_4$. Their relation can be expressed with the help of reflection coefficients $\mathcal{R}_{ii}$ and transmission coefficients $\mathcal{T}_{ij}$ representing electron transition from the contact i to the contact j [16]. For the considered four terminal device we get

$$U_\perp = \frac{\mathcal{T}_{12}\mathcal{T}_{34} - \mathcal{T}_{14}\mathcal{T}_{32}}{(2N - \mathcal{R}_{22})(2N - \mathcal{R}_{44}) - \mathcal{T}_{24}\mathcal{T}_{42}} U_\parallel, \tag{9}$$

where N denotes the number of subbands at a given energy. To get a nonzero value of $U_\perp$ it is necessary to inject a polarized electron wave into the cross-junction device. To ensure it, let us place a filter into the lead 1 which is assumed to be fully transparent for spin-up electrons, $s_z = \frac{1}{2}$. Spin-down electrons are supposed to be reflected by the filter. Only injected spin-up electrons are thus allowed to reach the region between filter and crossing denoted in Fig. 1 as $1'$. Those spin-up electrons that are reflected back by crossing into a spin-down channel are not allowed to leave region $1'$ immediately. They are reflected by filter and can try to escape from region $1'$ again. The followed multiple-reflection process is controlled by reflection coefficient $R_{1\downarrow,1\downarrow}$. For transmission coefficients entering the numerator of the right-hand side of Eq. (9) we get

$$\begin{aligned} \mathcal{T}_{12} &= T_L^{(\uparrow)} + T_L^{(\downarrow)} - \gamma_1 T_L^{(\downarrow)}, \\ \mathcal{T}_{14} &= T_R^{(\uparrow)} + T_R^{(\downarrow)} - \gamma_1 T_R^{(\downarrow)}, \\ \mathcal{T}_{32} &= T_R^{(\uparrow)} + T_R^{(\downarrow)} + \gamma_3 T_L^{(\downarrow)}, \\ \mathcal{T}_{34} &= T_L^{(\uparrow)} + T_L^{(\downarrow)} + \gamma_3 T_R^{(\downarrow)}. \end{aligned} \tag{10}$$

Coefficients γ_i represent the effect of the filter, and they have the following form:

$$\gamma_1 = \frac{T_R^{(\downarrow)} + T_S^{(\downarrow)} + T_L^{(\downarrow)}}{N - R_{1\downarrow,1\downarrow}}; \qquad \gamma_3 = \frac{T_{3\uparrow,1\downarrow} + T_{3\downarrow,1\downarrow}}{N - R_{1\downarrow,1\downarrow}}. \tag{11}$$

For simplicity, we have assumed that there is no spin-flip process associated with reflections at the filter boundary. We have also neglected any interference effects due to multiple scattering processes in the region $1'$ between filter and crossing assuming an inelastic equilibration process in the filter vicinity leading to equal occupation of spin-down channels.

Inserting expressions for scattering coefficients $\mathcal{R}_{ii}$ and $\mathcal{T}_{ij}$ into Eq. (9) and making use of the symmetry relations, Eqs. (7) and (8), we get

$$U_\perp = \frac{\frac{1}{4}(T_L^{(\uparrow)} - T_R^{(\uparrow)})}{N - R_{1\downarrow,1\downarrow} - \frac{1}{2}\frac{T_S^{(\uparrow)}(T_R^{(\uparrow)} + T_L^{(\uparrow)}) + 2T_R^{(\uparrow)}T_L^{(\uparrow)}}{T_R^{(\uparrow)} + 2T_S^{(\uparrow)} + T_L^{(\uparrow)}}} U_\parallel. \tag{12}$$

The voltage $U_\perp$ is proportional to the difference $T_L^{(\uparrow)} - T_R^{(\uparrow)} \equiv T_R^{(\downarrow)} - T_L^{(\downarrow)}$ similarly as in the case of the standard Hall resistance [16]. Because of the spin-orbit coupling this difference might be nonzero as can be seen in Fig. 3. Without the presence of the polarization filter, no perpendicular voltage drop arises as can be easily shown by assigning zero values to the coefficients γ_i. To obtain

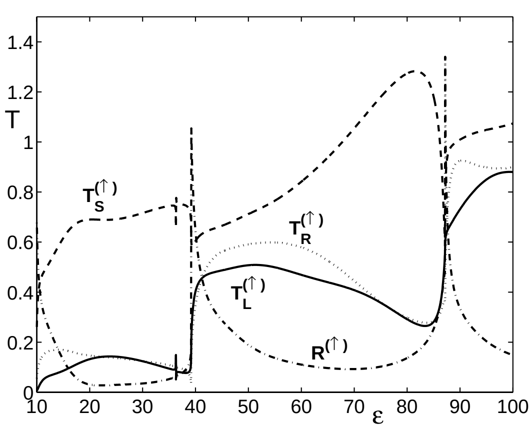

FIG. 3. Scattering coefficients $T_L^{(\uparrow)}$, $T_S^{(\uparrow)}$, $T_R^{(\uparrow)}$, and $R^{(\uparrow)}$ for spin-up polarized electron wave injected from the source (lead 1) as a function of the energy ε.

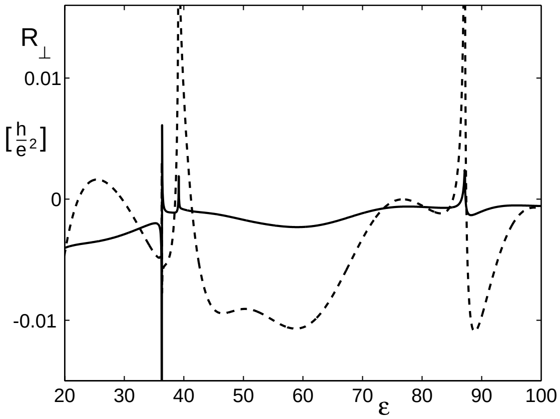

FIG. 4. Hall-like resistance induced by polarization filter transparent for electron waves polarized along the $\hat{z}$ direction (full line) and along the $\hat{x}$ direction (dotted line) as a function of the energy.

Hall-like resistance $R_{\perp} = U_{\perp}/J$ all other scattering coefficients have to be evaluated. The numerical results for the parameters already used to evaluate partial transmission coefficients are shown in Fig. 4.

Two other types of polarization filters have also been considered. We have found that no Hall-like resistance appears if polarization filter is transparent for waves polarized along the $\hat{y}$ direction. On the other side, the filter which is transparent for electron waves polarized along the current direction, $\hat{x}$ direction, Hall-like resistance becomes even larger as shown in Fig. 4.

In the vicinity of bound states and subband edges the Hall-like effect becomes stronger and changes its sign. It indicates that there appear circulating currents in the crossing region changing their orientation with energy. Their origin in the vicinity of the second bound state with energy $\varepsilon_b \approx 36.72$ is understandable. Because of spin-orbit coupling this state is split into two states with opposite orbital momentum similarly as in devices of the Aharonov-Bohm geometry [3,4]. Splitting of subband edges has a similar effect. However, for most energies, perpendicular voltage appears due to deformed current lines within cross junction only. Results of the model calculation slightly depend on the position of the boundaries between regions with turn-on and turn-off spin-orbit coupling. However, no qualitative changes have been observed.

The obtained values of the Hall-like resistance are measurable. However, available real cross junctions are of larger dimensions than that used in our calculation. For this reason we have calculated $R_{\perp}$ until energies ε four times larger than those presented in Fig. 4. As expected, with an increasing number of channels the effect decreases but $R_{\perp}$ is still of the order of $10^{-3}h/e^2$. Polarization filters might be realized by a locally applied magnetic field across the cross-junction arm, e.g., making use of a ferromagnetic top layer. Also a ferromagnetic injector [17] can be used to induce Hall-like voltage. Polarization effects in real systems will be hardly so effective as supposed in our model calculation. Especially possible spin-flip processes in the vicinity of spin reflecting boundaries or due to imperfections within device leads would partially suppress the described Hall-like effect. Nevertheless, the asymmetry induced by spin-orbit coupling was verified a long time ago in particle scattering experiments and we believe that the discussed effects will be ones observed in nanostructures as well.

This work was supported by the Grant Agency of the Czech Republic under Grant No. 202/98/0085 and by the Academy of Sciences of the Czech Republic under Grant No. A1048804. It has been also partially supported by the INTAS-RFBR Grant No. 95-IN-RU-657, RFFI Grant No. 97-02-16305, Krasnoyarsk Science Foundation Grant No. 7F0130, and by the "Foundation for Theoretical Physics" in Slemeno, Czech Republic. The authors acknowledge stimulating discussions with Pavel Exner.

[1] A. S. Davydov, *Quantum Mechanics* (Pergamon Press, Oxford, 1965), Chap. XI.
[2] S. Hikami, A. I. Larkin, and Y. Nagaoka, Progr. Theor. Phys. **63**, 707 (1980).
[3] Y. Meir, Y. Gefen, and O. Entin-Wohlman, Phys. Rev. Lett. **63**, 798 (1989).
[4] O. Entin-Wohlman, Y. Gefen, Y. Meir, and Y. Oreg, Phys. Rev. B **45**, 11 890 (1992).
[5] A. G. Aronov and Y. B. Lyanda-Geller, Phys. Rev. Lett. **70**, 343 (1993).
[6] A. F. Morpurgo, J. P. Heida, T. M. Kwapwijk, B. J. van Wees, and G. Borghs, Phys. Rev. Lett. **80**, 1050 (1998).
[7] Yu. A. Bychkov and E. I. Rashba, JETP Lett. **39**, 78 (1984).
[8] E. I. Rashba and E. Ya. Sherman, Phys. Lett. A **129**, 175 (1988).
[9] B. Das, D. C. Miller, S. Datta, R. Reifenberger, W. P. Hong, P. K. Bhattacharya, J. Singh, and M. Jaffe, Phys. Rev. B **39**, 1411 (1989).
[10] S. I. Dorozhkin, Phys. Rev. B **41**, 3235 (1990).
[11] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78**, 1335 (1997).
[12] Y. B. Lyanda-Geller, Phys. Rev. Lett. **71**, 657 (1993).
[13] T. Ando, Phys. Rev. B **44**, 8017 (1991).
[14] R. L. Schult, H. W. Wyld, and D. G. Ravenhall, Phys. Rev. B **41**, 12 760 (1990).
[15] E. N. Bulgakov and A. F. Sadreev, JETP Lett. **66**, 431 (1997).
[16] M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).
[17] S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1989).