Spin-wave susceptibility of partially randomized ferromagnetic superlattices

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This paper is a theoretical investigation of the effect of inhomogeneities in the period of a ferromagnetic superlattice on the high-frequency superlattice susceptibility. The calculations are done for a model in which the uniaxial magnetic anisotropy is taken as the physical parameter that characterizes both the ideal superlattice and a partially randomized superlattice. It is found that as the inhomogeneities become more intense, the two resonance peaks corresponding to the splitting of the spectrum at the edge of the Brillouin zone of the superlattice broaden, move closer to each other, and finally merge into one. The height of this peak increases and the peak width decreases as the intensity of the inhomogeneities increases further. The effect of inhomogeneities on the susceptibility differs dramatically in the two limits of short- and long-wave inhomogeneities: in the latter case (in contrast to the former) the dependence of the separation of the susceptibility peaks on the intensity and correlation properties of the inhomogeneities is nonmonotonic. The possibility of observing these effects in spin-wave resonance experiments involving multilayer magnetic films is also discussed. © *1999 American Institute of Physics.* [S1063-7761(99)01510-3]

1. INTRODUCTION

The problem of propagation of waves in partially or completely randomized multilayer structures (one-dimensional superlattices) has lately received much attention. There are several approaches in developing a theory: modeling stochastization by a random arrangement of layers of two different materials;[1] computer simulation of random deviations of the surface between layers from their initial periodic arrangement;[2] and introduction of a doubly periodic dependence (with incommensurate periods) of physical parameters on the coordinate along the superlattice axis[3] (only some typical papers on this subject have been cited here, since there are many publications devoted to each approach).

One approach has been developed in our two papers, Refs. 4 and 5. A brief discussion of the results obtained for one-dimensional inhomogeneities described by a correlation function with an exponential decay of correlations can be found in Ref. 4. In Ref. 5, we systematically develop a method for one-, two-, and three-dimensional inhomogeneities of the sublattice period. Our approach differs from methods used earlier in that we do not postulate the correlation properties of the superlattice—we derive them from very general assumptions concerning the nature of stochastic spatial modulation of the sublattice period. We then find the spectrum and decay of waves by studying the averaged Green's function containing the correlation function established earlier. The theory is developed for spin, elastic, and electromagnetic waves.

During recent years extensive experimental research has been in progress in which spin-wave resonance is observed in multilayer ferromagnetic films.[6,7] As is known, the spin wavelength λ_s in thin films is determined by a size effect, which allows generating spin waves with $\lambda_s \lesssim d$, where d is the thickness of the ferromagnetic film, by using electromagnetic fields with a wavelength $\lambda \gg d$. This makes it possible to meet the conditions in which both the frequency and the wavelength of the generated wave coincide with the corresponding parameters of the edge of the superlattice Brillouin zone. It is at this edge that the spectrum of spin waves is most sensitive to inhomogeneities in the superlattice structure. Since the physical parameter observed in the experiments is the high-frequency magnetic susceptibility, it would be interesting to theoretically study this characteristic of a multilayer system. In the present paper we investigate the high-frequency susceptibility of a superlattice for a model system, whose correspondence to a real system is discussed below. We assume that the initial superlattice is a magnetic structure with a harmonic dependence of uniaxial magnetic anisotropy along the z axis, with the direction of the anisotropy axis remaining constant and parallel to the z axis. This is the simplest model for a theoretical study. It also provides the possibility of demonstrating the main features of the modification of the spin-wave spectrum in superlattices.

2. HIGH-FREQUENCY SUSCEPTIBILITY

The dynamics of a ferromagnetic system is described by the Landau–Lifshitz equation

$$\dot{\mathbf{M}} = -g(\mathbf{M}\times\mathbf{H}_{\mathrm{eff}}), \tag{1}$$

where $\mathbf{M}$ is the magnetization, g is the gyromagnetic ratio, and $\mathbf{H}_{\mathrm{eff}}$ is the effective magnetic field.

In the geometry corresponding to spin-wave resonance, the external magnetic field $\mathbf{H}$ is directed along the reciprocal superlattice vector $\mathbf{q}$ parallel to the z axis, i.e., perpendicular to the magnetic film, while the high-frequency field $\mathbf{h}$ lies in the xy plane of the film. Accordingly, the components of the effective magnetic field $\mathbf{H}_{\text{eff}}$ are

$$H_{\text{eff}}^x = \alpha \frac{d^2 M_x}{dz^2} + h_x, \quad H_{\text{eff}}^y = \alpha \frac{d^2 M_y}{dz^2} + h_y,$$

$$H_{\text{eff}}^z = \alpha \frac{d^2 M_z}{dz^2} + H_i + \beta(z) M_z, \tag{2}$$

where α is the exchange parameter, β is the magnetic anisotropy parameter, and $H_i = H - 4\pi M_z$ is the internal constant magnetic field, which allows for the demagnetizing field of the film. If we take into account the symmetry of the current problem, all dynamic demagnetizing fields vanish.

We write the anisotropy parameter $\beta(z)$ in the form

$$\beta(z) = \beta[1 + \gamma\rho(z)], \tag{3}$$

where β is the average value of anisotropy, γ is its rms fluctuation, and $\rho(z)$ is the centered ($\langle\rho\rangle = 0$) and normalized ($\langle\rho^2\rangle = 1$) function describing a superlattice with a stochastically modulated period.

Linearizing Eq. (1) in the usual manner ($M_z \approx M$ and $M_x, M_y \ll M$), we obtain an equation for the resonant circular projections $m = M_x + iM_y$ and $h = H_x + iH_y$:

$$\nabla^2 m + (\nu - \epsilon\rho(z)) m = -\frac{\hbar}{\alpha}, \tag{4}$$

where we have introduced the notation

$$\nu = \frac{\omega - g(H + \beta M - 4\pi M)}{\alpha g M}, \quad \epsilon = \frac{\gamma\beta}{\alpha}, \tag{5}$$

with ω being the frequency of the external electromagnetic field. At frequencies used in spin-wave experiments ($\omega/2\pi \sim 10^{10}$ Hz), the wavelength ($\lambda \sim 1$ cm) is much greater than the thickness of films being studied ($d \sim 10^{-5}$ cm). Hence the amplitude h of the high-frequency field on the right-hand side of Eq. (4) may be assumed constant (time-independent). For such a field to excite standing spin waves in the film, the magnetic moment must be at least partially fixed at the surfaces of the film. We assume that this moment is completely fixed at the surfaces:

$$m(z)|_{z=\pm d/2} = 0; \tag{6}$$

the origin of the z axis is chosen at the center of the film. Such conditions may be created by depositing additional layers of a magnetically hard alloy on both surfaces of the film.

Kittel[8] was the first to solve for the spectrum of spin waves in a thin homogeneous film (see, e.g., Gurevich and Melkov's monograph[9]), essentially by solving Eq. (4) with the boundary conditions (6) at $\epsilon = 0$. The spectrum is given by the expressions

$$\nu = k_n^2, \quad k_n = \frac{\pi n}{d}, \tag{7}$$

with the field h exciting only symmetric vibrations $m \sim \cos k_n z$ corresponding to an odd number of half-waves that fit into the film thickness, i.e., $n = 1, 3, 5, \ldots$.

To investigate Eq. (4) for $\epsilon \neq 0$, we expand m, ρ, h in the eigenfunctions of the unperturbed problem:

$$m(z) = \sum_{n=-\infty}^{+\infty} m_n \cos k_n z, \quad \text{etc.} \tag{8}$$

Then for the Fourier transforms of the corresponding functions, m_n, ρ_n, and h_n, we obtain the equation

$$(\nu - k_n^2) m_n = \epsilon \sum_{l=-\infty}^{+\infty} m_l \rho_{n-l} + \frac{h_n}{\alpha}, \tag{9}$$

where the Fourier transforms of the high-frequency field are given by the expression

$$h_n = \begin{cases} \dfrac{2h}{\pi n} \sin \dfrac{\pi n}{2}, & n \neq 0, \\ 0, & n = 0. \end{cases} \tag{10}$$

We see that the equation does not contain the term in the series with $l = n$, since $\rho(z)$ is a centered function. Substituting the formal solution of Eq. (9) into the right-hand side of the same equation and averaging over the ensemble of random realizations of the function $\rho(z)$, we obtain

$$(\nu - k_n^2)\langle m_n\rangle = \epsilon^2 \sum_{l \neq n} \sum_{l_1 \neq l} \frac{\langle m_l \rho_{l-l_1} \rho_{n-l}\rangle}{\nu - k_l^2} + \frac{h_n}{\alpha}. \tag{11}$$

Now we decouple the correlator on the right-hand side of the equation in the approximation of the first nonvanishing perturbative term (the Bourret approximation[10]),

$$\langle m_l \rho_{l-l_1} \rho_{n-l}\rangle \approx \langle m_l\rangle \langle \rho_{l-l_1} \rho_{n-l}\rangle, \tag{12}$$

and use an identity valid for all homogeneous random functions (see, e.g., Ref. 11),

$$\langle \rho_{l_1} \rho_{l_2}\rangle = \langle |\rho_{l_1}|^2\rangle \delta_{l_1 l_2}. \tag{13}$$

This leads us to a solution of the form

$$\langle m_m\rangle = \frac{h_n}{\alpha} \left\{ \nu - k_n^2 - \epsilon^2 \sum_{l \neq n} \frac{\langle |\rho_{n-l}|^2\rangle}{\nu - k_l^2} \right\}^{-1}. \tag{14}$$

Experimenters measure the response, averaged over the film volume, of the system to the high-frequency field:

$$\overline{m} = \frac{1}{d} \int_{-d/2}^{d/2} m(z)\, dz = \sum_{n=-\infty}^{+\infty}{}' \left(\frac{2}{k_n d} \sin \frac{\pi n}{2} \right) m_n, \tag{15}$$

where the prime on the sum indicates the absence of the term with $n = 0$.

Thus, the average susceptibility observed in spin-wave experiments is the sum of partial susceptibilities,

$$\chi = \frac{\langle \overline{m}\rangle}{h} = \sum_{n=-\infty}^{+\infty}{}' \chi_n, \tag{16}$$

where

$$\chi_n = \left(\frac{2}{\pi n} \sin \frac{\pi n}{2} \right)^2 \frac{1}{\alpha} \left\{ \nu - k_n^2 - \epsilon^2 \sum_{l \neq n} \frac{\langle |\rho_{n-l}^2|\rangle}{\nu - k_l^2} \right\}^{-1}. \tag{17}$$

The study of an expression containing a sum over discrete k_n poses a serious problem. We therefore limit ourselves to the study of the continuous analog of this expression (replacing summation by integration):

$$\chi(\nu,k)=a(k)\left\{\nu-k^2-\epsilon^2\int\frac{S(k-k_1)dk_1}{\nu-k_1^2}\right\}^{-1}, \qquad (18)$$

where $a(k)=\alpha^{-1}(2/kd)^2\sin^2(kd/2)$. Here we must bear in mind that this expression describes the frequency dependence of χ only near the discrete values of the wave number, $k=k_n$. The function $S(k)$ is the spectral density of the random function $\rho(z)$ and is related to the superlattice correlation function $K(r)$ via the Fourier transform (the Wiener–Khinchin theorem)

$$K(r)\equiv\langle\rho(z)\rho(z-r)\rangle=\int S(k)\exp\{ikr\}dk. \qquad (19)$$

So far we have not made any assumptions concerning the function $\rho(z)$, except that it is a centered and normalized homogeneous random function. In accordance with Ref. 5, we write

$$\rho(z)=\sqrt{2}\cos[q(z-u(z))], \qquad (20)$$

where $q=|\mathbf{q}|$ is the wave number of the initial superlattice, and $u(z)$ is a random function describing the inhomogeneity of the period of this superlattice.

Thus, we will examine a model in which the physical parameter characterizing the superlattice has, in this initial state (at $u(z)\equiv 0$), a harmonic dependence on z. A method for finding the correlation function of such a superlattice was developed in Ref. 5. It amounted to a generalization to the case of partially randomized superlattices of the theory of stochastic frequency (or phase) modulation of a periodic radio signal, well-known in radiophysics.[11,12] The correlation properties of the superlattice are expressed in this method in terms of the stochastic characteristics of the function $u(z)$ (more precisely, of the derivative du/dz). Here the shape of the correlation function $K(r)$ of the superlattice is independent, in the limits of long- and short-wave inhomogeneities, of the shape of the correlation functions modulating the initial inhomogeneities du/dz. At the same time, the shape of $K(r)$ is extremely sensitive to the correlation length of the inhomogeneities. The correlation function obtained in Ref. 5 for the two limits, long- and short-wave inhomogeneities, corresponding to a random shift in the boundaries separating the layers, has the form

$$K(r)=\cos(qr)\begin{cases}\exp\left\{-\dfrac{k_{c1}^2r^2}{2}\right\}, & p_0\ll 1,\\ \exp\{-k_{c2}r\}, & p_0\gg 1,\end{cases} \qquad (21)$$

where $p_0=k_\parallel/\sigma q$, and $k_{c1}=\sigma q$ and $k_{c2}=(\sigma q)^2/k_\parallel$ are the effective correlation wave numbers of the superlattice, with σ and $k_\parallel$ the rms fluctuation and the correlation wave number of the random function du/dz. Thus, irrespective of the shape of the correlation function modeling the properties of the initial inhomogeneities du/dz, the correlation function of the superlattice has a Gaussian decay of correlations for long-wave inhomogeneities and an exponential decay for short-wave inhomogeneities.

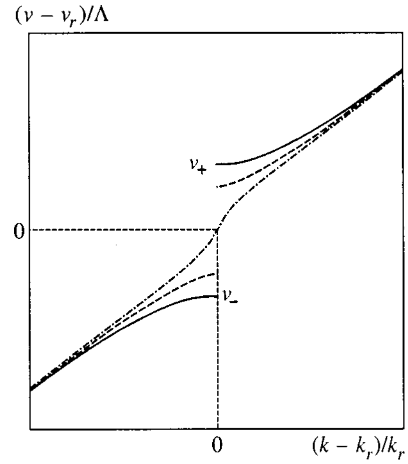

FIG. 1. Dispersion relation for the superlattice near the edge of the Brillouin zone (for more details see the main body of the text).

Using the Fourier transform to express the spectral density $S(k)$ corresponding to the correlation function (21) and substituting the result into (18), we obtain a formula for the superlattice susceptibility:

$$\chi=\frac{a(k)}{\nu-k^2-(\Lambda^2/4)F(L_-+L_+)}, \qquad (22)$$

where $\Lambda=\epsilon\sqrt{2}$, and the functions F and $L_\pm$ are determined for each limit, $p_0\gg 1$ and $p_0\ll 1$, by different expressions. We begin with the limit $p_0\gg 1$, corresponding to short-wave inhomogeneities. Then

$$F=1-\frac{ik_{c2}}{\sqrt{\nu}},\quad L_\pm=\frac{1}{(\sqrt{\nu}-ik_{c2})^2-(k\pm q)^2}, \qquad (23)$$

By equating the denominator in (22) to zero we obtain the dispersion equation for averaged spin waves in the superlattice (this equation has been studied in Refs. 4 and 5). The qualitative behavior of the results is schematically depicted in Fig. 1. At the edge of the Brillouin zone, corresponding to $k=k_r\equiv q/2$, the spectrum of the initial ideal ($k_{c2}=0$) superlattice exhibits a gap, $\Delta\nu\equiv\nu_+-\nu_-=\Lambda$ (the solid curve in Fig. 1). As k_{c2} increases, the gap decreases (the dashed curve) according to

$$\Delta\nu=\sqrt{\Lambda^2-G_2^2}, \qquad (24)$$

where $G_2=qk_{c2}=\sigma^2q^3/k_\parallel$ is the parameter characterizing the decay due to inhomogeneities. When $G_2>\Lambda$, the dispersion relation for the averaged waves is continuous and has a point of inflection at $k=k_r$ (the dot–dash curve in Fig. 1).

We study the dependence of the susceptibility (22) on the frequency ν at the edge of the Brillouin zone ($k=k_r$). Here we can limit ourselves to the two-wave approximation and discard the nonresonant term L_+. The susceptibility becomes

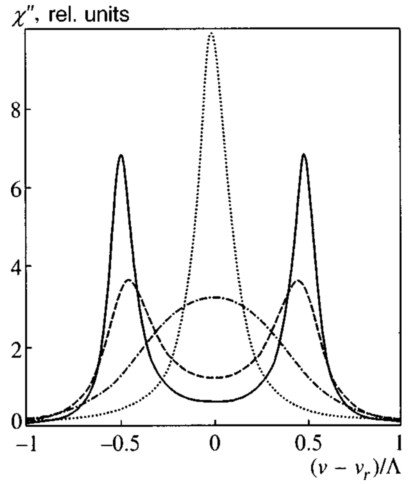

FIG. 2. The imaginary part of susceptibility, χ'', for the case of short-wave inhomogeneities at $G_2/\Lambda = 0.15$ (solid curve), 0.3 (dashed curve), 0.8 (dot–dash curve), and 2.5 (dotted curve).

$$\chi = \frac{a(k_r)(\nu - \nu_r - k_{c2}^2 - 2ik_{c2}\sqrt{\nu})}{(\nu-\nu_r)(\nu-\nu_r-k_{c2}^2-2ik_{c2}\sqrt{\nu}) - (\Lambda^2/4)(1 - ik_{c2}/\sqrt{\nu})}. \tag{25}$$

The frequency dependence of the imaginary part of the susceptibility, χ'', is depicted in Fig. 2 for four values of the ratio G_2/Λ. Clearly, for small values of G_2 there are two narrow peaks (the solid curve). As G_2 increases (the dashed curve), the height of the peaks decreases, the widths increase, and the peaks move closer to each other, and at a certain value of G_2 they merge into one broad resonance peak (the dot–dash curve). A further increase in G_2 leads to a decrease in the peak width as the peak becomes higher (dotted curve). Figure 3 depicts the decrease in the separation of the resonance peaks, $\Delta\nu_m$, with increasing G_2 (the solid curve). For the sake of comparison, we also give the dependence of the gap width $\Delta\nu$ in the spectrum of the average waves corresponding to (24) (the dashed curve). Clearly, the separation

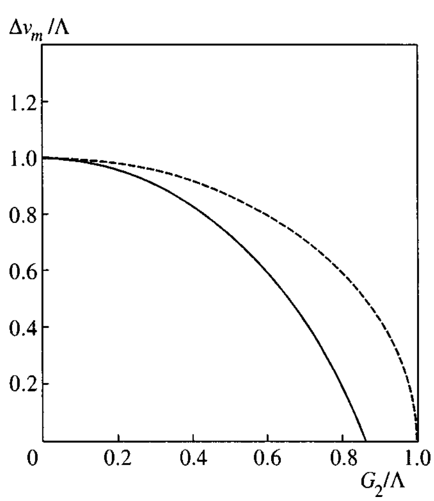

FIG. 3. Separation of the resonance peaks in the susceptibility, $\Delta\nu_m$ (solid curve), and the gap in the spectrum, $\Delta\nu$ (dashed curve) in the presence of short-wave inhomogeneities.

of the peaks, $\Delta\nu_m$, is always less than $\Delta\nu$ and the two peaks merge into one at values of G_2 less than $G_2 = \Lambda$, which corresponds to the collapse of the gap in the spectrum. Using these diagrams, we can determine the gap width in the spectrum, $\Delta\nu$, by observing the separation of the resonance peaks, $\Delta\nu_m$.

The imaginary part of the susceptibility can be represented by the sum of two resonances (if we neglect k_{c2}^2 in (25) and allow for the smallness of k_{c2}/k_r):

$$\chi'' = \frac{\Gamma_2 a(k_r)}{\Delta\nu}\left[\frac{\Delta\nu + \nu_r - \nu}{(\nu - \nu_r - \Delta\nu/2)^2 + \Gamma_2^2} + \frac{\Delta\nu - \nu_r + \nu}{(\nu - \nu_r + \Delta\nu/2)^2 + \Gamma_2^2}\right], \tag{26}$$

where $\Gamma_2 = G_2/2$ is the width of the resonance peaks, and $\Delta\nu$ is determined by (24).

The separation of these resonance peaks is given by the formula

$$\Delta\nu_m = 2\sqrt{\Lambda^2 - G_2^2} - \Lambda. \tag{27}$$

The expressions (26) and (27) provide a good approximation to the exact curves in Figs. 2 and 3.

If $G_2 \gg \Lambda$, i.e., when there is only one well-resolved central peak, χ'' can be expressed as

$$\chi'' = \frac{\Gamma_2' a(k_r)}{(\nu - \nu_r)^2 + \Gamma_2'^2}, \tag{28}$$

where the parameter $\Gamma_2' = \Lambda^2/4G_2$ acts as the effective decay constant. The value of this decay decreases with increasing G_2, with the result that the height of the resonance peak increases (Fig. 2).

Now we turn to long-wave one-dimensional inhomogeneities, corresponding to the Gaussian decay of correlations [the upper line in Eq. (21)]. Here

$$F = (2k_{c1}^2\nu)^{-1/2},$$

$$L_\pm = D(u_\pm) + D(v_\pm) + i\frac{\sqrt{\pi}}{2}(\exp\{-u_\pm^2\} + \exp\{-v_\pm^2\}), \tag{29}$$

where $D(x) = \exp(-x^2)\int_0^x \exp(t^2)dt$ is Dawson's integral, and

$$u_\pm = \frac{\sqrt{\nu} - |k \pm q|}{k_{c1}\sqrt{2}}, \quad v_\pm = \frac{\sqrt{\nu} + |k \pm q|}{k_{c1}\sqrt{2}}. \tag{30}$$

If we examine the susceptibility near the right-hand boundary of the Brillouin zone, we can neglect the term L_+ in the two-wave approximation, with the result that the susceptibility becomes (from now on we drop the subscript ''−'' in u_- and v_-)

$$\chi = \frac{a(k_r)}{\nu - k^2 - (\Lambda^2/4k_{c1}\sqrt{2\nu})[D(u)+D(v)+i(\sqrt{\pi}/2)(\exp\{u^2\}+\exp\{v^2\})]}. \tag{31}$$

The frequency dependence of the imaginary part of this expression (i.e., χ'') is depicted in Fig. 4 for four values of the parameter $G_1 = qk_{c1} = \sigma q^2$, which characterizes the decay, due to long-wave inhomogeneities. Clearly, in addition to the features common to both short-wave (Fig. 2) and long-wave (Fig. 4) inhomogeneities, there is an important difference between these two limits. Quantitatively, this difference manifests itself in the height and width of the resonance peaks for the same values of G_1/Λ and G_2/Λ for long- and short-wave inhomogeneities. Qualitatively, the difference amounts to the fact that for long-wave inhomogeneities, an increase in G_1 first leads to an increase in the separation of the peaks, and only after that do the peaks move closer to each other and merge into a single central resonance. A similar effect was obtained in Ref. 5 for the gap in the spectrum in the presence of long-wave inhomogeneities. Figure 5 shows that for the case of long-wave inhomogeneities, the peaks move closer to each other and finally merge at values of G_1 at which the gap $\Delta\nu$ is still only weakly modified by the inhomogeneities.

We studied the expression (31) numerically, but in the two limits the susceptibility is given by simple formulas. In particular, when G_1 is small, we have $u \gg 1$, and for Dawson's integral we have the simple expression

$$D(u) \approx \frac{1}{2u}\left(1+\frac{1}{2u^2}\right). \tag{32}$$

Ignoring the nonresonant terms containing v, we obtain a formula for the imaginary part of the susceptibility as a sum of two susceptibilities:

$$\chi'' = \frac{\Gamma_1 a(k_r)}{\Delta\nu}\left[\frac{\nu-\nu_r}{(\nu-\nu_r-\Delta\nu/2)^2+\Gamma_1^2} + \frac{\nu_r-\nu}{(\nu-\nu_r+\Delta\nu/2)^2+\Gamma_1^2}\right], \tag{33}$$

where the gap $\Delta\nu$ and the width Γ_1 of the resonance peaks are now given by the formulas

$$\Delta\nu = \sqrt{\Lambda^2+4G_1^2},$$
$$\Gamma_1 = \left(\frac{\pi}{2}\right)^{1/2}\frac{\Lambda^2-4G_1^2}{8G_1}\exp\left\{-\frac{\Lambda^2+4G_1^2}{8G_1^2}\right\}. \tag{34}$$

An increase in G_1 moves the resonance peaks apart, so that Eq. (33) corresponds to the initial section of the curve for $\Delta\nu_m$ in Fig. 5.

When the decay is large, Dawson's integral can be neglected in the denominator of (31). Then only an imaginary quantity is left as a factor of Λ^2, and the imaginary part of the susceptibility is given by an expression similar to (28) in which $\Gamma_1' = \sqrt{\pi/2}\Lambda^2/4G_1$ replaces Γ_2'.

3. DISCUSSION

What are the limitations that the simplifying assumptions used in our calculations impose on the comparison of the results obtained in this paper with the data of spin-wave experiments in thin films with a multilayer structure?

The calculations of the effect of inhomogeneities on the high-frequency magnetic susceptibility of the superlattice

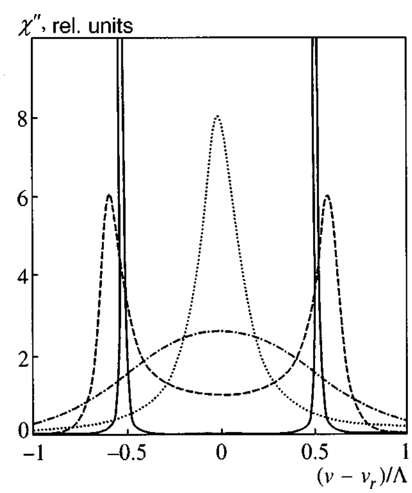

FIG. 4. The imaginary part of the susceptibility, χ'' (in relative units), for the case of long-wave inhomogeneities at $G_1/\Lambda = 0.15$ (solid curve), 0.3 (dashed curve), 0.8 (dot–dash curve), and 2.5 (dotted curve).

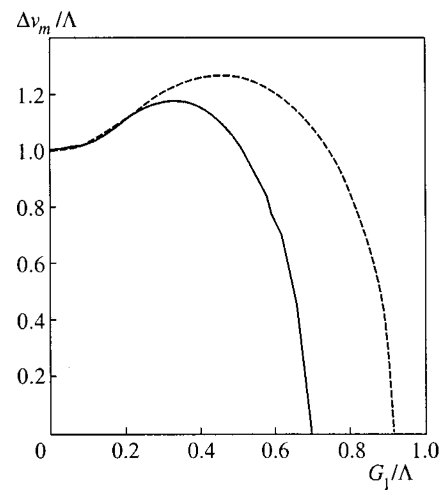

FIG. 5. Separation of the resonance peaks in the susceptibility, $\Delta\nu_m$ (solid curve), and the gap in the spectrum, $\Delta\nu$ (dashed curve) in the presence of long-wave inhomogeneities.

were done here for a model in which uniaxial magnetic anisotropy acts as the physical parameter characterizing both the ideal superlattice and a partially randomized superlattice. In real superlattices the magnetization may also be such a parameter, as well as the exchange parameter and the orientation of the anisotropy axis. The calculations are different for each of these inhomogeneity parameters. However, the comparison of the results obtained in calculations of the effect on the dispersion law of the averaged spin waves in a superlattice with anisotropy inhomogeneities and exchange inhomogeneities[4] show that the modifications of the spectrum at the edge of the Brillouin zone do not differ too much in these two cases. Hence we can expect that the differences will not be too large for the high-frequency susceptibility for the inhomogeneities of the various physical parameters either, with the result that basically only Λ will need to be redefined.

In spin-wave experiments only discrete values of the wave vector, $k_n = \pi n/d$, $n = 1,3,5$, can be observed. Consequently, the dispersion relation of the waves, depicted schematically in Fig. 1, can be obtained only at distinct, fairly distant, points, and generally among these there is not a single one that coincides with the edge of the Brillouin zone $k_r = q/2$. Note that our investigation of the frequency dependence of χ'' in this paper was done for $k = k_r$, so a comparison of the results of calculations is valid only with the data of an experiment in which coincidence of k_r with one of the wave numbers from the set $\{k_n\}$ is achieved only by special selection of the film thickness d and the superlattice period $l = 2\pi/q$.

In the present paper, as in Ref. 5, we used a model in which the physical parameter characterizing the superlattice varies along the z axis of the initial ideal superlattice by the harmonic law $\rho(z) = \sqrt{2}\cos qz$. This corresponds to the limit of smooth boundaries between the layers of the superlattice, with the thickness of the ''boundary'' being equal to the thickness of the ''layer.'' On the other hand, experiments are done with multilayer structures, in which the boundary-to-layer thickness ratio is usually much less than unity. In this case the function $\rho(z)$ for the initial superlattice is much closer in shape to a train of rectangular pulses of different polarity than to a harmonic function. A theoretical study of the modification of the dispersion law and decay due to inhomogeneities has been carried out for all odd Brillouin zones of a superlattice in Refs. 13–15. It was found that the results obtained for this model differ substantially from those obtained for the model of Ref. 5 with a harmonic ρ vs. z dependence for all Brillouin zones except the first. For the first Brillouin zone the modification is determined by the first harmonic $\rho \sim \cos qz$ in the Fourier expansion of $\rho(z)$, so that the results for the two limiting models differ only by the numerical normalization factor $8/\pi^2$. Since the experimental investigations of spin-wave resonance and the theoretical calculations of χ'' in the present paper were done for the first Brillouin zone, the discrepancy in the models considered here should not have a strong influence on the precision of the comparison of theory and experiment.

Our calculations were done on the assumption that the intrinsic decay of the spin system is much weaker than the decay due to the inhomogeneities in the sublattice period. Only in this situation can the effects described in this paper manifest themselves, and by comparing the theoretical results and the experimental data one can determine the real gap in the spectrum of spin waves (using the diagrams in Figs. 3 and 5) and measure the parameter that determine the correlation properties of the inhomogeneities.

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