

# Superconductivity of strongly correlated electrons in copper and ruthenium oxides within the $t$ - $J$ - $I$ model

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We propose a  $t$ - $J$ - $I$  model with direct ferromagnetic exchange  $I$  to explain the superconductivity of copper oxides and the ruthenate  $\text{Sr}_2\text{RuO}_4$  on the basis of the analysis of the electronic structure of these substances. We analyze the possible  $p$ - and  $d$ -type superconducting solutions. Solutions of the  $s$  type with singlet pairings are impossible in the strong-electron-correlations regime, and  $p$ -type solutions correspond to triplet superconductivity and is formed near the ferromagnetic instability threshold in ruthenates. The solution with the  $d_{x^2-y^2}$  symmetry near the antiferromagnetic instability threshold corresponds to copper oxides. We also discuss the reason for the high values of the superconducting transition temperature ( $T_c \sim 100$  K) in copper oxides and the low values ( $T_c \sim 1$  K) in ruthenates. © 1999 American Institute of Physics. [S1063-7761(99)02208-8]

## 1. INTRODUCTION

Even in the earliest research on superconductivity in  $\text{Sr}_2\text{RuO}_4$  (Ref. 1) attempts were made to compare the results with those for copper oxides, the reason being that  $\text{Sr}_2\text{RuO}_4$  and  $\text{La}_2\text{CuO}_4$  have similar structures. At the same time, the values of the superconducting transition temperature  $T_c$  differ dramatically:  $T_c \sim 1$  K for ruthenates and  $T_c \sim 100$  K for cuprates. Latter it was found that magnetic and superconducting properties differ, too. While in cuprates superconductivity occurs against the background of strong antiferromagnetic fluctuations, ruthenates are close to the ferromagnetic instability threshold.<sup>2</sup> The Cooper pairs in the superconducting state in  $\text{Sr}_2\text{RuO}_4$  are of the  $p$  type,<sup>3</sup> in contrast to the  $d$ -type pairing in cuprates. For  $\text{Sr}_2\text{RuO}_4$ , solutions of the  $p$  type were obtained by Mazin and Singh<sup>4</sup> as a result of band-structure calculations. The literature on superconductivity in the strong-electron-correlation regime within the Hubbard and  $t$ - $J$  models is vast, and this is reflected in Refs. 5 and 6.

In the present paper we propose a  $t$ - $J$ - $I$  model that takes into account the antiferromagnetic ( $J$ ) and ferromagnetic ( $I$ ) exchange interactions simultaneously. Three possible types of superconducting state are studied by this model: single states of the  $s$  and  $d$  types and triple states of the  $p$  type. In addition to the ordinary self-consistency equations for the gap and the chemical potential, in the strong-electron-correlation regime there is an additional self-consistency condition (constraint) that excludes two-particle states ("doubles") at a single site. Solutions of the  $s$  type do not meet this condition, but solutions of the  $d_{x^2-y^2}$  and  $p$ -types do. The coupling constant of  $d$ -type solutions is determined primarily by antiferromagnetic exchange interaction,

while the coupling constant of  $p$ -type solutions is determined primarily by ferromagnetic exchange interaction. The special features of the momentum-dependence of the gap in the equations for  $T_c$  lead to  $T_c \sim 100$  K for  $d$ -type pairings and to  $T_c \sim 1$  K for  $p$ -type pairings, with the coupling constants being equal.

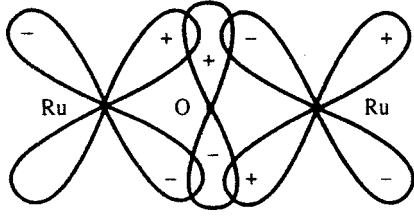
## 2. THE $t$ - $J$ - $I$ -MODEL HAMILTONIAN

The Hamiltonian of the  $t$ - $J$ - $I$  model can be written

$$H = \sum_{\mathbf{f}\sigma} (\varepsilon - \mu) X_{\mathbf{f}}^{\sigma\sigma} - t \sum_{\mathbf{f}\delta\sigma} X_{\mathbf{f}}^{\sigma 0} X_{\mathbf{f}+\delta}^{0\sigma} + J \sum_{\mathbf{f}\delta} K_{\mathbf{f},\mathbf{f}+\delta}^{(-)} - I \sum_{\mathbf{f}\delta} K_{\mathbf{f},\mathbf{f}+\delta}^{(+)}, \quad (1)$$

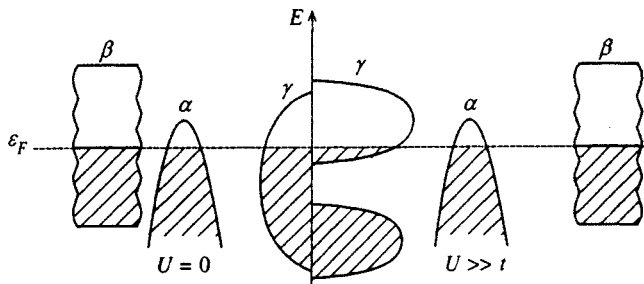
$$K_{\mathbf{f}\mathbf{m}}^{(\pm)} = \mathbf{S}_{\mathbf{f}} \cdot \mathbf{S}_{\mathbf{m}} \pm \frac{1}{4} n_{\mathbf{f}} n_{\mathbf{m}}, \quad X_{\mathbf{f}}^{\uparrow\uparrow} + X_{\mathbf{f}}^{\downarrow\downarrow} + X_{\mathbf{f}}^{00} = 1. \quad (2)$$

The Hamiltonian (1) is given on a lattice of  $N$  sites ( $\mathbf{f}$  and  $\mathbf{m}$  are the lattice sites) with periodic boundary conditions and with  $z$  nearest neighbors, and  $\delta$  is the vector connecting the nearest neighbors. The Hamiltonian describes a system of  $N_e$  electrons in the subspace of local states  $|0\rangle$  (holes or vacant lattice sites) and  $|\sigma\rangle$  (one-electron states with a spin projection  $\sigma = \uparrow$  or  $\sigma = \downarrow$ ), so that  $0 \leq N_e \leq N$ . In this basis the states and the transitions between them are described by the Hubbard  $X$ -operators  $X_{\mathbf{f}}^{pq} = |p\rangle\langle q|$  acting on the states  $|0\rangle$  and  $|\sigma\rangle$  (doubles are excluded automatically), and  $\mathbf{S}_{\mathbf{f}}$  and  $n_{\mathbf{f}}$  are the operators of the spin and number of particles at site  $\mathbf{f}$ . The signs in (1) are chosen so that all the parameters,  $t$ ,  $J$ , and  $I$ , are positive. The energy  $\varepsilon$  of the one-electron level will be assumed to be zero.

FIG. 1. The  $(d_{xy}-p)-\pi$  bond in  $\text{Sr}_2\text{RuO}_4$ .

As in the  $t$ - $J$  model, the antiferromagnetic exchange is an indirect cation-anion superexchange. The ferromagnetic exchange  $I$  has the form common to this model and is generated by direct overlap of cation  $d$ -orbitals of neighboring sites. In cuprates, where the electronic states near the Fermi level are formed primarily by the  $d_{x^2-y^2}-p-\sigma$  bond, direct overlap can be ignored,  $J \gg I$ . A characteristic feature of the electronic structure of  $\text{Sr}_2\text{RuO}_4$  is that near the Fermi surface the states are formed by the  $(t_{2g}-p)-\pi$  bond. In this case there is indirect 180-degree cation-anion-cation interaction  $J$  and direct overlap of  $t_{2g}$ -orbitals of neighboring cations (Fig. 1). According to Goodenough,<sup>7</sup> for  $\text{Ru}^{4+}$  the antiferromagnetic exchange is small, so that the model with  $I > J$  corresponds to  $\text{Sr}_2\text{RuO}_4$ . The importance of strong electron correlations for cuprates is well known, and for ruthenates the need to allow for such correlations stems from the large effective mass of the electrons belonging to the  $\gamma$ -band formed by  $d_{xy}$ -orbitals,  $m \approx 12m_e$ , a fact corroborated by experiments involving quantum oscillations.<sup>8</sup> According to Rise-man *et al.*,<sup>9</sup> it is the  $\gamma$ -band that forms the superconducting state. The other  $d$ -orbitals of ruthenium form the hole  $\alpha$ -band and the electron  $\beta$ -band, which in our model act as a reservoir for electrons. According to the band calculations of Singh,<sup>10</sup> the number of electrons in the  $\gamma$ -band,  $n_\gamma$ , is 1.28. After the  $\gamma$ -band splits into two Hubbard subbands due to strong electron correlations, the lower subband becomes completely filled by one electron per atom, while the upper subband is partially filled by "additional" electrons (i.e., doubles) with a concentration  $n_2 = 0.28$  (Fig. 2). In view of the hole-double symmetry, it is convenient to go over to the hole representation: the upper Hubbard subband with a concentration  $n_2$  of DOUBLES is equivalent to the lower subband with the same concentration of holes,  $n_0 = n_2$ , or the electron concentration  $n = 1 - n_0$ .

The Hamiltonian (1) describes states in the lower Hub-

FIG. 2. Band structure of  $\text{Sr}_2\text{RuO}_4$  in the limit of free electrons ( $U=0$ ) and in the strong correlation regime ( $U \gg t$ ).

bard band and makes it possible to study two very different scenarios of the system's behavior.

1. If  $J > I$ , as is the case for cuprates, then at  $n = 1$  ( $n_0 = 0$ ) the ground state has long-range antiferromagnetic (AF) order. As is known, the AF state is destroyed at low hole concentrations,  $(n_0)_{AF} \sim 0.05$ . In the region where  $n_0 > (n_0)_{AF}$ , superconductivity may emerge due to antiferromagnetic ( $J$ ) exchange interactions.

2. If  $I > J$ , which is the case for  $\text{Sr}_2\text{RuO}_4$ , then at  $n = 1$  the ground insulator state is ferromagnetic ( $F$ ). In the presence of holes there is competition between the saturated  $F$  state and the normal (nonmagnetic)  $N$  state, whose chemical potential is lower. As a result of this competition, for electron concentrations  $n < n_F$ , which is equivalent to hole concentrations  $n_0 > (n_0)_F$ , the system passes to the  $N$  state, and in this region superconductivity may form due to ferromagnetic ( $I$ ) exchange interactions.

It is convenient to normalize the Hamiltonian (1) to the halfwidth of the initial electron band,  $w = zt$ . If we introduce the Fourier transforms of the Hubbard operators,

$$X_{\mathbf{k}\sigma} = \frac{1}{\sqrt{N}} \sum_{\mathbf{f}} e^{i\mathbf{k}\cdot\mathbf{f}} X_{\mathbf{f}}^{0\sigma}, \quad X_{\mathbf{q}}^{\sigma\sigma'} = \frac{1}{\sqrt{N}} \sum_{\mathbf{f}} e^{i\mathbf{q}\cdot\mathbf{f}} X_{\mathbf{f}}^{\sigma\sigma'}, \quad (3)$$

where the vectors  $\mathbf{k}$  and  $\mathbf{q}$  belong to the first Brillouin zone, we arrive at the model Hamiltonian in the form

$$\frac{H}{zt} \equiv h = h_{\text{kin}} + h_{\text{int}}, \quad (4)$$

where

$$h_{\text{kin}} = \sum_{\mathbf{k}\sigma} (\omega_{\mathbf{k}} - \tilde{\mu}) X_{\mathbf{k}\sigma}^\dagger X_{\mathbf{k}\sigma},$$

$$\omega_{\mathbf{k}} = -\frac{1}{z} \sum_{\delta} e^{i\mathbf{k}\cdot\delta} = -\gamma_{\mathbf{k}}, \quad (5)$$

$$h_{\text{int}} = \frac{1}{2} \sum_{\mathbf{q}\sigma} \gamma_{\mathbf{q}} \{ g (X_{\mathbf{q}}^{\sigma\bar{\sigma}} X_{-\mathbf{q}}^{\bar{\sigma}\sigma} - X_{\mathbf{q}}^{\sigma\sigma} X_{-\mathbf{q}}^{\bar{\sigma}\bar{\sigma}}) - \lambda (X_{\mathbf{q}}^{\sigma\bar{\sigma}} X_{-\mathbf{q}}^{\bar{\sigma}\sigma} + X_{\mathbf{q}}^{\sigma\sigma} X_{-\mathbf{q}}^{\bar{\sigma}\bar{\sigma}}) \}, \quad (6)$$

with  $\bar{\sigma} = -\sigma$ ,  $g = J/t$ ,  $\lambda = I/t$ , and  $\mu/zt = \tilde{\mu}$  the dimensionless chemical potential. The Hamiltonian  $h_{\text{kin}}$  [Eq. (4)] describes the kinetic energy of the electrons and at  $U = \infty$  is the Hamiltonian of the Hubbard model.

### 3. EQUATIONS OF MOTION

Using the algebra of  $X$ -operators, we arrive at the equation of motion for a quasi-Fermi operator ( $\hbar = 1$ ):

$$i\dot{X}_{\mathbf{k}\sigma} = [K_{\mathbf{k}\sigma}, h] = (\omega_{\mathbf{k}} - \mu) X_{\mathbf{k}\sigma} + L_{\mathbf{k}\sigma},$$

$$L_{\mathbf{k}\sigma} = L_{\mathbf{k}\sigma}^{(\text{kin})} + L_{\mathbf{k}\sigma}^{(\text{int})}, \quad (7)$$

$$L_{\mathbf{k}\sigma}^{(\text{kin})} = \frac{1}{\sqrt{N}} \sum_{\mathbf{p}} \omega_{\mathbf{p}} (X_{\mathbf{k}-\mathbf{p}}^{\bar{\sigma}\sigma} X_{\mathbf{p}\bar{\sigma}} - X_{\mathbf{k}-\mathbf{p}}^{\bar{\sigma}\bar{\sigma}} X_{\mathbf{p}\sigma}), \quad (8)$$

$$L_{\mathbf{k}\sigma}^{(\text{int})} = \frac{1}{\sqrt{N}} \sum_{\mathbf{p}} \omega_{\mathbf{k}-\mathbf{p}} \{ (\lambda - g) X_{\mathbf{k}-\mathbf{p}}^{\bar{\sigma}\sigma} X_{\mathbf{p}\bar{\sigma}} + g X_{\mathbf{k}-\mathbf{p}}^{\bar{\sigma}\bar{\sigma}} X_{\mathbf{p}\sigma} + \lambda X_{\mathbf{k}-\mathbf{p}}^{\sigma\sigma} X_{\mathbf{p}\sigma} \}, \quad (9)$$

where the nonlinear operator  $L_{\mathbf{k}\sigma}$  describes the correlations of electrons with spin projections in the opposite directions and in the same direction.

We introduce the irreducible operator (see Ref. 11)

$$\bar{L}_{\mathbf{k}\sigma} = L_{\mathbf{k}\sigma} - \frac{\langle \{ L_{\mathbf{k}\sigma}, X_{\mathbf{k}\sigma}^\dagger \} \rangle}{\langle \{ X_{\mathbf{k}\sigma}, X_{\mathbf{k}\sigma}^\dagger \} \rangle} X_{\mathbf{k}\sigma} - \frac{\langle \{ L_{\mathbf{k}\sigma}, X_{-\mathbf{k}\bar{\sigma}} \} \rangle}{\langle \{ X_{-\mathbf{k}\bar{\sigma}}^\dagger, X_{-\mathbf{k}\bar{\sigma}} \} \rangle} X_{-\mathbf{k}\bar{\sigma}}^\dagger, \quad (10)$$

which has the property of being ‘‘mean-orthogonal’’:  $\langle \{ \bar{L}_{\mathbf{k}\sigma}, X_{\mathbf{k}\sigma}^\dagger \} \rangle = \langle \{ \bar{L}_{\mathbf{k}\sigma}, X_{-\mathbf{k}\bar{\sigma}} \} \rangle = 0$ . Then Eq. (7) becomes

$$i\dot{X}_{\mathbf{k}\sigma} = \left( \omega_{\mathbf{k}} - \tilde{\mu} + \frac{C_{\mathbf{k}\sigma}}{1 - n_{\bar{\sigma}}} \right) X_{\mathbf{k}\sigma} + \frac{\Delta_{\mathbf{k}\sigma}}{1 - n_{\sigma}} X_{-\mathbf{k}\bar{\sigma}}^\dagger + \bar{L}_{\mathbf{k}\sigma}, \quad (11)$$

where  $X_{\mathbf{k}\sigma} = \langle \{ L_{\mathbf{k}\sigma}, X_{\mathbf{k}\sigma}^\dagger \} \rangle$  and  $\Delta_{\mathbf{k}\sigma} = \langle \{ L_{\mathbf{k}\sigma}, X_{-\mathbf{k}\bar{\sigma}} \} \rangle$ .

The generalized Hartree–Fock approximation, or the mean-field approximation, corresponds to the linear part of Eq. (11), i.e., we ignore the irreducible operator  $\bar{L}_{\mathbf{k}\sigma}$ . It is in this approximation that we will study the possibility of superconductivity manifesting itself. In Eq. (11),  $C_{\mathbf{k}\sigma}/(1 - n_{\bar{\sigma}})$  describes the renormalization of the spectrum, and  $\Delta_{\mathbf{k}\sigma}$  is the possible superconducting gap. Spectrum renormalization can be calculated in general form, but it is sufficient to limit ourselves to an approximation of the Hubbard I type:

$$\xi_{\mathbf{k}\sigma} = \omega_{\mathbf{k}} - \tilde{\mu} + \frac{C_{\mathbf{k}\sigma}}{1 - n_{\bar{\sigma}}} \approx (1 - n_{\bar{\sigma}}) \omega_{\mathbf{k}} - g n_{\bar{\sigma}} - \lambda n_{\sigma} - \tilde{\mu}.$$

In the nonmagnetic ground state,  $n_{\uparrow} = n_{\downarrow} = n/2$ , the dependence on the spin projection disappears and the modified spectrum can be written

$$\xi_{\mathbf{k}} = c(n)(\omega_{\mathbf{k}} - m),$$

$$m = \frac{(g + \lambda)n/2 + \tilde{\mu}}{c(n)}, \quad c(n) = 1 - \frac{n}{2}, \quad (12)$$

where  $m$  is the effective chemical potential.

The expression for the gap  $\Delta_{\mathbf{k}\sigma}$  has the form

$$\Delta_{\mathbf{k}\sigma} = \frac{1}{N} \sum_{\mathbf{p}} \omega_{\mathbf{p}} (\langle X_{-\mathbf{p}\sigma} X_{\mathbf{p}\bar{\sigma}} \rangle - \langle X_{-\mathbf{p}\bar{\sigma}} X_{\mathbf{p}\sigma} \rangle) + \frac{1}{N} \sum_{\mathbf{p}} \omega_{\mathbf{k}-\mathbf{p}} \{ (\lambda - g) \langle X_{-\mathbf{p}\sigma} X_{\mathbf{p}\bar{\sigma}} \rangle + g \langle X_{-\mathbf{p}\bar{\sigma}} X_{\mathbf{p}\sigma} \rangle \}. \quad (13)$$

We introduce the anomalous means

$$B_{\mathbf{p}} = \langle X_{-\mathbf{p}\downarrow} X_{\mathbf{p}\uparrow} \rangle. \quad (14)$$

Then, using the symmetry property  $\omega_{\mathbf{p}} = \omega_{-\mathbf{p}}$ , we find that

$$\Delta_{-\mathbf{k}\downarrow} = -\Delta_{\mathbf{k}\uparrow} = \Delta_{\mathbf{k}}$$

$$= \frac{1}{N} \sum_{\mathbf{p}} [2\omega_{\mathbf{p}} - g(\omega_{\mathbf{k}+\mathbf{p}} + \omega_{\mathbf{k}-\mathbf{p}}) + \lambda\omega_{\mathbf{k}+\mathbf{p}}] B_{\mathbf{p}}. \quad (15)$$

The first term on the right-hand side of Eq. (15) reflects the presence of kinematic electron correlations and originates in the kinetic term in the Hamiltonian (this is known as kinematic pairing<sup>12</sup>), and the other terms are the consequence of exchange interactions.<sup>1)</sup>

Using Eq. (11) (with the irreducible operator  $\bar{L}_{\mathbf{k}\sigma}$  discarded) and the relationships (12) and (15), we arrive in the mean-field approximation at the following system of equations:

$$i\dot{X}_{\mathbf{k}\uparrow} = \xi_{\mathbf{k}} X_{\mathbf{k}\uparrow} = \frac{\Delta_{\mathbf{k}}}{c(n)} X_{-\mathbf{k}\downarrow}^\dagger,$$

$$i\dot{X}_{-\mathbf{k}\downarrow}^\dagger = -\xi_{\mathbf{k}} X_{-\mathbf{k}\downarrow}^\dagger - \frac{\Delta_{\mathbf{k}}^*}{c(n)} X_{\mathbf{k}\uparrow}. \quad (16)$$

#### 4. MEANS AND SELF-CONSISTENCY EQUATIONS

Using the system of equations (16), we arrive at a system of equations for the two-time retarded anticommuting Green’s functions<sup>11</sup> and its solution:

$$\langle \langle X_{\mathbf{k}\uparrow} | X_{\mathbf{k}\uparrow}^\dagger \rangle \rangle_E = c(n) \frac{E + \xi_{\mathbf{k}}}{E^2 - E_{\mathbf{k}}^2},$$

$$\langle \langle X_{-\mathbf{k}\downarrow}^\dagger | X_{\mathbf{k}\uparrow}^\dagger \rangle \rangle_E = -\frac{\Delta_{\mathbf{k}}^*}{E^2 - E_{\mathbf{k}}^2}, \quad (17)$$

where

$$E_{\mathbf{k}}^2 = \xi_{\mathbf{k}}^2 + \frac{|\Delta_{\mathbf{k}}|^2}{c^2(n)}. \quad (18)$$

The spectral theorem<sup>11</sup> yields the following expressions for the means:

$$n_{\mathbf{k}} = \langle X_{\mathbf{k}\uparrow}^\dagger X_{\mathbf{k}\uparrow} \rangle = \langle X_{\mathbf{k}\downarrow}^\dagger X_{\mathbf{k}\downarrow} \rangle$$

$$= \frac{c(n)}{2} \left( 1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} \tanh \frac{E_{\mathbf{k}}}{2\tau} \right) \equiv c(n) f_{\mathbf{k}}, \quad (19)$$

$$B_{\mathbf{k}}^* = \langle X_{\mathbf{k}\uparrow}^\dagger X_{-\mathbf{k}\downarrow}^\dagger \rangle = \frac{\Delta_{\mathbf{k}}^*}{2E_{\mathbf{k}}} \tanh \frac{E_{\mathbf{k}}}{2\tau}, \quad (20)$$

where  $E_{\mathbf{k}} > 0$ , and  $\tau = k_B T / zt$  is the dimensionless temperature.

In the superconducting phase we have three self-consistency equations.

1. An equation that links the electron concentration  $n$  with the effective chemical potential  $m$ :

$$n = \frac{1}{N} \sum_{\mathbf{k}\sigma} n_{\mathbf{k}\sigma} = \frac{2c(n)}{N} \sum_{\mathbf{k}} f_{\mathbf{k}}, \quad f_{\mathbf{k}} = \frac{1}{2} \left( 1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} \right), \quad (21)$$

where  $f_{\mathbf{k}}$  is the distribution function (19) at  $T = 0$ .

2. The constraint condition, or the sum rule for anomalous means (the exclusion of doubles as a consequence of the algebra of  $X$ -operators):

$$\frac{1}{N} \sum_{\mathbf{k}} B_{\mathbf{k}} = \frac{1}{N} \sum_{\mathbf{k}} B_{\mathbf{k}}^* = 0. \quad (22)$$

3. The equation for the energy gap  $\Delta_{\mathbf{k}}$  (see below), which has meaning only if condition (22) is met.

But first we must examine the possible nonsuperconducting states.

*N state.* We begin with the normal (nonmagnetic) phase, or the  $N$  state ( $\Delta_{\mathbf{k}}=0$ ). The distribution function  $f_{\mathbf{k}}$  becomes the Fermi step  $f_{\mathbf{k}}^0 = \theta(m - \omega_{\mathbf{k}})$  and Eq. (21) becomes

$$\frac{n}{2-n} = \frac{1}{N} \sum_{\mathbf{k}} \theta(m - \omega_{\mathbf{k}}) = \int_{-1}^m \rho(\omega) d\omega \equiv g(m), \quad (23)$$

where  $\rho(\omega) = \rho(-\omega)$  is the density of states corresponding to the dispersion law for  $\omega_{\mathbf{k}}$ . The system energy (per lattice site) is

$$\begin{aligned} \epsilon_0 &= \frac{1}{N} \sum_{\mathbf{k}\sigma} \omega_{\mathbf{k}} \left(1 - \frac{n}{2}\right) f_{\mathbf{k}}^0 - \frac{1}{4} (g + \lambda) n^2 \\ &= (2-n) \int_{-1}^m \omega \rho(\omega) d\omega - \frac{1}{4} (g + \lambda) n^2. \end{aligned} \quad (24)$$

*F state.* In the model with  $U = \infty$  ( $J=0$ ), long-range ferromagnetic ( $F$ ) order sets in in the region of high concentrations,  $n > n_F(\lambda)$ . This critical concentration can easily be found by comparing the energies of the saturated ferromagnetic state [ $\epsilon_F(n, \lambda)$ ] and the normal state [ $\epsilon_0(n, \lambda)$ ]. The energy of the  $F$  state (per lattice site) is

$$\begin{aligned} \epsilon_F(n, \lambda) &= v(m_F) - \frac{1}{2} \lambda n^2, \\ n = g(m_F) &= \int_{-1}^{m_F} \rho(\omega) d\omega, \quad v(m_F) = \int_{-1}^{m_F} \omega \rho(\omega) d\omega, \end{aligned} \quad (25)$$

where  $m_F$  is the chemical potential in the  $F$  state, and  $n$  is the electron concentration. By comparing the energies of the  $N$  and  $F$  states we can find  $n_F(\lambda)$ . For instance, for a square lattice,  $n_F \approx 0.91$  at  $\lambda = 0.3$ , and the domain of existence of the ground  $F$  state grows as  $\lambda$  increases: at  $\lambda = 1$  we have  $n_F \approx 0.6$ . A similar situation occurs in the three-dimensional case.

*AF state.* When  $J \gg I$  and  $n \rightarrow 1$ , the system exhibits long-range antiferromagnetic ( $AF$ ) order. As noted in Sec. 2, the  $AF$  state is destroyed at low hole concentrations,  $n_0 \sim 0.05$  (see Refs. 5 and 6). We do not discuss this state in detail in this paper.

## 5. SYMMETRY PROPERTIES OF ANOMALOUS MEANS AND SOLUTIONS FOR THE GAP

We analyze the structure and symmetry properties of the anomalous averages  $B_{\mathbf{k}}$  and the gap  $\Delta_{\mathbf{k}}$ . We represent the anomalous averages  $B_{\mathbf{p}} \equiv \langle X_{-\mathbf{p}\downarrow} X_{\mathbf{p}\uparrow} \rangle$  as

$$\begin{aligned} B_{\mathbf{k}} &= B_{\mathbf{k}}^{(s)} + B_{\mathbf{k}}^{(a)}, \quad B_{\mathbf{k}}^{(s)} = \frac{1}{2} (B_{\mathbf{k}} + B_{-\mathbf{k}}) = B_{-\mathbf{k}}^{(s)}, \\ B_{\mathbf{k}}^{(a)} &= \frac{1}{2} (B_{\mathbf{k}} - B_{-\mathbf{k}}) = -B_{-\mathbf{k}}^{(a)}, \end{aligned} \quad (26)$$

i.e., as the sum of the symmetric ( $s$ ) and antisymmetric ( $a$ ) parts. We immediately note that the sum rule (22) for the antisymmetric part  $B_{\mathbf{k}}^{(a)}$  is satisfied automatically. Performing an inverse Fourier transformation, we obtain an expression for the symmetric part:

$$\begin{aligned} (B_{\mathbf{k}}^{(s)})^* &= \frac{1}{2} \langle X_{\mathbf{k}\uparrow}^\dagger X_{-\mathbf{k}\downarrow}^\dagger + X_{-\mathbf{k}\uparrow}^\dagger X_{\mathbf{k}\downarrow}^\dagger \rangle \\ &= \frac{1}{\sqrt{2}} \sum_{\mathbf{r}} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{1}{N} \sum_{\mathbf{f}} \langle Z_{\mathbf{f}, \mathbf{f}+\mathbf{r}}^\dagger \rangle, \end{aligned} \quad (27)$$

$$Z_{\mathbf{f}\mathbf{m}}^\dagger = \frac{1}{\sqrt{2}} (X_{\mathbf{f}}^\dagger X_{\mathbf{m}}^{\downarrow 0} + X_{\mathbf{m}}^{\uparrow 0} X_{\mathbf{f}}^{\downarrow 0}) = Z_{\mathbf{m}\mathbf{f}}^\dagger, \quad (28)$$

where  $Z_{\mathbf{f}\mathbf{m}}^\dagger$  is the operator of creation of a singlet pair at an arbitrary pair of sites  $\mathbf{f}$  and  $\mathbf{m}$ . Thus, the symmetric part of an anomalous mean corresponds to singlet pairings.

Similarly, for the antisymmetric part we have

$$\begin{aligned} (B_{\mathbf{k}}^{(a)})^* &= \frac{1}{2} \langle X_{\mathbf{k}\uparrow}^\dagger X_{-\mathbf{k}\downarrow}^\dagger - X_{-\mathbf{k}\uparrow}^\dagger X_{\mathbf{k}\downarrow}^\dagger \rangle \\ &= \frac{1}{\sqrt{2}} \sum_{\mathbf{r}} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{1}{N} \sum_{\mathbf{f}} \langle T_{\mathbf{f}, \mathbf{f}+\mathbf{r}}^\dagger \rangle, \end{aligned} \quad (29)$$

$$T_{\mathbf{f}\mathbf{m}}^\dagger = \frac{1}{\sqrt{2}} (X_{\mathbf{f}}^{\uparrow 0} X_{\mathbf{m}}^{\downarrow 0} - X_{\mathbf{m}}^{\uparrow 0} X_{\mathbf{f}}^{\downarrow 0}) = -T_{\mathbf{m}\mathbf{f}}^\dagger, \quad (30)$$

where  $T_{\mathbf{f}\mathbf{m}}^\dagger$  is the operator of creation of a triplet pair with  $S^z = 0$  at an arbitrary pair of sites  $\mathbf{f}$  and  $\mathbf{m}$ . Thus, the antisymmetric part of an anomalous mean corresponds to triplet pairings.

Let us examine two alternative lattices: a square lattice ( $d=2$ ) and a simple cubic lattice ( $d=3$ ). For these lattices we have

$$\gamma_{\mathbf{k}} = \frac{1}{d} \sum_j^d \cos k_j, \quad \omega_{\mathbf{k}} = -\gamma_{\mathbf{k}} \quad (31)$$

(the lattice constant  $a = 1$ ). We introduce two functions,

$$C_j = \frac{1}{N} \sum_{\mathbf{p}} \cos p_j B_{\mathbf{p}}^{(s)}, \quad S_j = \frac{1}{N} \sum_{\mathbf{p}} \sin p_j B_{\mathbf{p}}^{(a)}. \quad (32)$$

Since

$$\gamma_{\mathbf{k} \pm \mathbf{p}} = \frac{1}{d} \sum_j^d (\cos k_j \cos p_j \mp \sin k_j \sin p_j),$$

the gap (15) can be written

$$\Delta_{\mathbf{k}} = \Delta_{\mathbf{k}}^{(s)} + \Delta_{\mathbf{k}}^{(a)}, \quad \Delta_{\mathbf{k}}^{(s)} = 2\Delta_0 + \frac{2g - \lambda}{d} \sum_j \cos k_j C_j, \quad \Delta_0 = \frac{1}{N} \sum_{\mathbf{p}} \omega_{\mathbf{p}} B_{\mathbf{p}}^{(s)}, \quad \Delta_{\mathbf{k}}^{(a)} = \lambda \frac{1}{d} \sum_j \sin k_j S_j \quad (33)$$

and contains a symmetric (*s*) part in the momenta and an antisymmetric (*a*) part. The expression (18) for the spectrum  $E_{\mathbf{k}}$  also contains  $|\Delta_{\mathbf{k}}|^2$  with mixed symmetry. From the general physical requirement imposed on the spectrum,  $E_{\mathbf{k}} = E_{-\mathbf{k}}$ , it follows that  $|\Delta_{\mathbf{k}}|^2 = |\Delta_{-\mathbf{k}}|^2$ , which leads either to the class of symmetric solutions  $\Delta_{\mathbf{k}}^{(s)} = \Delta_{-\mathbf{k}}^{(s)}$  (singlet pairings) or to the class of antisymmetric solutions  $\Delta_{\mathbf{k}}^{(a)} = -\Delta_{-\mathbf{k}}^{(a)}$  (triplet pairings).

Actually, there can be several solutions within one class, with each solution corresponding to a linear combination of cosines (for symmetric solutions) or sines (for antisymmetric solutions). In the general case we number the solutions by the label *l*, denote the gap of the *l*th type by  $\Delta_{\mathbf{k}l}$ , and the spectrum (18) with such a gap by  $E_{\mathbf{k}l}$ . The expression for the anomalous means becomes

$$B_{\mathbf{k}l} = \frac{\Delta_{\mathbf{k}l}}{2E_{\mathbf{k}l}} \tanh \frac{E_{\mathbf{k}l}}{2\tau}, \quad E_{\mathbf{k}l} = \sqrt{\xi_{\mathbf{p}}^2 + \frac{|\Delta_{\mathbf{k}l}|^2}{c^2(n)}}. \quad (34)$$

Denoting the corresponding linear combination of trigonometric functions by  $\psi_l(\mathbf{k})$  and the dimensionless coupling constant of the interaction that forms a gap of the *l*th type by  $\alpha_l$ , we arrive at the following types of solution.

*Symmetric solutions of the s type (singlet pairings), l=0:*

$$\Delta_{\mathbf{k}0} = (2 + \alpha \omega_{\mathbf{k}}) \Delta_0, \quad \Delta_0 = \frac{1}{N} \sum_{\mathbf{p}} \omega_{\mathbf{p}} B_{\mathbf{p}}^{(s)}, \quad \psi_0(\mathbf{k}) = \omega_{\mathbf{k}}, \quad \alpha = 2g - \lambda. \quad (35)$$

The constraint condition (22) for the anomalous means  $B_{\mathbf{k}0}$  with the gap  $\Delta_{\mathbf{k}0}$  is not met, which implies that there can be no solutions of the *s* type. Note that many equations for the gap of the *s* type have already been proposed (see, e.g., Ref. 5), but the constraint condition was not taken into account.

Antisymmetric solutions of the *p* type (*l*=1) and symmetric solutions of the *d* type (*l*=2) can be written in a unique form:

$$\Delta_{\mathbf{k}l} = \alpha_l \psi_l(\mathbf{k}) \Delta_l, \quad \Delta_l = \frac{1}{N} \sum_{\mathbf{p}} \psi_l(\mathbf{p}) B_{\mathbf{p}l}. \quad (36)$$

Combining (34) and (36), we arrive at the gap equation

$$\frac{1}{\alpha_l} = \frac{1}{N} \sum_{\mathbf{p}} \frac{\psi_l^2(\mathbf{p})}{2E_{\mathbf{p}l}} \tanh \frac{E_{\mathbf{p}l}}{2\tau}. \quad (37)$$

The solutions of this equation are meaningful only if the sum rule (22) holds, and at  $T=0$  this sum rule can be written

$$\frac{1}{N} \sum_{\mathbf{p}} \frac{\psi_l(\mathbf{p})}{E_{\mathbf{p}l}} = 0. \quad (38)$$

In explicit form, we have the following types of solution.

1. *Antisymmetric solutions of the p-type (triplet pairings), l=1:*

$$\psi_p(\mathbf{k}) = \frac{1}{d} \sum_j \sin k_j, \quad \alpha_p = \lambda. \quad (39)$$

2. *Symmetric solutions of the d-type (singlet pairings), l=2:*

$$\psi_d(\mathbf{k}) = \frac{\cos k_x - \cos k_y}{2}, \quad \alpha_d \equiv \alpha = 2g - \lambda \quad (40)$$

for the square lattice. For *p*-type solutions the sum rule is satisfied automatically, while for *d*-type solutions the validity of (40) follows from the symmetry properties.

In conclusion of this section we examine the symmetry properties of the solutions from a general position. The system Hamiltonian (1) is written in terms of the exchange operators (2). The operator  $K_{\mathbf{fm}}^{(-)}$  has an eigenvalue equal to  $-1$  when it acts on a singlet pair and a zero eigenvalue when it acts on a triplet pair. Antiferromagnetic exchange ( $J > 0$  and  $J/t = g > 0$ ) ensures attraction between the electrons in a singlet pair and ‘‘ignores’’ triplet pairs. For this reason, *J*-exchange takes no part in the formation of *p*-type triplet superconductivity. This fact is reflected by the presence in the expression (15) for the gap of the momentum-symmetric contribution  $g(\gamma_{\mathbf{k}+\mathbf{p}} + \gamma_{\mathbf{k}-\mathbf{p}})$ , which yields only symmetric solutions corresponding to singlet pairings. On contrast to *J*-exchange, ferromagnetic direct exchange ( $I > 0$  and  $I/t = \lambda > 0$ ) acts on any pairs: the operator  $K_{\mathbf{fm}}^{(+)}$  has an eigenvalue equal to  $+1/2$  when it acts on a triplet pair and an eigenvalue equal to  $-1/2$  when it acts on a singlet pair. With allowance for the sign in the Hamiltonian, *I*-exchange leads to attraction between the electrons in a triplet pair and the only term responsible for the formation of *p*-type triplet superconductivity. On the other hand, since in the singlet state of a pair this type of exchange leads to repulsion, competition between the exchange interactions emerges in this case, and this is reflected by the coupling constant  $\alpha = 2g - \lambda$ .

Our equations for the gap and  $T_c$  in the mean-field approximation coincide in structure with similar equations obtained by the diagrammatic technique for *X*-operators in the *t*-*J* model<sup>5,6</sup> when Cooper instability of the normal phase is examined.

## 6. COMPARISON OF *p*- AND *d*-TYPE SOLUTIONS

We write the spectrum  $E_{\mathbf{k}l}$  in the form ( $l=p, d$ )

$$E_{\mathbf{k}l} = c(n) \sqrt{(\omega_{\mathbf{k}} - m)^2 + \psi_l^2(\mathbf{k}) D_l^2}, \quad D_l^2 = \frac{\alpha_l^2 |\Delta_l|^2}{c^4(n)}. \quad (41)$$

Then from (37) we obtain in explicit form the equations for the effective gap  $D_l$  as a function of concentration (chemical potential) and the dimensionless temperature  $\tau$ ,

$$\frac{2c(n)}{\alpha_l} = \frac{1}{N} \sum_{\mathbf{p}} \frac{\psi_l^2(\mathbf{p})}{\sqrt{(\omega_{\mathbf{p}} - m)^2 + \psi_l^2(\mathbf{p}) D_l^2}} \times \tanh \frac{c(n) \sqrt{(\omega_{\mathbf{p}} - m)^2 + \psi_l^2(\mathbf{p}) D_l^2}}{2\tau}, \quad (42)$$

and an equation for the transition temperature  $\tau_c^{(l)}$  ( $D_l^2 \rightarrow 0$  as  $\tau \rightarrow \tau_c^{(l)}$ ),



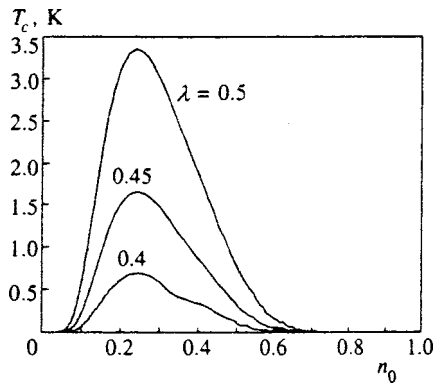


FIG. 3. Concentration dependence of the transition temperature  $T_c$  in  $\text{Sr}_2\text{RuO}_4$  with triplet pairing of the  $p$  type;  $\lambda = I/t$ , with  $I$  the ferromagnetic exchange.

$$\frac{2c(n)}{\alpha_l} = \frac{1}{N} \sum_{\mathbf{p}} \frac{\psi_l^2(\mathbf{p})}{|\omega_{\mathbf{p}} - m|} \tanh \frac{c(n)|\omega_{\mathbf{p}} - m|}{2\tau_c^{(l)}}. \quad (43)$$

When stating the problem, we can use these equations with  $l=d$  to describe cuprates, which at a certain concentration of holes (or electrons) pass from the  $AF$  state to the superconducting ( $SC$ ) state with singlet pairing of the  $d$  type, and at  $l=p$  they should describe superconductivity with triplet pairings of the  $p$  type in  $\text{Sr}_2\text{RuO}_4$ , which is above the ferromagnetic stability threshold. Although the dimensionless coupling constants  $\alpha_d = \alpha$  and  $\alpha_p = \lambda$  may differ in value, when they are equal, the values of the transition temperatures  $\tau_c^{(d)}$  and  $\tau_c^{(p)}$  depend significantly on the properties of the functions  $\psi_d^2(\mathbf{k})$  [Eq. (40)] and  $\psi_p^2(\mathbf{k})$  [Eq. (39)].

### 6.1. Results of calculations

The equations for the effective gap [Eqs. (42)] at  $\tau=0$  and for the transition temperature [Eqs. (43)] were solved numerically by summing over the momenta  $\mathbf{p}$  in the first Brillouin zone of a square lattice (it contained  $10^6$  values of  $\mathbf{p}$  and about 100 values of  $m$  in the interval from  $-1$  to  $+1$ ). We used the relationship that links the electron concentration  $n$  with the effective chemical potential  $m$  in the normal phase [Eq. (23)], i.e.,

$$n = \frac{2g(m)}{1+g(m)}, \quad g(m) = \int_{-1}^m \rho(\omega) d\omega, \quad (44)$$

where  $\rho(\omega)$  is the density of states. For a square lattice we have

$$\rho(\omega) = \frac{2}{\pi^2} K(\sqrt{1-\omega^2}) \approx \frac{1}{\pi} - \left(\frac{1}{2} - \frac{1}{\pi}\right) \ln|\omega|, \quad (45)$$

$$\int_{-1}^{+1} \rho(\omega) d\omega = 1,$$

where  $K$  is the complete elliptic integral of the first kind. The results of calculating  $T_c = zt\tau_c$ , which is considered a function of the hole concentration, are depicted in Figs. 3 and 4 for typical values of the coupling constants  $\lambda$  and  $\alpha$ . The fitting in the curves was to the experimental data on  $\text{Sr}_2\text{RuO}_4$ . At low concentrations of doubles in the upper

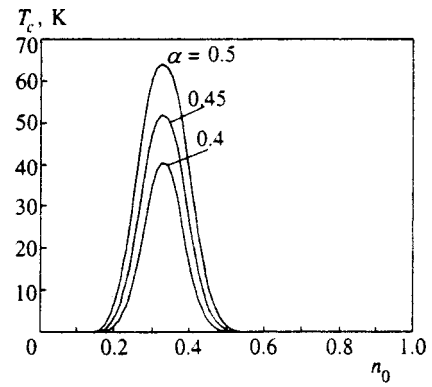


FIG. 4. Concentration dependence of the transition temperature  $T_c$  in copper oxides with singlet pairing of the  $d$  type;  $\alpha = 2g - \lambda$  and  $g = J/t$ , with  $J$  the antiferromagnetic exchange ( $J > I$ ).

Hubbard subband,  $n_2 = 0.28$  (this is equivalent to holes in the lower Hubbard subband), the dispersion law is described fairly well by the quadratic expression  $\varepsilon(\mathbf{k}) = -ztc(n)\omega_{\mathbf{k}} \approx \varepsilon(0) + p^2/2m^*$ , where  $1/m^* = (4/3)ta^2/\hbar^2$  for  $n_2 \sim 1/3$ . Plugging in the values of the effective mass  $m^* = 12m_e$  and of the atomic separation  $a = 1.93 \text{ \AA}$ , we find that  $t \sim 0.1 \text{ eV}$  and  $zt \sim 0.4 \text{ eV}$ . Within our theory, the maximum value of  $\tau_c(n_0)$  corresponds to  $n_0 \sim 1/3$  and the effective chemical potential  $m = 0$ . Note that in this sense ruthenates are ‘‘self-doped,’’ since the concentration  $n_2 = n_0 = 0.28$  is close to the optimal concentration with respect to the maximum in  $T_c$ . In cuprates the value of  $T_c$  depends on the degree of doping to which the hole concentration  $n_0$  corresponds. What is indicative is the fact that even if all the parameters are equal (including  $\alpha = \lambda$ ), the transition temperatures differ substantially:  $T_c^{(d)} \gg T_c^{(p)}$ . Let us analyze the reason for this difference.

### 6.2. Gap anisotropy

The functions  $\psi_p^2(\mathbf{k})$  and  $\psi_d^2(\mathbf{k})$  describe the anisotropy of the superconducting gap in  $p$ - and  $d$ -type pairings, respectively. Figure 5a depicts the Brillouin zone of a square lattice and the constant-energy surface  $\omega_{\mathbf{k}} = 0$  (the square  $ABCD$ ). If the constant-energy surface corresponds to the chemical potential  $m = 0$ , the states inside it in the nonsuperconducting phase at  $T = 0$  are filled, which is denoted by hatching. Note that for free electrons such filling corresponds to the concentration  $n = 1$ , for the saturate  $F$  state  $n = 1/2$ , and for the case of strongly correlated electrons  $n = 2/3$ , in accordance with (44) and (45). The behavior of the functions  $\psi_p^2(\mathbf{k})$  and  $\psi_d^2(\mathbf{k})$  at  $\omega_{\mathbf{k}} = 0$  is depicted in Figs. 5b and 5c. The gap ‘‘collapses’’ (vanishes) not only at isolated points of the constant-energy surface ( $d$  type) but also on its fragments  $BC$  and  $DA$  ( $p$  type). Reasoning in a similar manner, we can study gap anisotropy for an arbitrary constant-energy surface.

### 6.3. Mean values of $p$ - and $d$ -type functions on constant-energy surfaces

We now replace summation over momenta with integration over constant-energy surfaces corresponding to the law

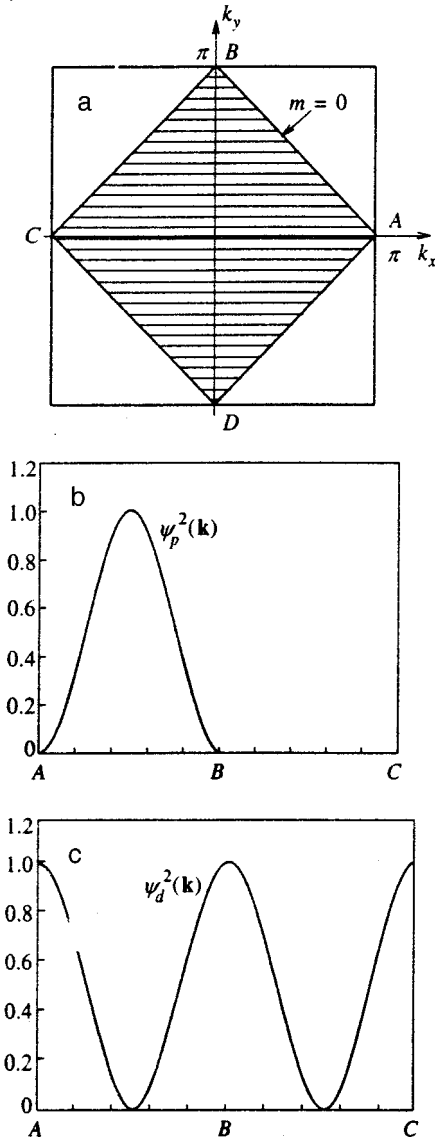


FIG. 5. Gap anisotropy on the constant-energy surface  $\omega_{\mathbf{k}}=0$  (the square ABCD): (a) the Brillouin zone of a square lattice and the filled states with the chemical potential  $m=\omega_{\mathbf{k}}=0$ ; (b) anisotropy of the  $p$  type; and (c) anisotropy of the  $d$  type (in both cases the behavior of the functions along the lines AB, CD, BC, and DA is the same).

of dispersion for  $\omega_{\mathbf{k}}$ . Let  $\omega_{\mathbf{k}}=\omega$  by the equation of the constant-energy surface ( $-1 \leq \omega_{\mathbf{k}} \leq +1$ ). Then for the function  $A_{\mathbf{k}}$ , which can be expressed explicitly in terms of the dispersion law for  $\omega_{\mathbf{k}}$ , i.e.,  $A_{\mathbf{k}}=A(\omega_{\mathbf{k}})$ , we have

$$\frac{1}{N} \sum_{\mathbf{k}} A(\omega_{\mathbf{k}}) = \int_{-1}^{+1} \rho(\omega) A(\omega) d\omega, \quad (46)$$

$$\begin{aligned} \rho(\omega) &= \frac{1}{N} \sum_{\mathbf{k}} \delta(\omega - \omega_{\mathbf{k}}) = \frac{1}{(2\pi)^d} \oint_{(\sigma_\omega)} \frac{d\sigma_\omega}{|\nabla_{\mathbf{k}}\omega_{\mathbf{k}}|} \\ &= \frac{d}{d\omega} \left( \frac{1}{(2\pi)^d} \int_{v_\omega} d^d\mathbf{k} \right). \end{aligned} \quad (47)$$

Here  $\rho(\omega)$  is the density of states corresponding to the dispersion law for  $\omega_{\mathbf{k}}$ ,  $\sigma_\omega$  is the surface area of the  $(d-1)$ -dimensional constant-energy surface, and  $v_\omega$  is the

$d$ -dimensional volume of the  $\mathbf{k}$ -space encompassed by the constant-energy surface  $\omega$  (the volume occupied by the unit cell is  $a^d=1$ ). In the two-dimensional case (the square lattice) considered here,

$$\omega_{\mathbf{k}} = -\frac{\cos k_x + \cos k_y}{2}, \quad |\nabla_{\mathbf{k}}\omega_{\mathbf{k}}| = \frac{\sqrt{\sin^2 k_x + \sin^2 k_y}}{2}, \quad (48)$$

and the density of states (45) has a logarithmic singularity. Unfortunately, the functions  $\psi_i^2(\mathbf{k})$  present in the theory cannot be expressed explicitly in terms of  $\omega_{\mathbf{k}}$ , and for this reason we introduce the mean values of these functions over the constant-energy surface  $\omega_{\mathbf{k}}=\omega$ :

$$\langle \psi_i^2(\mathbf{k}) \rangle_{\omega_{\mathbf{k}}=\omega} \equiv \psi_i^2(\omega) = \frac{1}{(2\pi)^2} \oint_{(\sigma_\omega)} \frac{\psi_i^2(\mathbf{k})}{|\nabla_{\mathbf{k}}\omega_{\mathbf{k}}|} d\sigma_\omega. \quad (49)$$

Note that direct summation of the functions  $\psi_i^2(\mathbf{k})$  over the Brillouin zone yields the obvious and same result  $1/2d=1/4$ , so that these functions must satisfy the integral condition

$$\frac{1}{N} \sum_{\mathbf{k}} \psi_i^2(\mathbf{k}) = \int_{-1}^{+1} \psi_i^2(\omega) d\omega = \frac{1}{4}. \quad (50)$$

We analyze the properties of these functions. The integrand for a  $p$ -type function in (49),

$$\frac{\psi_p^2(\mathbf{k})}{|\nabla_{\mathbf{k}}\omega_{\mathbf{k}}|} = |\nabla_{\mathbf{k}}\omega_{\mathbf{k}}| + \frac{\sin k_x \sin k_y}{\sqrt{\sin^2 k_x + \sin^2 k_y}}, \quad (51)$$

has no singularity, with the result that

$$\begin{aligned} \psi_p^2(\omega) &= \frac{1}{(2\pi)^2} \oint_{(\sigma_\omega)} |\nabla_{\mathbf{k}}\omega_{\mathbf{k}}| d\sigma_\omega \\ &= \frac{2}{\pi^2} (E(\sqrt{1-\omega^2}) - \omega^2 K(\sqrt{1-\omega^2})), \end{aligned} \quad (52)$$

where  $E$  is the complete elliptic integral of the second kind. Here we used the symmetry properties, in view of which the second term in (51) yields zero when it is integrated. The function (52) is smooth and is approximated very well by the expression

$$\psi_p^2(\omega) = \frac{2}{\pi^2} (1 - |\omega|^\nu), \quad \nu = \frac{\pi^2}{16 - \pi^2} \approx 1.61. \quad (53)$$

The integrand for a  $d$ -type function in (49),

$$\frac{\psi_d^2(\mathbf{k})}{|\nabla_{\mathbf{k}}\omega_{\mathbf{k}}|} = \frac{1}{2} \frac{(\cos k_x - \cos k_y)^2}{\sqrt{\sin^2 k_x + \sin^2 k_y}}, \quad (54)$$

has singularities of the same type as the density of states (45). Calculations yield

$$\psi_d^2(\omega) = (1 - \omega^2)\rho(\omega) - 2\psi_p^2(\omega). \quad (55)$$

For the sake of comparison, we depict the functions  $\psi_p^2(\omega)$  and  $\psi_d^2(\omega)$  in Fig. 6, with each normalized to  $1/4$  by (50).

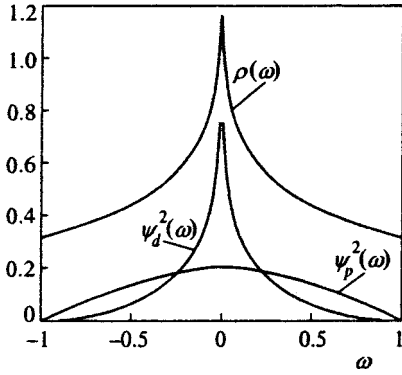


FIG. 6. Density of electronic states in a square lattice,  $\rho(\omega)$ , and the mean value of  $p$ - and  $d$ -type functions on constant-energy surfaces  $\omega$ .

#### 6.4. Transition temperature

Let us now turn to Eqs. (43) for the transition temperature. We replace summation over the Brillouin zone with integration over constant-energy surfaces. It is convenient to divide the integral into two parts: over a narrow layer of width  $2\delta$  near the chemical potential  $m$ , and over the remain part of the zone. The value of  $\delta$  can always be chosen so that the argument of the hyperbolic tangent is larger than 2.0 and the tangent is, to a high accuracy, equal to unity, i.e.,  $c(n)\delta/2\tau_c \geq 2$ . Thus, we have the integrals

$$\begin{aligned} I(m, \tau_c^{(l)}) &\approx \int_{-1}^{+1} \frac{d\omega \psi_l^2(\omega)}{|\omega - m|} \tanh \frac{c(n)|\omega - m|}{2\tau_c^{(l)}} \\ &= \int_{m-\delta}^{m+\delta} \frac{d\omega \psi_l^2(\omega)}{|\omega - m|} \tanh \frac{c(n)|\omega - m|}{2\tau_c^{(l)}} \\ &\quad + \int_{-1}^{m-\delta} \frac{d\omega \psi_l^2(\omega)}{|\omega - m|} + \int_{m+\delta}^{+1} \frac{d\omega \psi_l^2(\omega)}{|\omega - m|} \\ &\equiv I_S^{(l)} + I_B^{(l)}, \end{aligned} \quad (56)$$

which are divided into an integral over the thin lawyer ( $S$ ) and an integral over the part of the zone outside the layer ( $B$ ).

*Triplet pairings of the  $p$  type.* Since  $\psi_p^2(\omega)$  is a smooth and slowly varying function, the integral over the layer in the logarithmic approximation is

$$I_S^{(p)} = 2\psi_p^2(m) \ln \frac{1.14c(n)\delta}{\tau_c^{(p)}}, \quad (57)$$

where  $2\gamma/\pi = 1.14$ , with  $\gamma$  the Euler constant. Calculating the integral  $I_B^{(p)}$ , we can write the solution of Eq. (43):

$$\begin{aligned} \tau_c^{(p)} &\approx 1.14c(n)\delta \exp \left\{ - \frac{c(n)/\lambda - I_B^{(p)}/2}{\psi_p^2(m)} \right\} \\ &= 1.14c(n)\delta \exp \left\{ - \frac{c(n)}{(\lambda + \lambda_B)\psi_p^2(m)} \right\} \end{aligned} \quad (58)$$

with  $\delta \geq 2\tau_c^{(p)}/c(n)$ . In the second part of (58) we have introduced the notation

$$\frac{c(n)}{\lambda} - \frac{I_B^{(p)}}{2} \equiv \frac{c(n)}{\lambda + \lambda_B}, \quad \lambda_B = \lambda^2 \frac{I_B^{(p)}}{2c(n) - \lambda I_B^{(p)}}, \quad (59)$$

where the function  $\lambda_B$  describes the effect of ‘‘band enhancement’’ of the parameter  $\lambda$ . Since  $\psi_p^2(m) \leq 0.2$ , the exponential factor proves to be small, which leads to small values of  $\tau_c^{(p)}$  for triplet superconductivity.

*Singlet pairings of the  $d$  type.* From Eq. (55) and Fig. 6 it follows that  $\psi_d^2(\omega)$  changes rapidly in the vicinity of  $\omega=0$ . Hence in this region the integral over the layer will be calculated with allowance for the logarithmic singularity in  $\rho(\omega)$  as  $\omega \rightarrow 0$ . Below we estimate the integral over the layer by replacing the density of states  $\rho(\omega)$  with its mean value  $\rho(\omega, \delta)$  in the interval  $[\omega - \delta, \omega + \delta]$ . Then  $\psi_d^2(\omega) \rightarrow \langle \psi_d^2(\omega, \delta) \rangle$  and, as in the previous case, we find that

$$\tau_c^{(d)} \approx 1.14c(n)\delta \exp \left\{ - \frac{c(n)}{(\alpha + \alpha_B)\langle \psi_d^2(m, \delta) \rangle} \right\}, \quad (60)$$

with  $\delta \geq 2\tau_c^{(d)}/c(n)$ . Here we have also introduced the function  $\alpha_B$  according to the definition

$$\frac{c(n)}{\alpha} - \frac{I_B^{(d)}}{2} \equiv \frac{c(n)}{\alpha + \alpha_B}, \quad \alpha_B = \alpha^2 \frac{I_B^{(d)}}{2c(n) - \alpha I_B^{(d)}}; \quad (61)$$

$\alpha_B$  describes the effect of band enhancement of the parameter  $\alpha$ .

Representing the solutions in the form (58) or (60) is convenient when we wish to do a comparison with the BCS theory, in which  $T_c \propto \exp\{-1/N(\varepsilon_F)V\}$ , where  $N(\varepsilon_F)$  is the density of states at the Fermi level, and  $V$  is the effective attraction. The dimensionless parameter  $N(\varepsilon_F)V$  is similar to the expressions  $(\alpha + \alpha_B)\langle \psi_d^2(m, \delta) \rangle$  and  $(\lambda + \lambda_B)\psi_p^2(m)$  used in our theory. In contrast to the case of triplet superconductivity, the function  $\langle \psi_d^2(m, \delta) \rangle$  in the exponent of the exponential function is large compared to  $\psi_p^2(m)$  in the vicinity of  $m=0$ , which ensures, other things being equal, substantially larger values of  $\tau_c^{(d)}$ .

## 7. CONCLUSION

The proposed  $t$ - $J$ - $I$  model makes it possible to study the superconductivity of strongly correlated electrons with different symmetries ( $s$ ,  $p$ , or  $d$ ) of the order parameters, facilitates comparison of the high- $T_c$  superconductivity of copper oxides and the superconductivity of the ruthenate  $\text{Sr}_2\text{RuO}_4$  with low  $T_c$  within a unified approach. The above comparison answers the question of why there is high- $T_c$  superconductivity ( $T_c \sim 100$  K) in layered copper oxides. In light of our results, the features that copper oxides exhibit are as follows: in two-dimensional  $\text{CuO}_2$  layers, owing to the  $(d_{x^2-y^2-p})-\sigma$  bond and strong electron correlations, there form quasiparticles with particle-to-particle hops occurring against the background of strong antiferromagnetic fluctuations. These fluctuations lead to  $d_{x^2-y^2}$ -type pairing, with the gap anisotropy being such that the Van Hove singularity, which increases the ordinary logarithmic contribution in the equations for  $T_c$  and the order parameter, manifests itself in full.



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<sup>1</sup>Note that in their monograph,<sup>5</sup> Izyumov *et al.* used the diagrammatic technique in the  $t$ - $J$  model to derive a similar expression for the gap, where, however, the term  $-g\omega_{\mathbf{k}-\mathbf{p}}$  (in our notation) replaces the  $\mathbf{p}$ -symmetric combination  $-g(\omega_{\mathbf{k}+\mathbf{p}} + \omega_{\mathbf{k}-\mathbf{p}})$  in (15). This discrepancy has certain consequences, which we leave for Sec. 6.

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