# Direct-current generation due to wave mixing in semiconductors

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# Direct-current generation due to wave mixing in semiconductors

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**Abstract.** – We describe an effect of the generation of direct current which may arise in semiconductors or semiconductor microstructures due to a mixing of coherent electromagnetic radiations of commensurate frequencies. The effect is, in essence, due to a nonparabolicity of the electron energy bands and is stronger in systems where this nonparabolicity is greater. We have made exact calculations in the framework of the Kane model, applicable to narrow-gap semiconductors and the tight-binding model which we employ for a description of a semiconductor superlattice.

The problem of emission and reception of electromagnetic radiation has attracted the attention of the scientific community for a long time. Good sources of coherent electromagnetic radiation, its receivers and detectors exist for the radio-frequency, microwave and optical ranges of the spectrum. Many of these devices are based on semiconductor technology. Nowadays, the terahertz range is the last unexploited *terra incognita*.

In 1970, Esaki and Tsu [1] made a pioneering suggestion to use the semiconductor superlattice (SSL) for the generation of Bloch oscillations of frequency $\omega_{\mathrm{B}} = eaE_0/\hbar$, where $E_0$ is a constant electric field applied along the axis of a SSL with a spatial period $a$. For typical SSLs and bias $E_0$ of 1–10 kV/cm, the Bloch frequency belongs to the THz range [1]. This can be used for the generation of THz radiation [2]. The work of Esaki and Tsu [1] stimulated enormous theoretical activity devoted to the interaction of a high-frequency electric field with a SSL [3]. Moreover, recent progress in the development of THz radiation sources and coupling techniques allow the systematic experimental studies of many of the associated nonlinear effects [4]. Amongst others, one of the most interesting suggestions was that of Esaki and Tsu [5] and Romanov [6] to use the SSL as a new, artificial, nonlinear material for electromagnetic wave mixing and harmonic generation. The theory of wave mixing in SSLs, based on a solution of the Boltzmann equation, has been developed by Romanov and co-workers [7, 3].

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Recently, the effect inverse to the Bloch oscillations in SSL was found [8], namely, an alternating field without constant bias can create Bloch oscillations in a single miniband SSL. The appearance of induced Bloch oscillations means a spontaneous creation of constant voltage and corresponding direct current (DC) [8], *i.e.* a rectification of the THz field in SSLs which results from an interplay of dissipation (scattering of ballistic electrons with impurities and phonons) and generation of a self-consistent electric field along the SSL axis. Moreover, the generation of DC in [8] is a counterpart of chaos, which arises for the same ac field strength and frequency but for much weaker dissipation [8].

Independently, Goychuk and Hänggi [9] suggested another scheme of quantum rectification using a wave mixing of an alternating electric field and its second harmonic in a single miniband SSL. The approach of [9] is based on the theory of quantum ratchets and therefore the necessary conditions for the appearance of DC include a dissipation (quantum noise) and an extended periodic system.

In this work we have identified two different mechanisms leading to an effect of DC generation at wave mixing. The first mechanism is related to a nonparabolicity of the energy spectrum, which is always present in any semiconductor or its structure [10, 3] and the arising DC current may have a transient character. The transient DC could be generated during a time shorter than a characteristic scattering time of carriers or a dephasing time of a laser field. The best candidates for the described effect are semiconductors having wide bands and narrow gaps or doped semiconductors such as InSb and other analogous compounds. Indeed, wave-mixing has already been observed in such narrow-gap semiconductors [11]; its mechanism being mainly related to the nonparabolicity of the energy band.

The second mechanism for DC current generation is associated with dissipation or scattering of current carriers. Such DC have a stationary character and may arise in SSLs. Within the standard semiclassical Boltzmann equation approach [7, 3] we demonstrate that the value of the stationary DC is strongly dependent on the product of a characteristic scattering time, $\tau$, and field frequency, $\Omega$. In the experimentally relevant case of weak-field strength, the expression for DC can be represented in the form of the generalized Ohm's law with prefactor dependent on $\Omega\tau$. The value of DC is maximal at $\Omega \simeq \tau^{-1}$ (namely, $\Omega\tau \approx 0.7$) and it decreases quadratically for both small and large values of $\Omega\tau$. The effect we find is consistent with results obtained previously in a quantum approach [9] where a microscopic approach to dissipative nonlinear quantum system was used to show that the rectification is independent of the details of dissipation. Here, we use a simple physical picture associated with wave mixing in nonlinear media arising due to a nonparabolicity of the electron bands in semiconductors. In this framework the results of [8, 9], which at first glance look very different, may be unified.

*The rectification effect due to a nonparabolicity of the spectrum of the semiconductors.* – Consider, for simplicity, a cubic semiconductor subjected to the electric mixing harmonic fields

$$E(t) = E_1 \cos(\Omega t) + E_2 \cos(2\Omega t + \phi)\,. \tag{1}$$

With the aid of an effective mass method [10] the energy-momentum dispersion relation of a nondegenerate cubic semiconductor in the vicinity of the bottom of the conduction band may be represented as (see, for comparison, ref. [3])

$$\varepsilon(p_x, p_y, p_z) = \frac{p^2}{2m} + \frac{\eta}{4}(p_x^4 + p_y^4 + p_z^4), \quad \frac{1}{m} = \left.\frac{\partial^2 \varepsilon}{\partial p_i^2}\right|_{p_i=0}, \quad \eta = \left.\frac{1}{6}\frac{\partial^4 \varepsilon}{\partial p_i^4}\right|_{p_i=0}, \tag{2}$$

where $p^2 = p_x^2 + p_y^2 + p_z^2$, $m$ is an effective mass at the bottom of the conduction band, $\eta$ is a parameter of nonparabolicity. Following [3], for simplicity, we consider electron motion only along one direction, for example, along the **x**-direction and, therefore, we may limit ourselves to the one-dimensional approximation. From now on the index $x$ is suppressed. The electron velocity is given by $v = \partial\varepsilon/\partial p$, the DC related to nonparabolicity is $\tilde{j}_{\rm dc} = en\eta\langle p^3\rangle$, where $n$ is the number of electrons per unit volume and the angled brackets $\langle\ldots\rangle$ indicate the time averaging over a period of the electric field $2\pi/\Omega$. For pure ballistic electron motion without scattering by impurities or phonons, the electron dynamics is determined by the accelerating theorem $\dot{p} = eE(t)$. Combining this formula with eqs. (1) and (2), we have

$$\tilde{j}_{\rm dc} \propto -en\eta\frac{3}{8}\frac{e^3}{\Omega^3}E_1^2E_2. \tag{3}$$

Consider two specific examples where the value of the DC can be expressed explicitly through the well-established parameters of the semiconductor energy bands. The energy-momentum dispersion relations, $\varepsilon(\mathbf{p})$, of a cubic semiconductor is usually obtained within the effective mass method [10]. For a SSL, the dependence $\varepsilon(\mathbf{p})$ was obtained within a tight-binding model [3]. In both cases, in the weak-field limit when the excitation energy of electrons is small in comparison with the bandwidth, we may take into account only the first terms in the energy momentum relation (see the dependence (2)).

Now, consider the motion of an electron within a single miniband of the SSL with spatial period $a$, the miniband width $\Delta$ and the energy-momentum relation: $\varepsilon(p) = (\Delta/2)[1 - \cos(pa/\hbar)]$, where $p$ is the momentum of the electron along the SSL axis. For a weak electric field is the electrons oscillate in $p$-space near the center of the Brillouin zone $|p| \ll \pi\hbar/a$ or at the bottom of the miniband. Then, from $\varepsilon(p)$ we have (2) with $\eta = -a^2/6\hbar^2 m$ and $m = (2\hbar^2)/(\Delta a^2)$. The DC (eq. (3)) takes the form $\tilde{j}_{\rm dc} \simeq (en\hbar/am)\xi_1^2\xi_2$, where $\xi_l = (eaE_l/l\hbar\Omega)$, $(l = 1, 2)$. The condition of weak nonparabolicity, $|p| \ll \pi\hbar/a$, corresponds to $\xi_l \ll 1$.

*Influence of collisions of electrons with impurities and phonons dissipation effects.* – The electron transport properties in narrow miniband SSLs at temperatures above 40 K are known [12] to be well described by the semiclassical Boltzmann equation with a constant relaxation time, $\tau$. Starting from a formal solution of the Boltzmann equation with constant relaxation time, Romanov and co-workers found the exact expression for a time-dependent current $j(t)$ in a tight-binding lattice subjected to an electric field with two frequencies $\omega_1$ and $\omega_2$ [7, 13]. Taking $\omega_1 = \Omega$, $\omega_2 = 2\Omega$ and averaging $j(t)$ over the period of the ac field, $2\pi/\Omega$, we get for the rectified DC $j_{\rm dc} = \langle j(t)\rangle$, for $t \gg \tau$, the following formula:

$$j_{\rm dc} = j_0 \sum_{\mu_1,\mu_2=-\infty}^{+\infty} \sum_{\nu=-\infty}^{+\infty} \frac{(\mu_1 + 2\mu_2)x\cos(\nu\phi) + \sin(\nu\phi)}{1 + (\mu_1 + 2\mu_2)^2x^2} J_{\mu_1}(\xi_1)J_{\mu_2}(\xi_2)J_{\mu_1-2\nu}(\xi_1)J_{\mu_2+\nu}(\xi_2), \tag{4}$$

where $x = \Omega\tau$, $j_0 = \frac{\hbar\sigma}{e\tau a}$, with $\sigma$ being a static, Ohmic conductivity along the SSL axis, $J_\mu(\xi)$ is the Bessel function. The DC, eq. (4), strongly depends on the product $x = \Omega\tau$. Figure 1 illustrates the dependence of the DC on $\xi_1$ and $\xi_2$ for different values of $x$. For small $x$, the absolute value of the DC increases monotonically with an increase of both $\xi_1$ and $\xi_2$ (fig. 1a). The increasing slope of the DC graph changes dramatically when the value of $x$ increases. For instance, this slope can increase by almost five orders of magnitude with an increase in $x$ of only one order (compare figs. 1a and 1b). However, for further change of $x$ from $x = 0.2$ (fig. 1b) to $x = 1$ (fig. 1c), the corresponding increase in DC slows down and becomes nonmonotonical. The DC reaches its maximal value for some optimal value of the relationship between $\xi_1$ and $\xi_2$. So for the case $x = 1$, the DC is maximal when $\xi_1/\xi_2 \simeq 1.0$ (see fig. 1c). When the electric-field amplitudes are small, $\xi_{1,2} \ll 1$, we can use the Bessel function approximation $J_n(\xi) \approx (\xi/2)^n(1/n!)$ and obtain from (4) the following analytic expression for the DC [14]:

$$j_{\rm dc} = -1.5j_0\frac{x^3}{4x^4 + 5x^2 + 1}\xi_1^2\xi_2\cos\phi + {\rm O}(\xi^5). \tag{5}$$

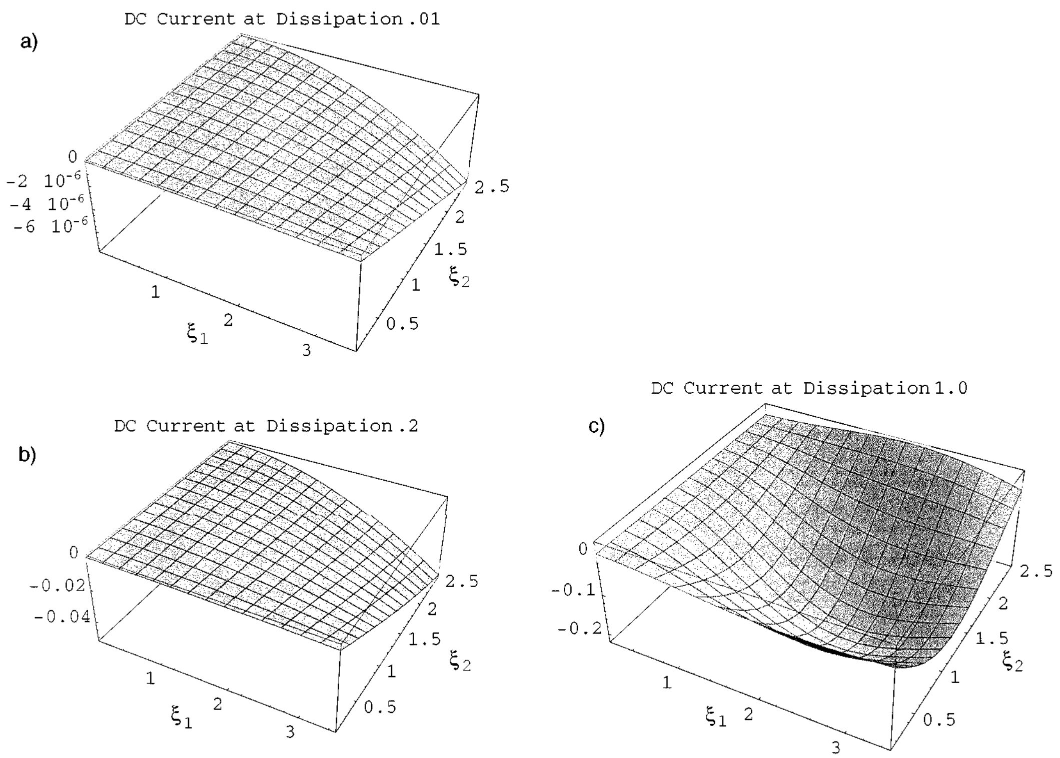


Fig. 1. – The dependence of scaled DC $j_{\mathrm{dc}}/j_0$ on scaled field amplitudes $\xi_1$ and $\xi_2$ (eq. (4)). The product $\Omega\tau$ has the values: $x = 0.01$ (a), $x = 0.2$ (b), and $x = 1.0$ (c). Phase $\phi = 0$.

Substituting the usual Drude conductivity $\sigma_{\mathrm{Drude}} = ne^2\tau/m$ into $j_0$, we get the factor $(en\hbar)/(am)$, which is similar to the prefactor for $\tilde{j}_{\mathrm{dc}}$ obtained within the collisionless approximation. Equation (5) agrees well with the asymptotic dependence of the DC on the field amplitudes, $j_{\mathrm{dc}} \propto \xi_1^2\xi_2 \cos\phi$ as obtained in ref. [9] in the framework of a different approach. However, our result (5) also gives the dependence of DC on the parameter $x = \Omega\tau$, as well as it indicates that the next significant contributions to the current appear only when the parameter, $\xi$, is of the order of $\xi^5$.

Let us estimate the value of the predicted DC for the experimental conditions of THz field driven SSLs [4]. For the miniband width $\Delta \simeq 10$ meV, superlattice period $a \simeq 10$ nm, electron density $n \simeq 10^{16}$ cm$^{-3}$, sample area $S \simeq 10\ (\mu\mathrm{m})^2$, electric field of amplitude $E \simeq 1$ kV/cm and frequency $\Omega$ of several THz ($\xi_1 = \xi_2 \simeq 0.1$) and the characteristic relaxation time of several picoseconds ($x = \Omega\tau \simeq 1$), we get the DC $I_{\mathrm{dc}} = j_{\mathrm{dc}}S \simeq 0.1\ \mu$A. This value is in very good agreement with an estimate for the DC obtained by Goychuk and Hänggi [9] from numerical calculations of the integral representations for DC within a different approach.

Note that DC density (5) can be represented in the form of the generalized nonlinear Ohm's law incorporating only the total electric field $E(t)$ (eq. (1)). Really, using the property $\langle E^3\rangle = (3/4)(E_1^2E_2)\cos\phi$, we have from (5) the following expression for the DC:

$$j_{\mathrm{dc}} = -\sigma f(x)\langle \overline{E}^2 E\rangle, \quad f(x) = \frac{x^2}{4x^4 + 5x^2 + 1}, \quad \overline{E} \equiv \frac{eaE(t)}{\hbar\Omega}. \tag{6}$$

The nontrivial prefactor $f(x)$ increases as $x^2$ in the limit $x \ll 1$, reaches its maximal value

$f(x^*) \approx 0.11$ at $x^* \approx 0.71$, and finally decreases as $\left(4x^2\right)^{-1}$ for high frequencies ($x = \Omega\tau \gg 1$). Note that in order to use the single miniband approximation and neglect the interminiband transitions, the electric-field frequency, in units of the energy $\hbar\Omega$, should be less than the interminiband distance or, in other words, it should be of the order of or less than the miniband width $\hbar\Omega \lesssim \Delta$. The case $\hbar\Omega \simeq \Delta$ is the typical case in experiments [4] (see also our estimates above). Thus, the dimensionless electric field, $\overline{E}$, (involved in the expression (6)) is of the order of $eaE/\Delta$. To apply the Boltzmann equation to the description of miniband transport in a quantum superlattice, the parameter $eaE/\Delta$ should always be small [3], *i.e.*, in our notation, $\overline{E} \ll 1$. This remark shows that the DC generated in the SSL under the action of the high-frequency electric field $E(t)$ is less than the corresponding current $j = \sigma E$ generated by a constant bias of the same strength $E$ in the factor $\simeq 0.1\overline{E}^2$. The numerical value of this factor is only $\simeq 10^{-3}$ for typical experiments in SSLs, which implies that the semiclassical approach is valid and at the same time gives serious grounds for the observation of DC generation due to wave mixing.

*The transient DC at harmonic mixing in the III-V type semiconductors.* – Mixing of mm-waves in $n$-doped InAs, InSb and GaAs semiconductors has been studied experimentally and theoretically in [11] a long time ago. The experiments were devoted only to the mixing of waves with similiar frequencies or third-harmonic generation. Semiconductors such as InAs, InSb can be described by the Kane four-band model [15,10]. Of course, for a proper description of the electron-hole dynamics we have to take into account all four bands, which is, in fact, a very tedious task. However, in a weak field assuming that, due to dominant donor doping, the electron concentration is larger than the hole concentration, we may take into account only one of the energy bands: the electron branch with the energy-momentum dispersion relation [15, 3, 10] in the form

$$\varepsilon(p) = \frac{\varepsilon_{\rm g}}{2}\left[\left(1 + \frac{2p^2}{m\varepsilon_{\rm g}}\right)^{1/2} - 1\right], \tag{7}$$

where $\varepsilon_{\rm g}$ is the width of the gap. Note that GaAs cannot be described by the dependence (7), however in the weak-field limit, the nonparabolicity, $\eta$, is roughly twice as great as follows from the Kane model. Note also that, due to a diamond structure of the III-V type semiconductors, into a nonparabolicity coefficient the cubic invariants will also contribute; however, these contributions should not exceed those which follow from the Kane model. In the limiting case $p/\sqrt{m\varepsilon_{\rm g}} \ll 1$, we have the dependence (2) with $\eta = -1/(m^2\varepsilon_{\rm g})$ and the DC generated at wave mixing (3) is of the order of

$$\tilde{j}_{\rm dc} \simeq \frac{3}{4} en\left(\frac{\varepsilon_{\rm g}}{m}\right)^{1/2} \tilde{E}_1^2\tilde{E}_2, \quad \tilde{E}_l \equiv \frac{eE_l}{l\Omega\left(m\varepsilon_{\rm g}\right)^{1/2}} \quad (l = 1, 2). \tag{8}$$

The condition of weak field $p/(m\varepsilon_{\rm g})^{1/2} \ll 1$ takes the form $\tilde{E}_l \ll 1$ ($l = 1, 2$). For the donor doped $n$-InSb with gap width $\varepsilon_{\rm g} \approx 0.2$ eV, effective mass $m \approx 0.016m_{\rm e}$, level of doping $n \simeq 10^{16}\,\text{cm}^{-3}$, sample area $S \simeq 10^{-4}\,\text{cm}^2$, field frequency $\Omega \simeq 10^{11}\,\text{s}^{-1}$ and field strength $\simeq 100$ V/cm, we get the DC $I_{\rm dc} = \tilde{j}_{\rm dc}S$ of the order of several mA. At the same time, the condition $\tilde{E} \simeq 0.1 \ll 1$ guarantees the applicability of the weak-field limit here.

Otherwise, the dependence of the DC will have a form similar to that obtained above for the case of a SSL, eq. (6). Finally, a directional photocurrent generation in an undoped bulk GaAs subjected by the femtosecond and picosecond laser pulses has been reported recently in [16]. However, DC at mixing of two light beams of frequencies $\Omega(= 0.775\ \mu\text{m})$ and $2\Omega$ was attributed to a different mechanism involving interband transitions and electron-hole plasma [16, 17].

In summary, with the use of the semiclassical Boltzmann equation we have found the novel effect of DC generation in semiconductors and semiconductor microstructures driven by two pure coherent electromagnetic waves of commensurate frequencies. The described effect originates from the nonlinearity associated with the nonparabolicity of the energy band and, therefore, it is universal and may be observed in any semiconductors. Thus, our findings indicate that any semiconductor may be considered as a particular type of nonlinear medium serving as a generator of DC [18]. Specifically, the effect of DC generation arising due to the mixing of mm-waves should exist in narrow-gap III-V and even some II-VI semiconductors. We hope that our results describing this new effect will attract the attention of experimentalists to this very intriguing issue.

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[13] See also ref. [3], p. 310, eq. (8.15).

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