Effect of conduction electrons on the law of approach to saturation of a metallic ferromagnet with surface pinning of the magnetic moment

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The effect of conduction electrons on the magnetization curve of a metallic ferromagnet with surface pinning of the magnetic moment is investigated theoretically. The electronic contribution is due to the rearrangement of the discrete spectrum of charge carriers trapped by the nonuniform magnetic induction of such a ferromagnet, and it is a kind of diamagnetic effect that appreciably decreases the volume-averaged magnetization of the ferromagnet. A power-law dependence $H^{-3/4}$ on the external magnetic field H is obtained according to the law of magnetization approach to saturation. This dependence is due to the contribution of the conduction electrons. © *1999 American Institute of Physics.* [S1063-7834(99)01804-3]

The effect of surface pinning of the magnetic moment of a ferromagnet on its physical properties continuously attracts investigators. The behavior of the magnetic subsystem of such materials,[1–23] specifically, the magnetization processes in a ferromagnet, is unique because of the nonuniform distribution of the magnetic moment over the thickness of the sample and is characterized by two main features: first, the displacement of the magnetization curve and — for irreversible magnetization reversal — of the hysteresis loop of the ferromagnet and, second, the law of approach to saturation.

Much less attention has been devoted to the investigation of the effect of surface pinning of the magnetic moment (surface anisotropy) on other subsystems of the ferromagnet. Nuclear magnetic resonance[7] and the electrical properties[24,25] of such materials have been studied. As far as I know, however, the interaction of the magnetic and any other subsystem has never been studied in ferromagnets with surface anisotropy. At the same time such an effect could be substantial, if the high-energy subsystem of the ferromagnet, for example, the electronic subsystem, is modified by surface pinning of the magnetic moment. In the present paper the effect of changes produced in the conduction-electron spectrum by a nonuniform magnetic induction of a ferromagnet with surface pinning of the magnetic moment on the magnetization curve of the ferromagnet is examined.

The restructuring of the discrete spectrum of the electrons in a magnetic field, due to the nonuniformity of the material or the magnetic field itself, is well known. It occurs near the surface of a normal metal (magnetic surface levels),[26] in a domain wall of a metallic ferromagnet,[27] or for electrons trapped by the domain structure of the ferromagnet.[28,29] The magnetic surface levels lead, as is well known,[30,31] to an additional contribution to the thermodynamic potential of a metallic sample and change its diamagnetic properties. In a ferromagnet the conduction electrons localized in a domain wall increase its energy and width.[32] The mechanism of these effects is common to systems with a nonuniform distribution of the magnetic induction. It consists in the fact that, for a magnetic uniformity much smaller in size than the electron cyclotron radius, some electrons are influenced by an effective potential which is much narrower than in the case of a uniform magnetic field. As a result, the electronic subsystem of a metallic ferromagnet acquires an additional energy.

In a ferromagnet with surface pinning of the magnetic moment the size of the magnetic nonuniformity or the width of the transitional region where the magnetic moment vector turns depends on the external magnetic field (see, for example, Refs. 1 and 5), so that the conduction electrons should influence mainly the magnetization curve of the metallic ferromagnet. In the present paper the electronic contribution to the law of magnetization approach to saturation is determined.

1. MAGNETIZATION DISTRIBUTION

Let us consider an isotropic metallic ferromagnet with a magnetic subsystem whose free energy has the form

$$\mathcal{F}_m = \int_V \left[\frac{1}{2} \alpha \left(\frac{\partial \mathbf{M}}{\partial \mathbf{r}} \right)^2 - \mathbf{M} \cdot \mathbf{H} \right] d\mathbf{r}. \tag{1}$$

Here $\mathbf{M}$ is the magnetization vector, $M \equiv |\mathbf{M}|$, V is the volume of the sample, $\mathbf{H}$ is the external magnetic field, and α is the exchange parameter. Let us consider a flat ferromagnetic layer of thickness d, where the magnetization vector $\mathbf{M}$ is pinned on one surface (bottom face in Fig. 1) and free on the other. In such samples, for magnetization in a magnetic field $\mathbf{H}$ directed opposite to the vector $\mathbf{M}$ fixed on the surface, there arises a magnetization distribution that is nonuniform over the thickness of the layer. The solution of the magnetostatic problem with the boundary conditions $M_z|_{y=0} = -M$ and $(\partial M_z/\partial y)|_{y=d} = 0$ has the form[1,2,5]

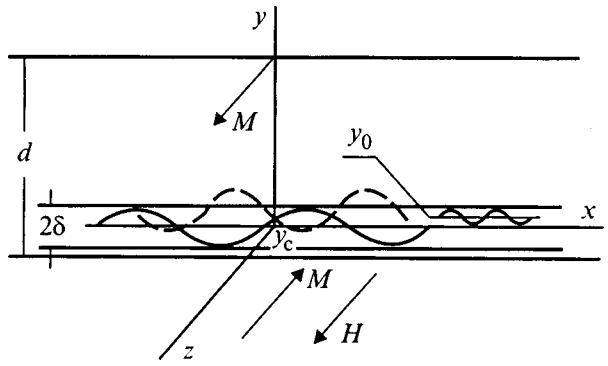

FIG. 1. Geometry of the problem and coordinate system. The wavy lines show the trajectories of the trapped electrons.

$$\frac{M_z}{M}=\begin{cases}-1+2k^2\mathrm{sn}^2[(h/\alpha)^{1/2}y], & h\geqslant h_u,\\ -1, & h<h_u.\end{cases} \tag{2}$$

Here, similarly to Ref. 5, the coordinate y is measured from the bottom face of the sample. The component $M_y=0$, just as in a Bloch domain wall, while M_x is determined by the relation $M_x^2+M_z^2=M^2$. In the expression (2) $h=H/M$, where $H=|\mathbf{H}|$; $h_u=(\pi/2)^2\alpha/d^2$ is the critical field or the field in which the magnetization curve is displaced. It is greatest for $d<10^{-6}$ cm, for which for typical values of α in metallic ferromagnets $\alpha/d^2\geqslant 1$. As d increases, the field h_u appreciably decreases ($h_u\propto d^{-2}$), becoming negligibly small at $d\approx 10^{-3}$ cm (for $\alpha=5\times 10^{-12}$ cm^2 the field $h_u\approx 10^{-5}$). The modulus k of the complete elliptic integral of the first kind $\mathbf{K}(k)$ as a function of the magnetic field can be found from the equation

$$\mathbf{K}^2(k)=hd^2/\alpha\sim h/h_u. \tag{3}$$

The magnetic field corresponding to the saturation region of the magnetization curve is determined in the present model by the condition $h\gg h_u$. In this limit the expression (2) assumes the form

$$\frac{M_z}{M}\approx -1+2\tanh^2\left[\frac{y}{\sqrt{2}\delta}\right]. \tag{4}$$

Here

$$\delta=\sqrt{\frac{\alpha}{2h}}. \tag{5}$$

is the half-width of the transitional region near the surface of the sample $y=0$, the main location where the magnetization vector turns ($\delta\ll d$).

Since **M** depends only on the coordinate y, the average magnetization $\langle M\rangle$ of the layer accompanying magnetization-reversal along the z axis is given by the expression

$$\langle M\rangle=\frac{1}{d}\int_0^d M_z dy. \tag{6}$$

Carrying out the integration with M_z in the form (4) we obtain the law of variation of the magnetization of a ferromagnet in an external magnetic field

$$\frac{\Delta M}{M}=\frac{2\sqrt{2}\delta}{d}\tanh\frac{d}{\sqrt{2}\delta}. \tag{7}$$

Thus, if the exponentially small difference of the hyperbolic tangent in Eq. (7) from 1 is neglected, then one can see that the change in the magnetization $\Delta M\equiv M-\langle M\rangle$ is proportional to the width δ of the transitional region, where the vector **M** turns. In accordance with the definition of δ (5) we obtain for the relative change in the magnetization of a ferromagnet with surface anisotropy the characteristic magnetic field dependence

$$\frac{\Delta M}{M}=\frac{2}{d}\sqrt{\frac{\alpha}{h}}\propto\frac{1}{d\sqrt{H}}. \tag{8}$$

Hence for $d=10^{-3}$ cm and $h=10^{-3}$ follows the estimate $\Delta M/M\approx 0.15$, i.e. for such a sample thickness the deviation of the average magnetization $\langle M\rangle$ from M is quite large (in contrast to h_u), which is due to the inverse proportionality of $\Delta M/M$ to d. In this case the satisfaction of the law (8) becomes the best evidence of the existence of surface pinning of a magnetic moment in a ferromagnetic layer.

If M_z in the form of the exact expression (2) is substituted into Eq. (6), then, as shown in Ref. 11, $\Delta M/M=2\mathbf{E}(k)/\mathbf{K}(k)$, where $\mathbf{E}(k)$ is a complete elliptic integral of the second kind. Hence we obtain once again the expression (8) for $h\gg h_u$. Experimentally, the dependence $\Delta M/M\propto 1/\sqrt{H}$ has been observed, for example, in Ref. 2, where thin ferromagnetic films on a magnetically hard substrate were investigated.

It should be noted that the $1/\sqrt{H}$ dependence in the law of approach of the magnetization to saturation holds in bulk samples, where it is due to the point localization of stresses,[33] and in amorphous ferromagnets with spatial fluctuations of the anisotropy constant.[34] In both cases the coefficient of $1/\sqrt{H}$ is independent of the sample thickness.

2. CONDUCTION ELECTRONS

We shall describe a charged quasiparticle of the electronic subsystem of the ferromagnet by the Hamiltonian

$$\mathcal{H}_s=\varepsilon_s\left(\mathbf{P}-\frac{e}{c}\mathbf{A}(\mathbf{r})\right), \tag{9}$$

where **P** is the canonical momentum of the quasiparticle with charge e in the band s. We assume an arbitrary dispersion law for charge carriers. The vector potential **A** can be expressed in terms of the magnetic induction vector of the ferromagnet $\mathbf{B}=\nabla\times\mathbf{A}$, the coordinate dependence of whose components is determined by the magnetization distribution in the sample ($\mathbf{B}(\mathbf{r})=\mathbf{H}+4\pi\mathbf{M}(\mathbf{r})$). If $\mathbf{B}(\mathbf{r})\equiv\mathbf{B}(y)$, as in our problem, then the vector potential can be written in the form

$$A_x=-\int_0^y B_z(y')dy',\quad A_y=0,\quad A_z=\int_0^y B_x(y')dy'. \tag{10}$$

Hence follows the Landau gauge in the case of a uniformly magnetized sample. For **A** in the form (10) the Hamiltonian $\mathcal{H}_s$ is independent of x and z (cyclic variables). Therefore the

momentum components P_x and P_z of the quasiparticles are conserved in the motion. Carrying out the semiclassical quantization procedure we obtain the condition for quantization of the quasiparticle energy in the form

$$S(P_x,P_z,\mathcal{E})=2\pi\frac{|e|B}{c}\hbar(n+\gamma), \tag{11}$$

where

$$\frac{|e|B}{c}\oint p_y dy=S(P_x,P_z,\mathcal{E}). \tag{12}$$

Here $\mathcal{E}$ is the quasiparticle energy, $B=4\pi M$, $\mathbf{P}=(P_x,p_y,P_z)$, $n=1,2,3,\ldots,0\leqslant\gamma\leqslant 1$, and $\hbar$ is Planck's constant divided by 2π.

A nonuniform induction $\mathbf{B}(y)$ does not affect different groups of charge carriers in the same way. Its nonuniformity has virtually no effect on quasiparticle trajectories far from the transitional region but, when $\delta\ll R$, where R is the average radius of a charge-carrier orbit in the field B, the transitional region is crossed at not too acute an angle. For such carriers $S(P_x,P_z,\mathcal{E})\approx S(P_z,\mathcal{E})$ and their spectrum is virtually identical to that in a uniform magnetic field. The rearrangement of the spectrum is maximal for quasiparticles moving inside the transitional region (Fig. 1). Let us examine their dynamics in greater detail.

For $h\gg h_u$ the projection M_z of the magnetization vector is given by the expression (4). Then B_z vanishes at the point $y=y_c\equiv u_c(\alpha/h)^{1/2}$, where $u_c=\tanh^{-1}(1/2-h/8\pi)^{1/2}$. One can see that this point exists only for $h<4\pi$. If $h\ll 4\pi$, then $y_c\sim\delta$ and, moreover, this approximation makes it possible to set $B\approx 4\pi M$. Only such values of the magnetic field will be considered below.

Taking as the origin on the y axis the point y_c, as shown in Fig. 1, we obtain for the components of the magnetic induction vector taking account of only the first term in the expansion near this point

$$B_x\approx B,\quad B_y=0,\quad B_z\approx By'/\delta, \tag{13}$$

where $y'=y-y_c$. To investigate the electronic subsystem we shall assume that this expansion holds in the entire transitional region $-\delta<y'<\delta$ and that outside this region the induction in the sample is uniform, i.e. according to Fig. 1, $B_z\approx\mp 4\pi M$ below and above the transitional region. For $\delta\ll R$ the function $\varepsilon_s(\mathbf{P}-e\mathbf{A}(\mathbf{r}))/c)$ in Eq. (9) can be expanded in a power series near the point $\mathbf{p}_0=(P_x,p_{y0},P_z)$, where p_{y0} is determined by the equation $v_y=0$. We note that, depending on the orientation and form of the Fermi surface, there may be no such point in the band s or there can be several such points. Let us consider the electronic band where there is one such point. To simplify the calculations we drop the index s. Simple transformations yield

$$\varepsilon(\mathbf{p})=\varepsilon_0+\frac{1}{2m_y}(p_y-p_{y0})^2+\frac{1}{2}m_y\omega_1^2(y'-y_0)^2, \tag{14}$$

where $\mathbf{p}=\mathbf{P}-e\mathbf{A}(\mathbf{r})/c$, $\varepsilon_0=\varepsilon(\mathbf{p}_0)$, $y_0=v_{z0}\delta/v_{x0}$, $|y_0|<\delta$, and v_{x0}, u_{z0}, and m_y are the components of the velocity and the effective mass of the conduction electrons near the point $\mathbf{p}_0$,

$$\omega_1=(eBv_{x0}/c\,\delta m_y)^{1/2}. \tag{15}$$

In what follows we shall be interested in the particles undergoing finite motion in the transitional region. For them ω_1 is real and it is the frequency of the motion in the transitional region. From expression (15) follows $ev_{x0}/m_y>0$, which determines the position of this group of carriers on the isoenergy surface. According to Eq. (14), the cross-sectional area corresponding to quasiparticles in the transitional region of the ferromagnet has the form

$$S(\mathcal{E},P_x,P_z)=2\pi(\mathcal{E}-\varepsilon_0)\sqrt{\frac{m_y eB\delta}{v_{x0}c}}. \tag{16}$$

Hence, using the quantization condition (11), we obtain for the charge-carrier spectrum the expression

$$\mathcal{E}_n(P_x,P_z)=\varepsilon_0+\hbar\left(n+\frac{1}{2}\right)\omega_1. \tag{17}$$

Since the Hamiltonian (14) describes a harmonic oscillator whose semiclassical spectrum is identical to the quantum spectrum, the quantity γ appearing in Eq. (11) is set equal to 1/2.

Equations (17) and (15) apply to the case of a ferromagnet with an arbitrary isoenergy surface the expressions for the electron spectrum in the transitional region, and the electron frequencies obtained previously in Ref. 24 for a spherical Fermi surface, as well as the expressions for the spectrum and frequency of electrons trapped by a domain wall.[27] In the latter case $\delta=\sqrt{\alpha/\beta}$, where β is the uniaxial anisotropy constant.

It is convenient to represent the frequency ω_1 in the form

$$\omega_1=\omega_B\sqrt{R_x/\delta}, \tag{18}$$

where $\omega_B=|eB/m_y c|$, $R_x=m_y v_{x0}c/eB$ is the radius of curvature of the electron trajectory at the point where $v_y=0$ and is determined by the radius of curvature $m_y v_{x0}$ of the isoenergy surface in a plane passing through the p_y axis and the velocity vector $\mathbf{v}$. In the case of a quadratic and isotropic electron dispersion law, R_x is the cyclotron radius of electrons with velocity v_{x0} and ω_B is the cyclotron frequency in the magnetic field B.

In multiband ferromagnets, there exists a set of generally different frequencies ω_{1s}. The index s then enumerates the sections of isoenergy surfaces in bands corresponding to trapped electrons.

In the foregoing discussion we neglected the conduction-electron spin. This is valid if $\hbar v_y/\delta\ll I_{sd}$, which means that the electron spin adiabatically follows the magnetization during the motion of an electron in the transitional layer. In a metallic ferromagnet the s–d exchange integral is $I_{sd}\sim 10^{-13}$ erg. Then the latter inequality can hold for $\delta\sim 10^{-4}$ cm, even if the electron Fermi velocity $v_F\sim 10^8$ cm/s is taken for v_y.

Finally, we note that, using in Eq. (1) the linear approximation (13) for M_z and then determining the parameter δ from the condition that the free energy $\mathcal{F}_m$ is minimum, gives an expression for $\Delta M/M$ that differs from the exact expression (8) by the numerical factor ≈ 0.9. This correspon-

dence of the relation obtained on the basis of an approximate description to the exact result can be explained by the fact that the average magnetization $\langle M \rangle$, being an integral characteristic, is insensitive to the details of the magnetization distribution.

3. THERMODYNAMIC POTENTIAL OF THE ELECTRONIC SUBSYSTEM

We write the thermodynamic potential of the conduction-electron gas in the form

$$\Omega = -2T\frac{L_x L_z}{(2\pi\hbar)^2} \times \sum_n \int_{S>0} dP_x dP_z \ln\left[1+\exp\frac{\zeta-\mathcal{E}_n(P_x,P_z)}{T}\right], \tag{19}$$

where $\mathcal{E}_n(P_x,P_z)$ is determined from the condition (11); L_x and L_z are the dimensions of the sample in the xz plane; and, ζ is the chemical potential. We shall use the Poisson equation to carry out the summation in Eq. (19). We make the substitution

$$dn = \frac{c}{2\pi\hbar eB}\frac{\partial S}{\partial \mathcal{E}}\, d\mathcal{E}, \tag{20}$$

in the integral over n arising in the process, and integrating over $\mathcal{E}$ by parts we obtain

$$\Omega = -2\frac{L_x L_z}{(2\pi\hbar)^3}\int_0^\infty d\mathcal{E} f(\mathcal{E}) \int_0^{\mathcal{E}} d\mathcal{E}' \times \int_{S>0} dP_x dP_z \frac{\partial S(\mathcal{E}',P_z,P_x)}{\partial \mathcal{E}'} \times \left\{1+2\mathrm{Re}\sum_{l=1}^\infty \exp\left[il\frac{c}{eB\hbar}S(\mathcal{E}',P_x,P_z)-i2\pi l\gamma\right]\right\}. \tag{21}$$

Here $f(\mathcal{E})=\{1+\exp[(\mathcal{E}-\zeta)/T]\}^{-1}$ is the Fermi distribution function. The first term in braces in Eq. (21) gives, after integrating, the thermodynamic potential Ω_0 of a free-electron gas, which is independent of the magnetic induction.[35]

We shall integrate the second term in braces in Eq. (21) by the stationary-phase method. Since the oscillation effects are of no interest to us in what follows, we take account of the contributions (21) only of the boundaries of the region of integration. For electrons, which move wholly or mainly outside the transitional region, the function S is essentially independent of P_x: $S(\mathcal{E}',P_z,P_x)\approx S(\mathcal{E}',P_z)$. In this case the peaks of the region of integration over P_z contribute to Eq. (21). The corresponding term Ω_L in the thermodynamic potential Ω describes, just as in a uniformly magnetized sample, the diamagnetism of the electron gas.[35] For us, however, the contribution of trapped electrons to the thermodynamic potential is of greatest interest. This contribution determines the peak region of the integration over P_x in Eq. (21), which we denote by $P_m(\mathcal{E},P_z)$. Here it is important that, as P_x approaches P_m, the spectrum of any trapped electrons is described by Eq. (17). For example, for the trajectory shown by the dashed line in Fig. 1, the limiting trajectory is the trajectory shown by the wavy line near $y_0\neq 0$. Thus, using Eq. (16) for S and performing the standard calculations,[35–37] we obtain for the thermodynamic potential of the electron gas in a ferromagnet with surface pinning of the magnetic moment

$$\Omega = \Omega_0 + \Omega_L + \frac{L_x L_z}{\sqrt{\delta}}\sum_s G_s, \tag{22}$$

where

$$G_s = \frac{1}{24(2\pi)^2\hbar}\int_0^\infty d\mathcal{E}\int_{y_0<\delta} dP_z \frac{f(\mathcal{E})}{\sqrt{R_{xs}(\mathcal{E},P_z,P_{ms})}}. \tag{23}$$

Here the radius of curvature R_x of the electronic trajectory is taken at the point $P_x=P_m$. The potential Ω_L in Eq. (22), generally speaking, has a factor $1-\ \mathrm{const}(\delta/d)\sqrt{\delta/R}$, where the quantity being subtracted is proportional to the number of trapped electrons. However, the δ-dependent part of Ω_L is several orders of magnitude smaller than the last term in Eq. (22), so that it can be ignored.

In summary, the part of the thermodynamic potential that depends on the width of the transitional region is inversely proportional to $\sqrt{\delta}$, i.e. the free energy of the electronic subsystem will decrease as δ increases. This behavior of the thermodynamic quantities is due to the fact that the spacing between the energy levels in the electron spectrum (17) in the transitional region will decrease with increasing δ.

Given the dispersion law of the conduction electrons, G_s can be calculated. Here we estimate this parameter making the assumption that the ferromagnet is a single-band magnet with an ellipsoidal isoenergy surface

$$\varepsilon(\mathbf{p}) = \frac{(P_x-e/cA_x)^2}{2m_x}+\frac{p_y^2}{2m_y}+\frac{(P_z-e/cA_z)^2}{2m_z}. \tag{24}$$

Then $\varepsilon_0 = P_x^2/2m_x + P_z^2/2m_z$, $v_{z0}=P_z/m_z$, and $P_m = \sqrt{2m_x\varepsilon - m_x P_z^2/m_z}$. Performing the integration in Eq. (23) with $u_{x0}=P_m/m_x$, we have for G_s the expression

$$G = \frac{1}{15(2\pi)^2}\left(\frac{eB}{c}\right)^{1/2}\frac{\zeta}{\hbar}(2\zeta m_x)^{1/4}\left(\frac{m_z}{m_y}\right)^{1/2} I. \tag{25}$$

Here the index s has been dropped [a single term appears instead of the sum over s in Eq. (22)]

$$I = \sqrt{2}[2E(\lambda,1/\sqrt{2})-F(\lambda,1/\sqrt{2})],$$

where F and E are elliptic integrals of the first and second kinds and $\lambda=\cos^{-1}[m_x/(m_x+m_z)]^{1/4}$. The limits of integration over P_x were determined from the condition $y_0=\delta$, i.e. it was assumed that the expansion (14) holds in the entire transitional region. For $m_x \ll m_z$ the parameter $I\approx\sqrt{2}[2\mathbf{E}(1/\sqrt{2})-\mathbf{K}(1/\sqrt{2})]\approx 1.2$. In the limit $m_x \gg m_z$ we have $I\approx(m_z/m_x)^{1/2}\ll 1$. Therefore G is greater when $m_y \ll m_x \ll m_z$, which corresponds to a Fermi surface that is

oblate in the p_y direction and prolate in the direction p_z in the $p_z p_x$ plane. Finally, for a spherical Fermi surface ($m_x = m_y = m_z = m_0$)

$$G = \frac{0.74}{15(2\pi)^2\hbar}\zeta m_0 \omega_B \sqrt{R}, \tag{26}$$

which, to within a constant, is identical to the coefficient of $1/\sqrt{\delta}$ in the electronic contribution to the free energy of a ferromagnet with a domain wall.[32]

4. EQUILIBRIUM WIDTH OF THE TRANSITION REGION AND THE MAGNETIZATION CURVE

In accordance with expression (22), an increase in δ is energetically favorable for the electronic subsystem. The increase in magnetic energy due to the deviation of δ from the equilibrium value (5), determined only by the magnetic subsystem of the ferromagnet, will be compensated by a decrease in the energy of the electronic subsystem. In other words, a new equilibrium width of the transition region arises. We shall calculate it, assuming that the distribution (4) of the magnetic moment is valid for a metallic ferromagnet with surface pinning of the magnetic moment, where δ is now an unknown parameter to be determined from the condition of a minimum of the free energy.

Substituting the expression (4) into Eq. (1) and integrating over the volume of the sample we obtain for the free energy of the isotropic metallic ferromagnet, using Eq. (22),

$$\mathcal{F} = -VMH + Vn_0\zeta + \Omega_0 + \Omega_L + L_x L_z \times \left[2M_0^2 \left(\frac{\alpha}{\sqrt{2\delta}} + \sqrt{2} h \delta \right) + \frac{1}{\sqrt{\delta}} \sum_s G_s \right]. \tag{27}$$

Here the surface energy density (the expression in brackets) is singled out. It is convenient to represent it in the form

$$F_w = 2M^2 \left(\frac{\alpha}{\sqrt{2\delta}} + \sqrt{2} h \delta + \frac{g}{\sqrt{\delta}} \right), \tag{28}$$

where $g = (1/2M^2)\Sigma_s G_s$. From the condition of a minimum of the function $F_w(\delta)$ we obtain the equation

$$\delta^2 - \frac{g}{2\sqrt{2h}}\sqrt{\delta} - \frac{\alpha}{2h} = 0, \tag{29}$$

whose solution determines the equilibrium value of the width of the transitional region. Allowing g in Eq. (29) to approach zero, the solution of the equation gives for δ the expression (5). For finite g the solution of Eq. (29) was obtained numerically. For small values of g, when the effect of the electronic subsystem can be treated as a perturbation, the solution of this equation can be written in the simple analytic form

$$\delta = \left(\frac{\alpha}{2h} \right)^{1/2} \left[1 + \frac{1}{2} \left(\frac{h_c}{h} \right)^{1/4} \right], \quad h_1 \ll h \ll h_2. \tag{30}$$

Here $h_c = g^4/8\alpha^3$ is the characteristic field, $h_1 = \max\{h_u, h_c\}$, and h_2 is the upper limit of the admissable values of the magnetic field, determined by the smallest of the limiting fields. Now there is only one limit on the magnetic field ($h \ll 4\pi$); a second limit will be obtained below. Comparing Eq. (30) with the numerical solution of Eq. (29) shows that this expression is a good approximation when $h \approx h_c$. Formally Eq. (9) possesses a solution for $h \ll h_c$ also (if, of course, $h_c \gg h_u$), but then there arises the question of the applicability of the expression (4) for describing the magnetization distribution in the sample.

Since, as follows from expression (7), the equilibrium value of δ determines the field-dependence of the magnetization, we have

$$\frac{\Delta M}{M} = \frac{2}{d} \left(\frac{\alpha}{h} \right)^{1/2} \left[1 + \frac{1}{2} \left(\frac{h_c}{h} \right)^{1/4} \right], \quad h_1 \ll h \ll h_2. \tag{31}$$

In the derivation of Eq. (31) we neglected in Eq. (7) the exponentially small deviation of the hyperbolic tangent from 1. If the law of approach of the magnetization to saturation is rewritten in a different form, which is also often used, representing $\langle M \rangle$ as a power-law function of H, then

$$\frac{\langle M \rangle}{M} = 1 - \frac{a^{1/2}}{H_{3/4}} - \frac{a^{3/4}}{H_{3/4}}. \tag{32}$$

The coefficients a_i can be expressed in terms of the physical parameters appearing in Eq. (31).

Graphically, the magnetization curve can be represented more clearly in the coordinates M' and $h^{-1/4}$, where

$$M' \equiv \frac{\Delta M d}{2M} \left(\frac{h}{\alpha} \right)^{1/2}. \tag{33}$$

In the absence of an electronic contribution to the magnetization of the ferromagnet the function $M'(h^{-1/4}) = 1$. Figure 2 (solid line) shows the dependence of M' on $h^{-1/4}$, following from the numerical solution of Eq. (29) with $g = 1.7 \times 10^{-9}$ cm$^{3/2}$. This value of g for an ellipsoidal Fermi surface can be obtained from Eq. (25) with $(m_x/m_y)^{1/2} \approx 10$, $\zeta = 7$ eV, $M = 500$ Gs, and m_x is equal to the free-electron mass. Here $h_c \approx 8 \times 10^{-3}$, which is three orders of magnitude greater than h_u with sample thickness $d = 10^{-3}$ cm. The dashed line in Fig. 2 shows the dependence following from the approximate relation (31). The slope angle of the straight line with respect to the abscissa is proportional to g. One can see that $\Delta M/M$ can be approximated well by Eq. (31) right up to $h \approx h_c$.

The results presented above can be extended to the case of a uniaxial ferromagnet with easy axis along z. We shall assume that a nonuniform distribution of the magnetization, for which the expression (4) holds, has been produced in a such sample. Then h can be replaced by $h_\pm = \beta \pm h$ in Eq. (5) as well as in all subsequent expressions. We use the $+$ sign when the vector **H** is directed opposite to the vector **M** pinned on the surface, as shown in Fig. 1, and we use the $-$ sign when **H** is parallel to **M**. In other words, the expression (31) with h in it replaced by $h_\pm$ describes a section of the hysteresis loop of the ferromagnet. The inequalities (31) remain valid when h is replaced by $h_\pm$ in them.

Thus, the restructuring of the conduction-electron spectrum in a nonuniform magnetic induction of a ferromagnet with surface pinning of the magnetic moment contributes to the magnetization process in such a material. The magneti-

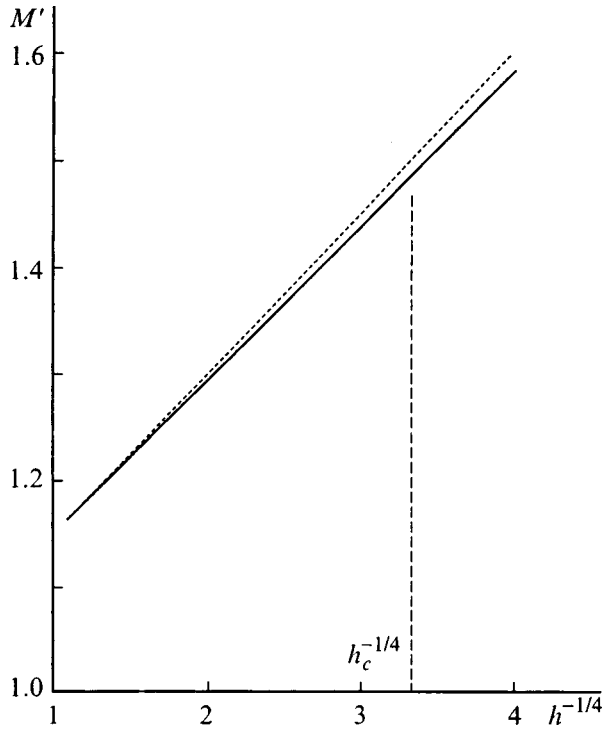

FIG. 2. Dependence of relative change in magnetization on the external magnetic field. The solid line was constructed using the numerical solution of Eq. (29) for δ; the dashed line corresponds to the expression (31).

zation curve becomes smoother, and its modification by the conduction electrons is greatest in weak magnetic fields, where $\Delta M/M$ is relatively large (Eq. (31)). This fact could facilitate experimental observation of the effect.

The results presented in this paper were obtained for a magnetic field in the interval $h_1 \ll h \ll 4\pi$, where the limits were discussed above. We note another limit on the magnetic field . This limit is due to the condition $\delta \gg \lambda_y$, where λ_y is the localization length of the wave function of a trapped electron. In accordance with Eq. (14) this inequality has the form

$$\delta \gg \sqrt{\frac{\hbar}{m_y \omega_1}} \tag{34}$$

or, taking account of the δ-dependence of ω_1,

$$\delta \gg R_x \left(\frac{\hbar}{m \omega_B} \right)^{2/3}. \tag{35}$$

Using the values of the physical parameters presented above for obtaining estimates gives $\delta \gg 10^{-6}$ cm. The inequalities determining the range of δ can be represented as a condition on the magnetic field

$$h \ll h_w = \frac{\alpha}{2R_x^2} \left(\frac{\zeta}{\hbar \omega_B} \right)^{4/3}. \tag{36}$$

According to this equation $h_w \approx 10$. Thus the limiting field entering in Eqs. (30) and (31) is determined by the expression $h_2 = \min\{h_w, 4\pi\}$. There is a limit on the external magnetic field that is due to the finite magnitude of the field of surface pinning of the magnetization. However, its value in the case, for example, of a film on an antiferromagnetic substrate reaches several tens of kilooersteds and certainly exceeds all characteristic fields.

Let us now examine the effect of scattering of the trapped electrons by impurities. If $l_y \gg \lambda_y$, where l_y is the electron mean free path length in the direction of the y axis ($l_y = v_y \tau$; τ is the electron free travel time), electron collisions do not change any of the results presented above. In other words, the spacing between the levels in the spectrum (17) should satisfy the condition $\omega_1 \gg 1/\tau$. For $h \approx 10^{-2}$ this inequality holds if the free path length $l = v_F \tau \approx 10^{-3}$ cm. Such values of l are characteristic for pure ferromagnets at low temperature.

The slope angle of the dashed line in Fig. 2 is proportional to the quantity g, determined in terms of the electronic parameters of the ferromagnet. Thus it is possible to determine the characteristics of local sections of the Fermi surface by means of magnetic measurements. However, at first even a simple observation of the electronic contribution to the magnetization curve would be of definite interest, since there are still no experimental proofs of the existence of electrons trapped by a domain wall or by the transition region in a ferromagnet with surface pinning of the magnetic moment.

For nickel $g \approx 10^{-10}$ cm$^{3/2}$ (the values of the exchange constant, magnetization, and chemical potential of this material were used above for estimates). Then, for sample thickness $d = 10^{-3}$ cm, the increase in $\Delta M/M$ due to trapped electrons is approximately 20% in a field $h = 10^{-3}$. The contribution to the magnetization $\langle M \rangle$, i.e., the value of the last term in Eq. (32), is approximately 0.03. For comparison, we note that Landau diamagnetism produces a relative decrease of the magnetization M of the order of 10^{-6}.

The results obtained here qualitatively describe also the magnetization curve of flat samples with antiparallel surface pinning of the magnetic field on opposite faces. Therefore it can be inferred that the effects studied can be intensified by producing a multilayer system having alternating layers of a ferromagnet and a material giving strong surface pinning.

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