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Propagation of two short pulses under conditions of electromagnetically induced transparency: adiabatic following

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ABSTRACT

Spatio-temporal dynamics of two short laser pulses propagating in an absorbing medium, which consists of one- and two-photon resonant Λ-atoms, is investigated in the adiabatic approximation. We give an analytical solution and compare it with numerical simulations. It is shown that pulses may propagate in a medium, whose thickness is considerably greater than the length of the linear absorption of a single weak probe wave. Finally, full transfer of the probe pulse energy into the coupling pulse takes place. A vector model of the adiabatic interaction of two pulses with three-level atom is proposed.

Keywords: electromagnetically-induced transparency, coherent population trapping, atomic coherence, quantum interference, adiabatic following, dressed field pulses

1. INTRODUCTION

The study of interaction of two laser radiation fields with a three-level Λ-system under simultaneous one-photon and two-photon resonance conditions represents one of "hotspots" of modern laser and optical physics. The effects of atomic coherence and quantum interference attract a great attention, for example, nonlinear interference effect (NIE) [1,2], electromagnatically-induced transparency (EIT) [3,4], coherent population trapping (CPT) [5,6], inversion-free amplification and generation [7]. These effects cardinally change optical characteristics of matter and allow to manipulate them. They are already used for control of an absorption and refraction indices [4,8], in engineering of ultrasensing phase measurements and optical interferometry [9], for measurements of weak magnetic fields [10], laser radiation frequency control [5], isotope separation [11], for increase of efficiency of resonant nonlinear frequency-mixing processes [3,12,13, 14].

Quantum interference phenomena also lead to interesting and curious effects at propagation of laser pulses in resonant three-level medium. Under certain conditions EIT and CPT are conserved in the temporal and space evolution of interacting pulses. Therefore pair of pulses can propagate without modification of the form, for example, matched pulses [4], "dressed-field" pulses [15], adiabatons [16], simultons [17, 18]. CPT generates also a maximal atomic coherence with simultaneous cancelation of resonant absorption of interacting waves. It may result in substantial increase of efficiency of nonlinear frequency conversion and parametric light generation [12a, 13,22].

Some aspects of pulse radiations propagation in resonant three-level medium were investigated, for example, in [15,16,23-27]. As usual there were considered the situations, when either both pulses have the identical form and duration, which exceeds the relaxation time of intermediate resonant state, or duration of one pulse is much more then duration of other pulse. In most cases situation is simplified by the adiabatic elimination of an intermediate level.

In this paper we study effects of spatial and temporal evolution of two short laser pulses, propagating through a resonant optically thick medium, which consists of three-level Λ-atoms. The pulses have an identical form, but different duration (see fig.1). The duration of pulses is small in comparison with all the relaxation times ($\Gamma_{ij}T_{1,2} << 1$, Γ_{ij} is the atomic relaxation rate; $T_{1,2}$ – the duration of the pulse, $T_2 > T_1$). Also, it is supposed that their envelopes satisfy to the adiabatic criterion [19]:

$$\sqrt{|G_1|^2 + |G_2|^2}T_{1,2} >> 1, \tag{1}$$

where $G_{1,2}$ are the Rabi frequencies of interacting pulses.

The conditions (1) can be fulfilled for short $\Gamma_{ij}T_{1,2} << 1$, but high-power pulses. Physically the adiabatic condition (1) means that envelopes of pulses are varied slowly with respect to time equal to inverse effective Rabi

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frequency $G = \sqrt{|G_1|^2 + |G_2|^2}$. Under the conditions the effect of CPT takes place. It results in a cancelation of absorption of propagating pulses. Also, the large coherence at the Raman transition, conserving over the length of a medium, which considerably exceeds the length of linear absorption of a single probe radiation.

The theoretical model consists of a system of equations for probability amplitudes and reduced wave equations for pulse envelopes, describing self-consistent spatio-temporal dynamics of atomic system and radiation fields. It is shown numerically and analytically, that under conditions (1) the pulses can propagate to distance, which is greater than the length of linear absorption of single probe pulse.

2. BASIC EQUATIONS AND THEIR SOLUTION

Let's consider a propagation of two pulses in a medium of three-level Λ-atoms, as shown in fig.1. The pulses propagate along the direction of z axis. Atomic levels $|0>$ and $|1>$, $|1>$ and $|2>$ have an opposite parity, the transition $|0> - |2>$ is dipole-forbidden, state $|0>$ is the ground state. The intermediate state $|1>$ is one-photon resonant to both fields, which interact only with the correspondent transitions. We'll name pulse with frequency ω_1 – probe, and another with ω_2 – coupling. The probe pulse duration is less than coupling one and much less than all atomic relaxation times of atomic subsystem.

In local time $\tau = t - z/c$ the slowly varying amplitude equations that describe the spatial and temporal evolution of two pulses in a three-level Λ-medium (fig.1) are:

$$\frac{\partial b_0}{\partial \tau} = iG_1^* b_1 \exp(-ik_1 z), \quad \frac{\partial b_2}{\partial \tau} = iG_2^* b_1 \exp(-ik_2 z), \qquad (2)$$
$$\frac{\partial b_1}{\partial \tau} = iG_1 b_0 \exp(ik_1 z) + iG_2 b_2 \exp(ik_2 z),$$

$$\frac{\partial G_1}{\partial z} = iK_1 b_1 b_0^* \exp(ik_1 z), \quad \frac{\partial G_2}{\partial z} = iK_2 b_1 b_2^* \exp(ik_2 z). \qquad (3)$$

The system of equations is written in the assumption that both carrier pulse frequencies are resonant to the appropriate transitions. $G_1 = d_{10}E_1(t)/2\hbar$, $G_2 = d_{12}E_2(t)/2\hbar$; $b_{0,1,2}$ – the probability amplitudes of states $|0>, |1>$ and $|2>$, respectively; $K_1 = \pi\omega_1 |d_{10}|^2 N/\hbar = \alpha_1\Gamma_{10}/4$, $K_2 = \pi\omega_2 |d_{21}|^2 N/\hbar = \alpha_2\Gamma_{12}/4$ – the propagation coefficients; $\alpha_{1,2}$ – the linear absorption indices of probe and coupling radiations with all the atoms in state $|0>$ or state $|2>$, respectively; Γ_{ij} – the transition half-widths. N – the atom concentration, d_{ij} – the dipole transition matrix elements; $k_{1,2}$ – the absolute values of wave vectors of interacting waves in vacuum. We consider, that all atoms are initially ($\tau = -\infty$) in the ground state $|0>$, and at the entrance of the medium ($z = 0$) both pulses have Gaussian envelopes $E_1(\tau) = E_1^0 \exp(-\tau^2/2T_1^2)$, $E_2(\tau) = E_2^0 \exp(-\tau^2/2T_2^2)$. The pulse amplitudes $E_{1,2}^0$ are considered to be real values. Parameters are selected to satisfy the adiabatic criterion (1) at the entrance of the medium $z = 0$.

In terms $a_0 = b_0 \exp(ik_1 z)$, $a_2 = b_2 \exp(ik_2 z)$, $a_1 = ib_1$, the equations (2) and (3) can be written as:

$$\frac{\partial a_0}{\partial \tau} = G_1^* a_1, \quad \frac{\partial a_2}{\partial \tau} = G_2^* a_1, \quad \frac{\partial a_1}{\partial \tau} = -G_1 a_0 - G_2 a_2 \qquad (4)$$

$$\frac{\partial G_1}{\partial z} = K_1 a_1 a_0^*, \quad \frac{\partial G_2}{\partial z} = K_2 a_1 a_2^* \qquad (5)$$

In the adiabatic approximation equations (4) have the solution for probability amplitudes:

$$a_0 \simeq \frac{G_2}{G}, \quad a_2 \simeq -\frac{G_1}{G}, \quad a_1 \simeq \frac{1}{G_1}\frac{\partial(G_2/G)}{\partial \tau} \simeq -\frac{1}{G_2}\frac{\partial(G_1/G)}{\partial \tau}, \qquad (6)$$

where $G = \sqrt{G_1^2 + G_2^2}$.

The expression for a_1 can be reduced to the following:

$$a_1 = (G_2\dot{G}_1 - G_1\dot{G}_2)/G^3. \qquad (7)$$

Using the condition (1), one can show that $|a_1| << 1$, i.e. the population of intermediate state $|1>$ is low during all the time of interaction. The last physically means, that the resonant absorption of pulses is decreasing. The population is distributed between initial and final states $|0>$ and $|2>$. So, there is an approximate equality:

$$|a_0|^2 + |a_2|^2 \simeq 1. \tag{8}$$

And solution for probability amplitudes $a_{0,2}$ is convenient to be presented as:

$$a_0 = \cos\theta, \qquad a_2 = -\sin\theta \tag{9}$$

Here θ is some angle, which sense will be clear from below.

It is useful to remark, that the equality (8) also reflects the fact, that atoms are trapped in a state of coherent population trapping, which probability amplitude is determined as $a_- = (G_2/G)a_0 - (G_1/G)a_2 = \cos(\theta)a_0 - \sin(\theta)a_2 = 1$. Thus the reduction of resonance absorption of interacting pulses is due to CPT effect.

The results can be interpreted in terms of a vector model (fig.2), using vectors $\vec{a} = (a_0, a_1, a_2)$ and $\vec{G} = (G_2, 0, -G_1)$ as variables. In the given notation the system (4) may be rewritten as:

$$\dot{\vec{a}} = \vec{G} \times \vec{a}, \tag{10}$$

where the sign $\times$ means a vector product. The solution of equation (10) is vector $\vec{a} = (G_2/G, 0, -G_1/G)$, which lies in plane $(\vec{i}, \vec{k})$, also including vector $\vec{G}$ under angle θ to axis $\vec{i}$ ($\cos\theta = G_2/G$). The components of vector $\vec{a}$ practically coincide with the adiabatic solution (6), as far as $|a_1| << 1$ and this value may be neglected. Therefore vector $\vec{a}$, corresponding to adiabatic solution (6), is practically parallel to vector $\vec{G}$. So, $\vec{a}$ adiabatically follows $\vec{G}$ and precesses around it with frequency $G = \sqrt{G_1^2 + G_2^2}$. Thus, there is an analogy with the case of adiabatic following at interaction of light pulse and two-level atom (see, for example, [28]). With account of propagation angle θ is a function of coordinates, as will be seen below.

Substituting the solution (6) in the field equations (5), we obtain a system of the connected nonlinear equations.

$$\frac{\partial G_1}{\partial z} = -(K_1/G)\frac{\partial(G_1/G)}{\partial \tau}, \qquad \frac{\partial G_2}{\partial z} = -(K_2/G)\frac{\partial(G_2/G)}{\partial \tau}. \tag{11}$$

One can show from (11), that the sum $K_2G_1^2(\tau, z) + K_1G_2^2(\tau, z)$ does not depend on coordinate z and is equal to $\tilde{G}^2(\tau) = K_2G_1^2(\tau, z = 0) + K_1G_2^2(\tau, z = 0)$. It reflects the fact, that the pulses propagate concordantly. Let's mark, that an integral $\int_{-\infty}^{\infty} d\tau[K_2G_1^2(\tau, z) + K_1G_2^2(\tau, z)] = const$.

When $K_1 = K_2 = K$ the equations can be solved analytically, for example, by characteristic method. The exact solutions are:

$$G_1 = G(0,\tau)\frac{G_1(0,p)}{G(0,p)}, \qquad G_2 = G(0,\tau)\frac{G_2(0,p)}{G(0,p)}. \tag{12}$$

Here $p = Z^{-1}(Z(\tau) - z)$, $Z(\tau) = K^{-1}\int_{-\infty}^{\tau} d\tau' G^2(0, \tau')$, $Z^{-1}(z)$ - inverse function of $Z(\tau)$.

It is seen from (12), that the sum $G_1^2(\tau, z) + G_2^2(\tau, z)$ does not depend on coordinate z and is equal to $G^2(\tau, 0) = G_1^2(\tau, z = 0) + G_2^2(\tau, z = 0)$. It is not difficult to show that $\sqrt{G_1^2(\tau, z) + G_2^2(\tau, z)}$ coincides with definition of dressed field pulses [15] $G_- = a_0G_2 - a_2G_1$.

3. DISCUSSION OF RESULTS

In fig.3 and fig.4 the level populations $\rho_{0,2} = |a_{0,2}|^2$ and the atomic coherence $|\rho_{20}| = |a_2a_0^*|$ versus time and length of medium are shown. It is seen that populations are varied nonmonotonously with the length. The reason is that the conditions of adiabatic population transfer from state $|0>$ to state $|2>$ are realized only on the defined length of a medium. Fig.4 demonstrates that maximal atomic coherence is conserved over the length of a medium, considerably exceeding the length of linear absorption of a single probe pulse.

Figure 5 shows the normalizes Rabi frequencies $g_{1,2} = G_{1,2}T_1$ versus time and length of a medium calculated from (12). The dependencies illustrate that in resonant medium the pulses can propagate over a distance, which is several orders greater than the length of the linear absorption of weak probe pulse.

Comparison of data of numerical calculation and data obtained from the formulas (6,12) shows, that they are practically indistinguishable them.

4. CONCLUSION

We have presented the results of investigations of the spatial propagation of pairs of short laser pulses under simultaneous one-photon and two-photon conditions in adiabatic approximation. Unlike [15], we have constructed the analytical solution for the case, when pulse durations $T_{1,2}$ are much less then the atomic relaxation times ($\Gamma_{ij}T_{1,2} << 1$). It was founded that the pulses can penetrate into a medium over distance considerably exceeding the length of linear absorption of a single weak probe radiation. They can be identified as "dressed-field" pulses, because combination of field states $G_- = a_0G_2 - a_2G_1$ does not depend on the spatial coordinate. In this sense our results extend the concept of "dressed-field pulses" and provides an additional information on EIT propagation in the case of short pulses.

Also, we have studied the spatial evolution of the level populations and atomic coherence induced at the dipole-forbidden transition. It is shown that maximal atomic coherence is conserved at the length exceeding the length of linear absorption. It may be used for increase of efficiency of resonant nonlinear frequency-mixing processes.

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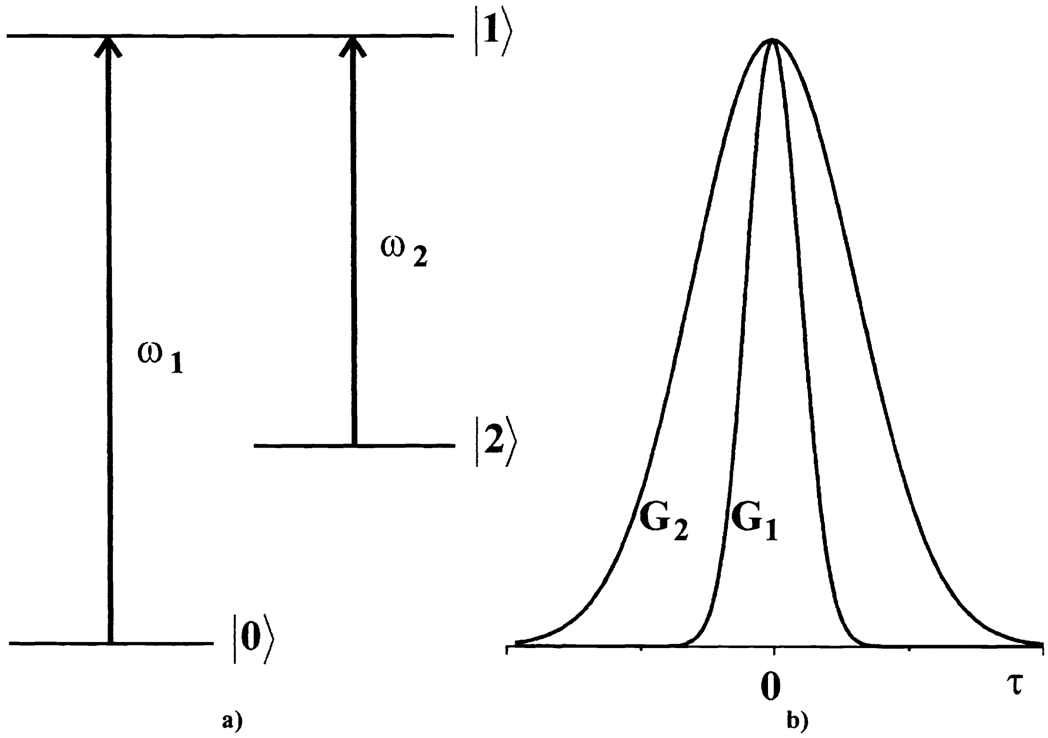

Fig.1. Λ -configuration of energy levels in atom (a) and envelopes of Rabi frequencies $G_{1,2}$ of interacting pulses at the entrance of a medium (b). $\omega_{1,2}$ -- carrier frequencies of probe and coupling pulses, respectively. The duration of coupling pulse G_2 is more, than the duration of probe G_1 ($T_2 > T_1$).

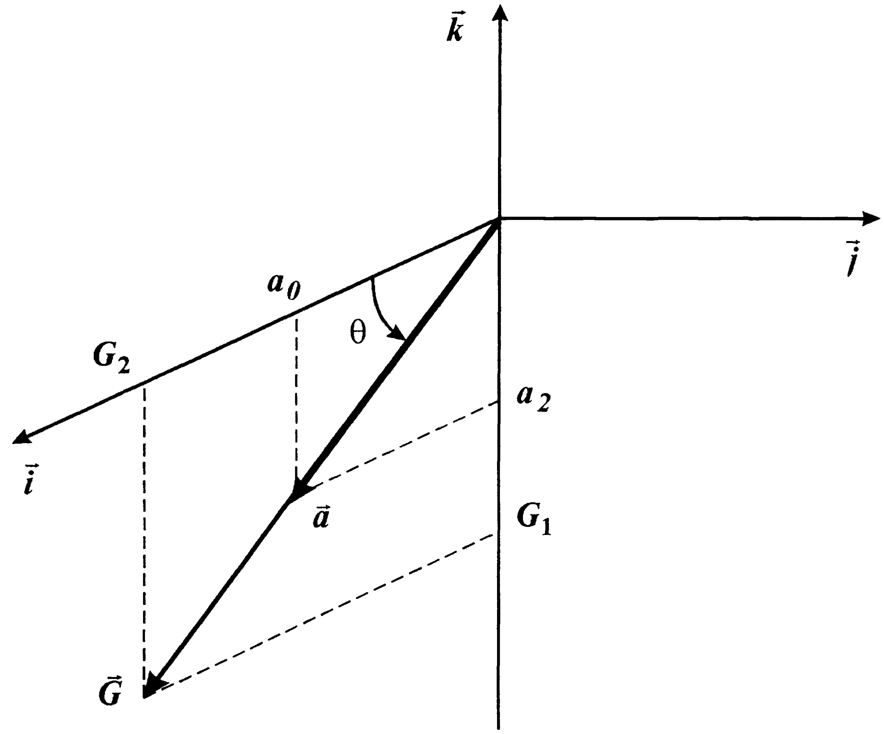

Fig.2. A vector model of adiabatic interaction of two short pulses with three-level Λ -system.

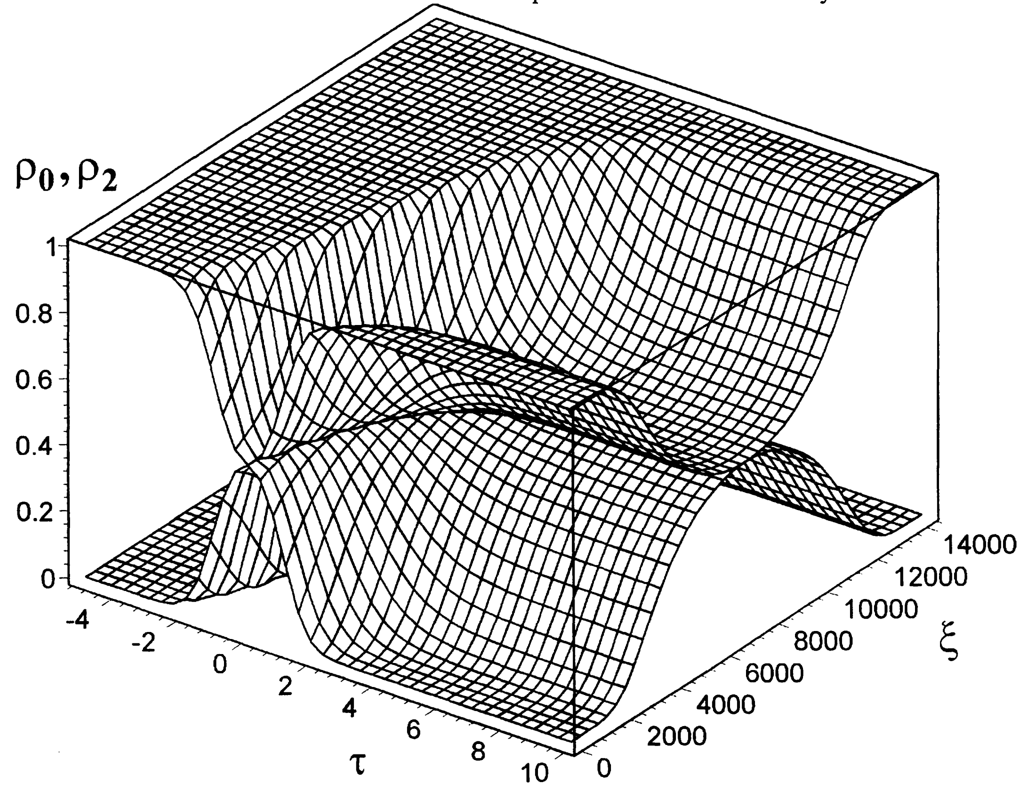

Fig.3. The dependencies of level populations $\rho_{0,2} = |a_{0,2}|^2$ from the time and the depth of penetration of radiation into a medium. The parameters are as follows: $T_2/T_1 = 3$, $G^0_{1,2}T_1 = 10$, $\Gamma_{10}T_1 = 0.1$, $\Gamma_{12}T_1 = 0.1$, $K_1 = K_2$.

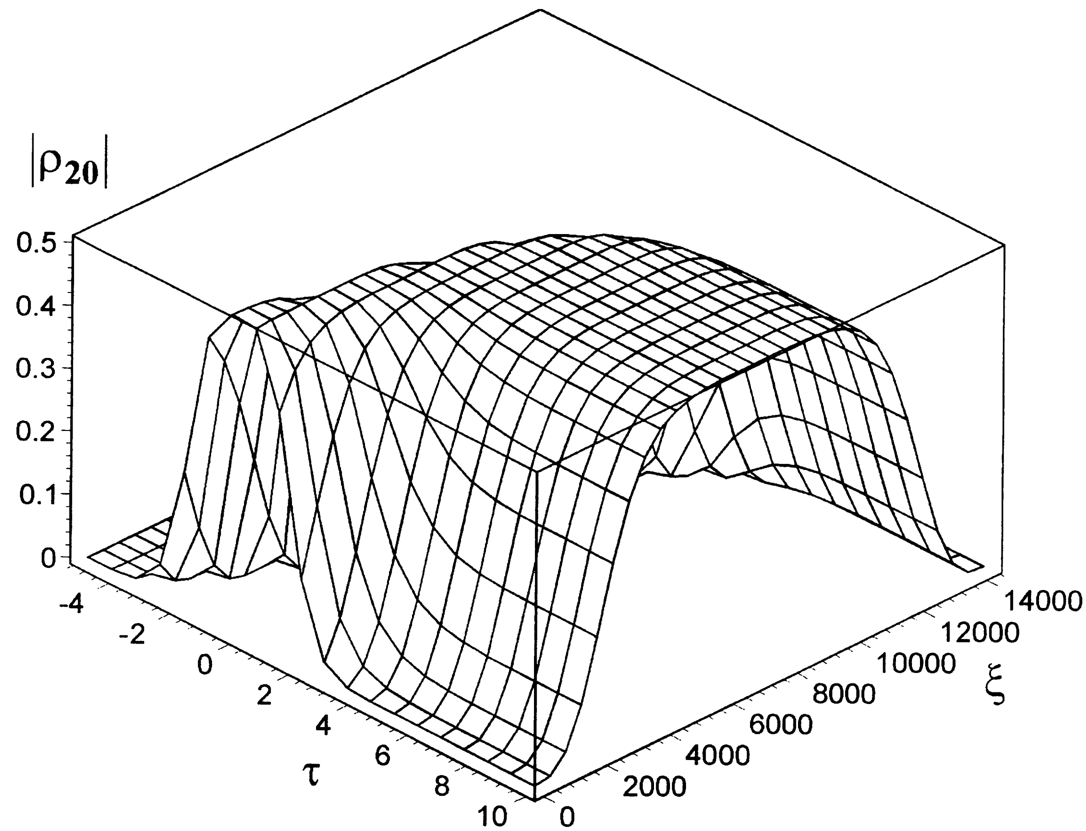

Fig.4. The dependencies of an atomic coherence $|\rho_{20}| = |a_2 a_0^*|$ from the time and the length of a medium. The parameters are the same as in fig.3

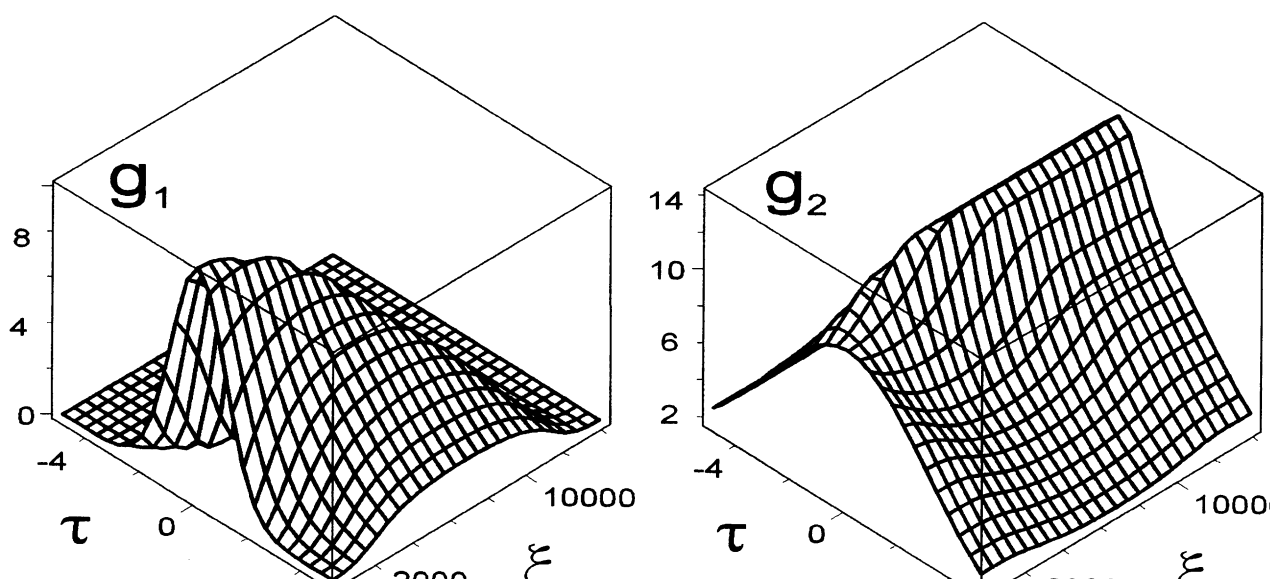

Fig.9. The dependencies of envelopes of Rabi frequencies $g_{1,2} = G_{1,2}T_1$ from the time and the depth of penetration of radiation into a medium. The parameters are the same as in fig.3