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Pulse pair propagation under conditions of induced transparency

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ABSTRACT

The spatial and temporal dynamics of two short laser pulses propagating in absorbing three-level medium under conditions of induced transparency is investigated in adiabatic approximation. We analyse two cases of electromagnetically induced transparency creating – coherent population trapping and adiabatic population transfer. It is shown that in both cases the probe pulse can penetrate into a medium at a distance considerably exceeding the length of linear absorption of a single weak probe pulse in absence of a coupling pulse at adjacent transition. The difference of spatial and temporal evolution of level populations in processes of coherent population trapping and adiabatic population transfer is although demonstrated.

Keywords: adiabatic population transfer, coherent population trapping electromagnetically induced transparency, counterintuitive pulse sequence

1. INTRODUCTION

The possibility to render optically thick media transparent for coherent laser radiation via electromagnetically induced transparency (EIT) gained considerable interest over last years. The EIT is achieved using quantum interference effects such as nonlinear interference effect,[1] coherent population trapping (CPT),[2,3] adiabatic population transfer (APT).[4] These effects cardinally change optical characteristics of a matter and allow to manipulate them. Many interesting applications have been proposed were in part experimentally realized.[2–5]

Quantum interference phenomena lead to interesting and curious effects at propagation of laser pulses in resonant three-level medium. The pulses propagating in conditions of EIT were investigated, for example, in.[6–11] As a rule, the situations are considered, when both pulses have an identical form, and their duration is more than the relaxation time of an intermediate resonant state (matched pulses[6]; dressed field pulses[7,8]) or the duration of a coupling radiation considerably exceeds the duration of probe (adiabatons[10,11]).

The process of APT is closely connected to the formation of trapped states (and therefore with CPT) and produces complete population transfer between two quantum states of atom or molecule. Such states lead to dark resonances when the sum of frequencies of two radiation fields is tuned to the two-photon resonance in a three-level system. The temporal evolution of APT is well investigated theoretically and experimentally (see, for example,[4,12–16]). As far as we know neither the problem of spatial evolution of interacting pulses in an optically thick medium, nor the features of space dynamics of population of target state excited in the course of APT were investigated.

Here we study the propagation of two short laser pulses in an absorbing three-level medium under conditions of CPT and APT, using the adiabatic approximation. It is assumed that envelopes of pulses vary slowly enough and satisfy the criterion of adiabaticity[3]:

$$|G_{1,2}|T_{1,2} \gg 1, \qquad (1)$$

The Rabi frequencies $G_{1,2}$ are of comparable strength; $T_{1,2}$ - the duration of interacting pulses.

This adiabaticity condition can be achieved for strong enough pulses even if the pulse duration is short ($\Gamma_{ij}T \ll 1$).[3,15] Further we shall consider just such a situation. Physically, this means that the pulse envelopes should vary

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slowly in a time interval equal to the reciprocal of the effective Rabi frequency $G = \sqrt{|G_1|^2 + |G_2|^2}$. The condition (1) can be fulfilled for cases depicted on Fig.1b,c. On the Fig.1b $T_2 > T_1$ and an interaction of pulses with such temporal configuration can lead to CPT. The effect of APT takes place for pulses of counterintuitive sequence (Fig.1c), when coupling pulse with envelope $G_2(t)$ is switched on and off earlier than the probe pulse $G_1(t)$.

For both cases the theoretical model consists of a system of coupled Schrödinger equations and reduced wave equations, describing simultaneously temporal and space evolution of atomic system and radiation. In adiabatic approximation (1) the analytic solution is constructed. It is shown, that the probe pulse can propagate over a distance considerably exceeding the length of linear absorption, but finally it is completely transferred into the coupling pulse. The difference of spatial and temporal evolution of level population in processes of CPT and APT is although demonstrated.

2. BASIC EQUATIONS AND SOLUTION

The slowly varying amplitude equations that describe the spatial and temporal evolution of two pulses in a three-level medium such as that shown in Fig.1a are:

$$\frac{\partial b_0}{\partial \tau} = iG_1^* b_1 \exp(-ik_1 z), \quad \frac{\partial b_2}{\partial \tau} = iG_2^* b_1 \exp(-ik_2 z), \frac{\partial b_1}{\partial \tau} = iG_1 b_0 \exp(ik_1 z) + iG_2 b_2 \exp(ik_2 z), \tag{2}$$

$$\frac{\partial G_1}{\partial z} = iK_1 b_1 b_0^* \exp(ik_1 z), \quad \frac{\partial G_2}{\partial z} = iK_2 b_1 b_2^* \exp(ik_2 z). \tag{3}$$

Here $b_{0,1,2}$ – the probability amplitudes of states $|0>, |1>$ and $|2>$, respectively; $K_{1,2} = \pi\omega_1 |d_{10,12}|^2 N/c\hbar$ - the propagation coefficients; N – the atom concentration, d_{ij} – the dipole transition matrix elements; $k_{1,2}$ – the absolute values of wave vectors of interacting waves in vacuum; $\tau = t - z/c$ – the local time. All the dynamical variables are functions of both z and τ. The intermediate state $|1\rangle$ is one-photon resonant to each field interacting only with "its own" transition.

We consider, that all atoms are initially ($\tau = -\infty$) in the ground state $|0>$, and at the entrance of the medium ($z = 0$) both pulses have envelopes, satisfying the adiabatic criterion (1).

In terms $a_0 = b_0 \exp(ik_1 z)$, $a_2 = b_2 \exp(ik_2 z)$, $a_1 = ib_1$, the equations (2) and (3) can be written as:

$$\frac{\partial a_0}{\partial \tau} = G_1^* a_1, \quad \frac{\partial a_2}{\partial \tau} = G_2^* a_1, \quad \frac{\partial a_1}{\partial \tau} = -G_1 a_0 - G_2 a_2, \tag{4}$$

$$\frac{\partial G_1}{\partial z} = K_1 a_1 a_0^*, \quad \frac{\partial G_2}{\partial z} = K_2 a_1 a_2^*. \tag{5}$$

In adiabatic approximation the solution of the system (4) has the form:

$$a_0 \simeq \frac{G_2}{G} = \cos\theta, \quad a_2 \simeq -\frac{G_1}{G} = -\sin\theta, \quad a_1 \simeq \frac{1}{G_1}\frac{\partial(G_2/G)}{\partial \tau} \simeq -\frac{1}{G_2}\frac{\partial(G_1/G)}{\partial \tau}, \tag{6}$$

where $G = \sqrt{G_1^2 + G_2^2}$, and θ - some angle, which sense will be cleared below. In adiabatic approximation this solution does not depend on specific form of pulses.

The expression for a_1 can be reduced in a form:

$$a_1 = (G_2 \dot{G}_1 - G_1 \dot{G}_2)/G^3. \tag{7}$$

It is easy to show, that in approximation (1) $|a_1| << 1$, i.e. the population of an intermediate condition $|1\rangle$ is negligible during all time of interaction with pulses. The last physically means, that the resonant absorption of pulses is low. It is the electromagnetically induced transparency. Thus pulses will propagate over the distance essentially exceeding the length of linear absorption of a single weak probe radiation.

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The population is distributed between initial $|0>$ and final $|2>$ states. So, there is an approximate equality:

$$|a_0|^2 + |a_2|^2 \simeq 1. \tag{8}$$

And solution for probability amplitudes $a_{0,2}$ is convenient to be presented as:

$$a_0 = \cos\theta, \qquad a_2 = -\sin\theta \tag{9}$$

It is interesting to remark, that the equality (8) also reflects the fact, that atoms are trapped in a state of coherent population trapping with probability amplitude $a_- = (G_2/G)a_0 - (G_1/G)a_2 = \cos(\theta)a_0 - \sin(\theta)a_2 = 1$. Thus the reduction of resonance absorption of interacting pulses is due to coherent population trapping.

Substituting the solution (6) in the field equations (5), we obtain a system of the connected nonlinear equations:

$$\frac{\partial G_1}{\partial z} = -(K_1/G)\frac{\partial(G_1/G)}{\partial\tau}, \qquad \frac{\partial G_2}{\partial z} = -(K_2/G)\frac{\partial(G_2/G)}{\partial\tau}. \tag{10}$$

One can show from (10), that the value $K_2G_1^2(\tau,z) + K_1G_2^2(\tau,z)$ does not depend on coordinate z and is equal to $\tilde{G}^2(\tau) = K_2G_1^2(\tau,z=0) + K_1G_2^2(\tau,z=0)$. It is not difficult to show that $K_2G_1^2(\tau,z) + K_1G_2^2(\tau,z) = n_1(\tau,z) + n_2(\tau,z)$, where $n_{1,2}(\tau,z)$ is the photons density. It is the Manley-Raw relation. Let's mark, that an integral $\int_{-\infty}^{\infty} d\tau[K_2G_1^2(\tau,z) + K_1G_2^2(\tau,z)] = constant$.

If $K_1 = K_2 = K$, the equations simplify and can be solved analytically, for example, by characteristic method. The exact solutions are:

$$G_1 = G(0,\tau)\frac{G_1(0,p)}{G(0,p)}, \qquad G_2 = G(0,\tau)\frac{G_2(0,p)}{G(0,p)}. \tag{11}$$

Here $p = Z^{-1}(Z(\tau) - z)$, $Z(\tau) = K^{-1}\int_{-\infty}^{\tau} d\tau' G^2(0,\tau')$, $Z^{-1}(z)$ - inverse function of $Z(\tau)$.

It is seen from (11), that the sum $G_1^2(\tau,z) + G_2^2(\tau,z)$ does not depend on coordinate z and is equal to $G^2(\tau,0) = G_1^2(\tau,z=0) + G_2^2(\tau,z=0)$. It is not difficult to show that $\sqrt{G_1^2(\tau,z) + G_2^2(\tau,z)}$ coincides with definition of dressed field pulses[7,8]: $G_- = a_0G_2 - a_2G_1$. Thus, in this case the pulses may be identified as dressed field states. Let's notice that the other combination $G_+ = a_0G_1 + a_2G_2 \equiv 0$ (see also[7]). When $K_1 \neq K_2$ it is not true. However in both cases the Manley-Raw relations take place, because their formulation is more general.

3. DISCUSSION OF RESULTS

3.1. The case of coherent population trapping

For demonstration of the main features of propagation of pulses under conditions of CPT we used Gaussian envelope at the entrance of the medium: $G_{1,2}(\tau) = G_{1,2}^0\exp(-\tau^2/2T_{1,2}^2)$.

In Fig.2 and Fig.3 the level populations $\rho_{0,2} = |a_{0,2}|^2$ and the atomic coherence $|\rho_{20}| = |a_2a_0^*|$ versus time and length of medium are shown. It is seen that populations are varied nonmonotonously with the length. This spatial dependence is similar to the temporal dependence. Thus, we can speak about spatio-temporal analogy at propagation of such pulses. Fig.3 demonstrates that maximal atomic coherence is conserved over the length of a medium, considerably exceeding the length of linear absorption of a single probe pulse.

Fig.4 shows the normalizes Rabi frequencies $g_{1,2} = G_{1,2}T_1$ versus time and length of a medium calculated from (11). The dependencies illustrate that in resonant medium the pulses can propagate over a distance, which is several orders greater than the length of the linear absorption of weak probe pulse.

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3.2. The case of adiabatic population transfer

Here we although used Gaussian pulses with the counterintuitive sequence: $G_1(\tau) = G_1^0 \exp(-\tau^2/2T^2)$, $G_2(\tau) = G_2^0 \exp[-(\tau-\tau_0)^2/2T^2]$, τ_0 - the delay time between pulses. It results from (6) that in the tail of the coupling pulse $|a_0|^2 \simeq 0$, and $|a_2|^2 \simeq 1$, in other words, the population of the ground state $|0\rangle$ is transferred into state $|2\rangle$. In this process angle θ varies from zero up to $\pi/2$. Actually the value of this angle also depends on a coordinate, as will be shown below.

In Fig.5 the normalized Rabi frequencies $g_{1,2} = G_{1,2}T$ versus time and depth of penetration of radiation in a medium calculated from formula (11) are shown. They demonstrate, that in a resonant medium the probe pulse can propagate over a distance, which is several orders greater than the length of linear absorption of weak probe pulse. However the energy of the leading edge of the probe pulse is partially absorbed, and the energy of the coupling pulse is amplified. The absorbed energy is used in adiabatic transfer of the atomic system to an excited state and in amplification of the coupling pulse. Eventually the probe pulse is completely transferred into the coupling pulse.

In Fig.6 the populations $\rho_{0,2} = |a_{0,2}|^2$ versus time and length of medium are shown. APT thus makes it possible to achieve practically 100% inversion on dipole-forbidden transition in extensive media.

In both cases obtained analytic results coincide with the results of the numeric analysis of system of equations (4) and (5) (see also[17]).

4. GEOMETRIC INTERPRETATION

These results can be interpreted in terms of a three-dimensional vector model, using the vectors $\vec{a} = (a_0, a_1, a_2)$ and $\vec{G} = (G_2, 0, -G_1)$ as a variables. In the given terms the system (4) may be rewritten as:

$$\dot{\vec{a}} = \vec{G} \times \vec{a}, \tag{12}$$

where the sign $(\times)$ means a vector product. The vector $\vec{G}$ lies in the plane $(\vec{i}, \vec{k})$ under angle θ to axis $\vec{i}$ ($\cos\theta = G_2/G$, see (6)). The solution of equation (12) is vector $\vec{a} = (G_2/G, a_1, -G_1/G)$, which lies practically in the plane $(\vec{i}, \vec{k})$, while $|a_1| << 1$. The angle between vectors $\vec{G}$ and $\vec{a}$ is very small. Therefore vector $\vec{a}$, corresponding to adiabatic solution (6), precesses around $\vec{G}$ with frequency $G = \sqrt{G_1^2 + G_2^2}$ and adiabatically follows it. During interaction $\vec{G}$ turns to certain angle, which depends on the process. For CPT the maximum angle is $\pi/4$ and for APT it is $\pi/2$. But under propagation θ although depends on the coordinate z. These dependencies are shown on Fig.7.

5. CONCLUSION

We have presented the results of analytical and numerical calculation of spatial propagation of short laser pulses pairs in absorbing three-level media under conditions of coherent population trapping and adiabatic population transfer. The results show that in both cases a transparency can be maintained over several thousands one-photon absorption lengths. But complete transfer of energy from probe pulse to coupling pulse takes place at some pre-defined optical length. When the propagation coefficients are equal, i.e. $K_1 = K_2$, the process can be understood in terms of dressed fields pulses. For the case $K_1 \neq K_2$ it is not true, but the pulses comply with Manley-Raw relation which have more general character than conception of dressed fields pulses.

It is shown also how the population in initial and target states can evolve spatially, i.e. in the course of pulses propagation. APT leads to practically complete inversion at dipole-forbidden transition at a characteristic propagation distance of the probe pulse which can be over several thousands Beers lengths. In the case of CPT the maximal atomic coherence is although maintained on the large length. In this sense our results provide additional information on EIT propagation in case of short pulses with duration much less then atomic relaxation times.

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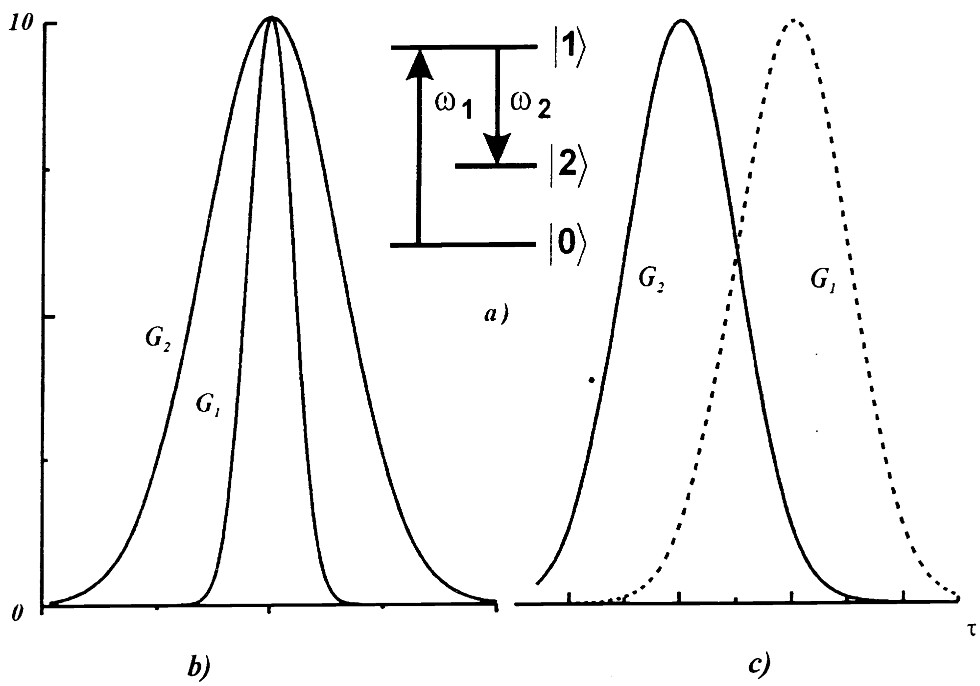

Fig.1. Configurations of energy levels in atom and Rabi frequencies at an input of a medium. $\omega_{1,2}$ — carrier frequencies of probe G_1 and coupling G_2 pulses, consequently. (a) — atom of Λ-scheme, (b) — case of coherent population trapping (the duration of coupling pulse is more, than the duration of probe ($T_2 > T_1$)), (c) - case of adiabatic population transfer.

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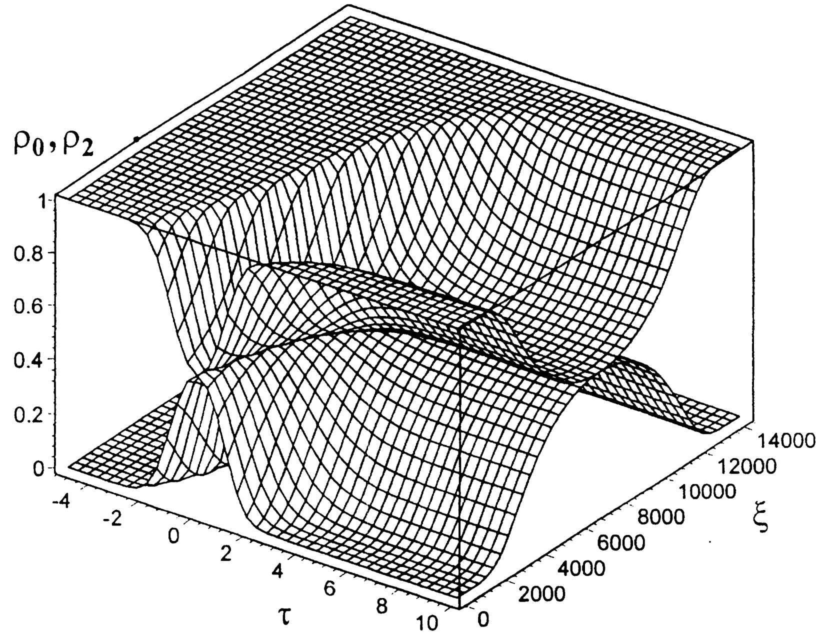

Fig.2. The dependencies of level populations $\rho_{0,2} = |a_{0,2}|^2$ from the time and the depth of penetration of radiation into a medium. The parameters are as follows: $T_2/T_1 = 3$, $G^0_{1,2}T_1 = 10$, $\Gamma_{10}T_1 = 0.1$, $\Gamma_{12}T_1 = 0.1$, $K_1 = K_2$.

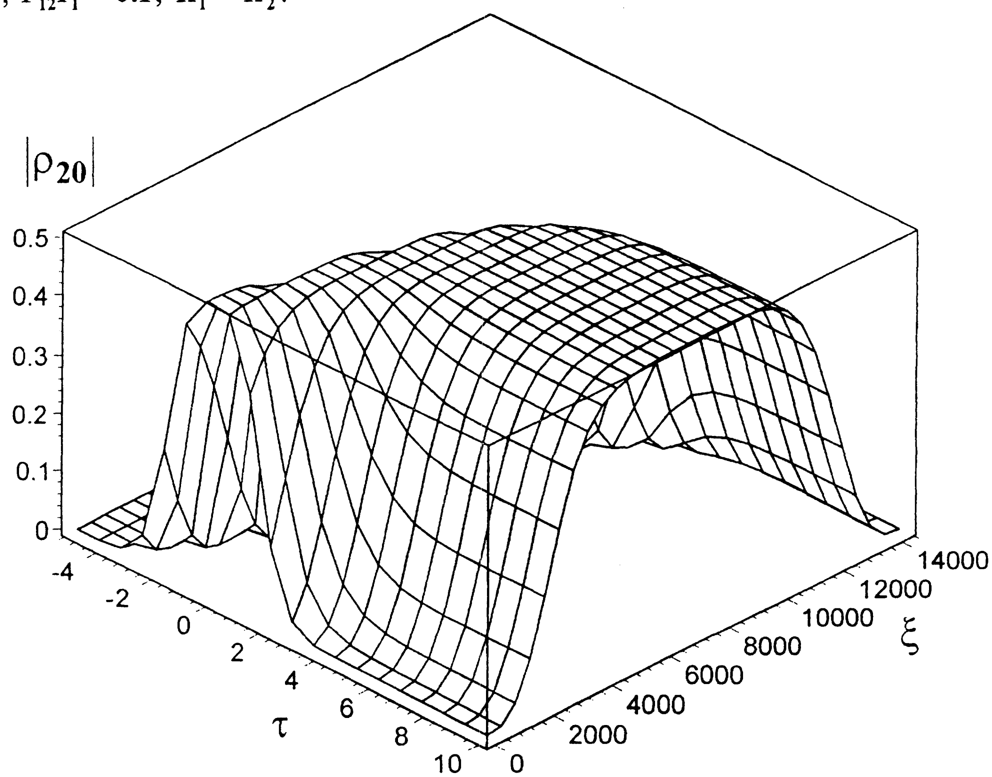

Fig.3. The dependencies of an atomic coherence $|\rho_{20}| = |a_2 a_0^*|$ from the time and the length of a medium. The parameters are the same as in fig.2

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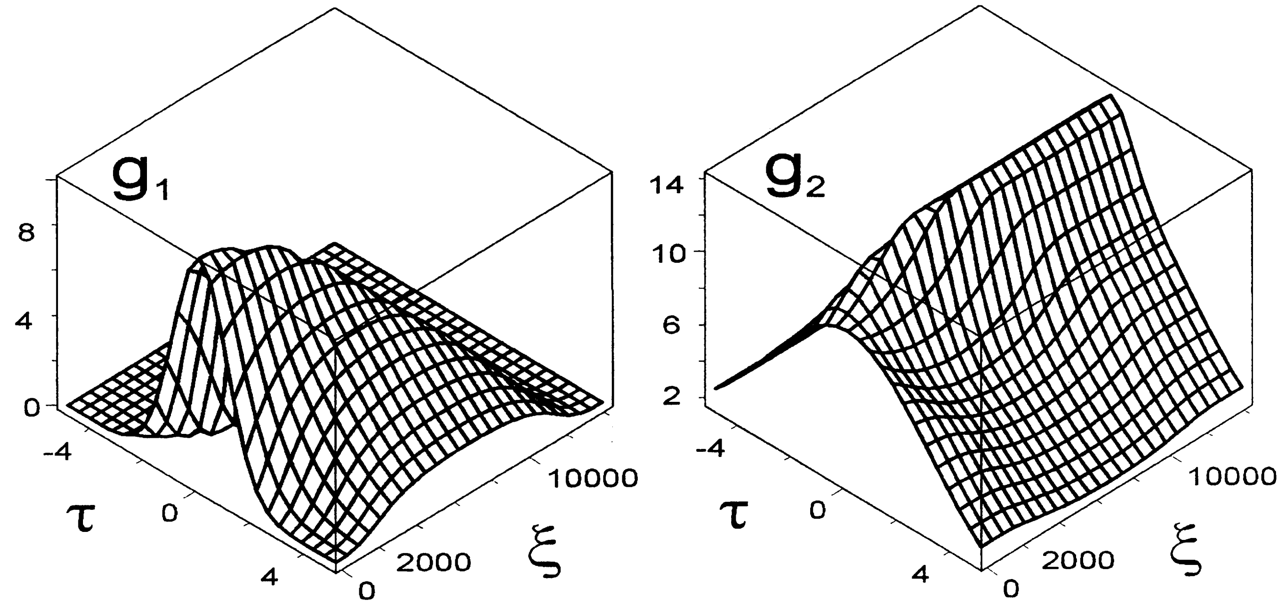

Fig.4. The dependencies of envelopes of Rabi frequencies $g_{1,2} = G_{1,2}T_1$ from the time and the depth of penetration of radiation into a medium. The parameters are the same as in fig.2

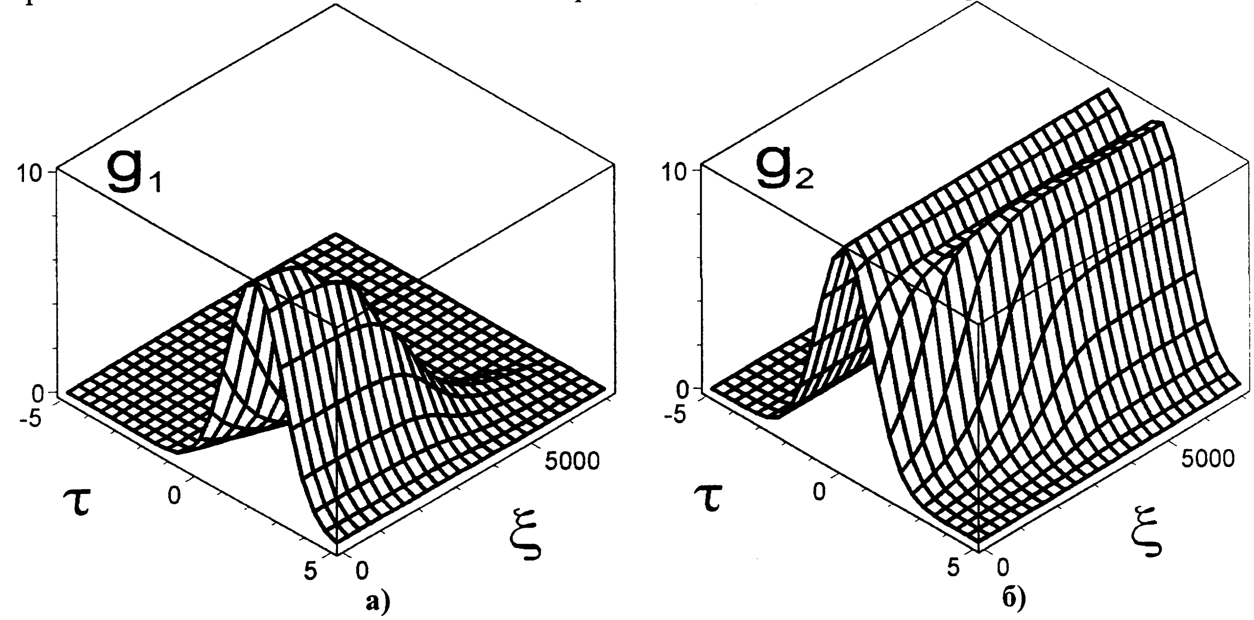

Fig.5. The normalized Rabi frequencies $g_1 = G_1T$ (a) and $g_2 = G_2T$ (b) versus time and length of a medium. The parameters are as follows: $t_0/T = 2$, $G_1^0T = 10$, $G_2^0T = 10$ ($G_{1,2}^0$— value of Rabi frequency $G_{1,2}$ in a maximum); $\Gamma_{10}T = 0.1$, $\Gamma_{12}T = 0.1$, $K_1 = K_2$.The time τ is measured in terms of pulse duration T, and length of a medium ξ — in terms of the length of linear absorption of a probe radiation with frequency ω_1

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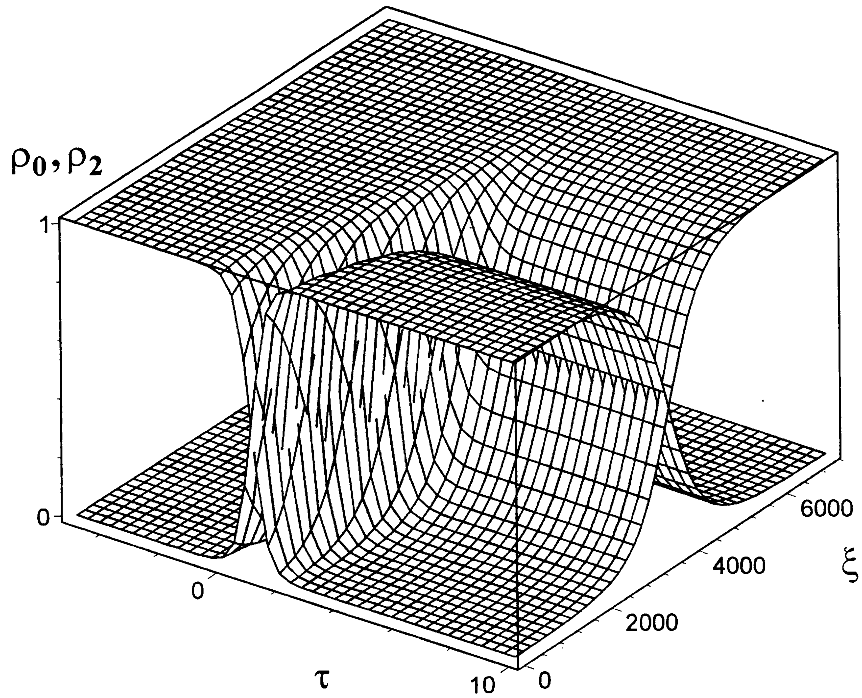

Fig.6. The level populations $\rho_{0,2} = |a_{0,2}|^2$ versus time and length of a medium. The parameters are the same as in Fig.5.

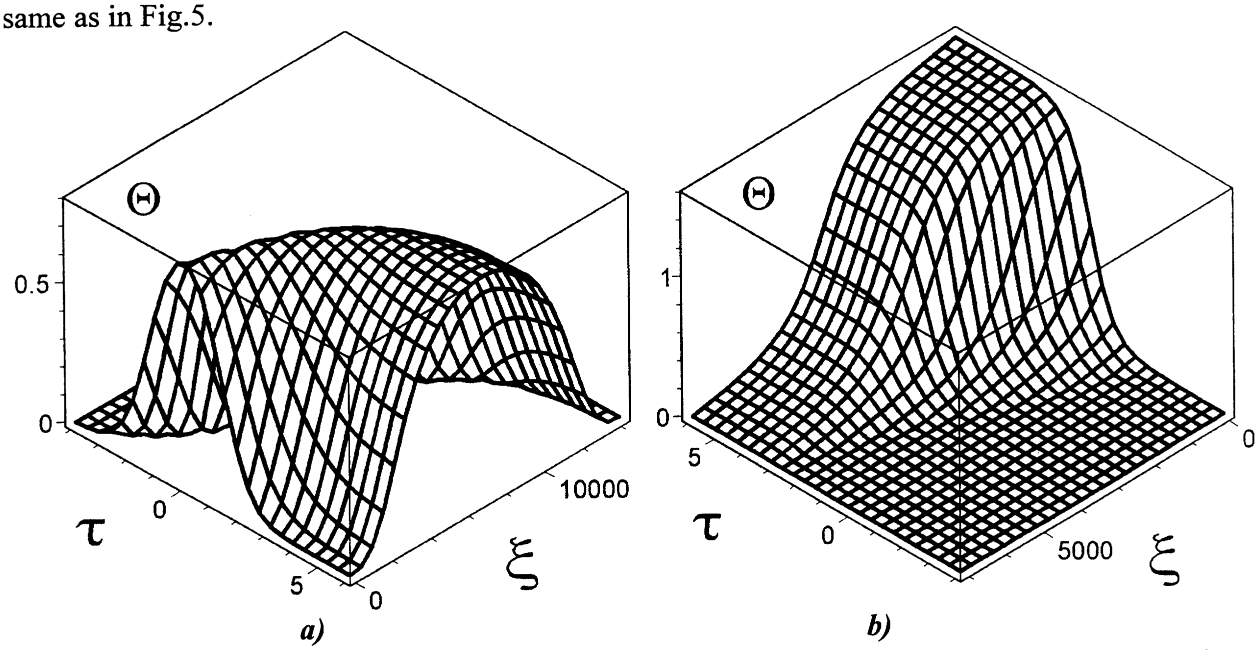

Fig.7. Angle Θ versus time and length of a medium. (a) — case of coherent population trapping. The parameters are the same as in Fig.2. (b) - case of adiabatic population transfer. The parameters are the same as in Fig.5.

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