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Pulse pair propagation under conditions of induced transparency: adiabatic approximation

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ABSTRACT

The features of spatial and temporal evolution of two short laser pulses propagating in three-level medium under conditions of coherent population trapping and adiabatic population transfer is investigated in adiabatic approximation. It is shown that in both cases pulses can penetrate into a medium at a distance considerably exceeding the length of linear absorption of a single weak probe pulse in absence of a coupling pulse at adjacent transition. The difference of spatial and temporal evolution of level populations in processes of coherent population trapping and adiabatic population transfer is demonstrated. Also we show that the concept of dressed-field pulses is consequence of Manley-Raw relation.

Keywords: adiabatic population transfer, coherent population trapping, electromagnetically induced transparency, counterintuitive pulse sequence

1. INTRODUCTION

The possibility to render optically thick media transparent for coherent laser radiation via electromagnetically induced transparency (EIT) gained considerable interest over last years. The EIT is achieved using quantum interference effects such as nonlinear interference effect,[1] coherent population trapping (CPT),[2,3] adiabatic population transfer (APT).[4] These effects cardinally change optical characteristics of a matter and allow to manipulate them. Many interesting proposed applications were in part experimentally realized.[2–5]

Quantum interference phenomena lead to interesting and curious effects at propagation of laser pulses in resonant three-level medium. The pulses propagating in conditions of EIT were investigated.[6–11] As a rule, the situations are considered, when both pulses have an identical form, and their duration is more than the relaxation time of an intermediate resonant state (matched pulses[6]; dressed field pulses[7,8]) or the duration of a coupling radiation considerably exceeds the duration of probe (adiabatons[10,11]).

The process of APT is closely connected to the formation of trapped states (and therefore with CPT) and produces complete population transfer between two quantum states of atom or molecule. Such states lead to dark resonances when the sum of frequencies of two radiation fields is tuned to the two-photon resonance in a three-level system. The temporal evolution of APT is well investigated theoretically and experimentally.[4,12–16]

Here we study the propagation of two short laser pulses in resonant three-level medium under conditions of CPT and APT, using the adiabatic approximation. It is assumed that envelopes of pulses satisfy the criterion of adiabaticity[3]:

$$|G_{1,2}|T_{1,2} \gg 1, \tag{1}$$

The Rabi frequencies $G_{1,2}$ are of comparable strength; $T_{1,2}$ are the durations of interacting pulses.

This adiabaticity condition can be achieved for strong enough pulses even if the pulse duration is short ($\Gamma_{ij}T_{1,2} <<$ 1).[3,15] Physically, this means that the pulse envelopes should vary slowly in a time interval equal to the reciprocal of the effective Rabi frequency $G = \sqrt{|G_1|^2 + |G_2|^2}$. The condition (1) can be fulfilled for cases of pulse switching depicted on Fig.1b,c. An interaction of pulses with such temporal configuration can lead to CPT. The effect of APT

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takes place for pulses of counterintuitive sequence (Fig.1b), when coupling pulse with envelope $G_2(t)$ is switched on and off earlier than the probe pulse $G_1(t)$.

For both cases the theoretical model consists of a system of coupled Schrödinger equations and reduced wave equations, describing simultaneously temporal and space evolution of atomic system and radiation. In adiabatic approximation (1) the analytic solution is constructed. It is shown, that the probe pulse can propagate over a distance considerably exceeding the length of linear absorption, but finally it is completely transferred into the coupling pulse. The difference of spatial and temporal evolution of level population in processes of CPT and APT is also demonstrated.

2. BASIC EQUATIONS

We consider the three-level Λ system shown schematically in Fig.1a. Transition $|2\rangle$-$|1\rangle$ is driven by a strong field with Rabi frequency $G_2(t)$ (coupling field). The second strong field (probe) with Rabi frequency $G_1(t)$ is applied on the transition $|0\rangle$-$|1\rangle$. The waves are assumed to be plane with time envelopes, satisfying the adiabatic criterion (1). They propagate in the medium in the same direction. All atoms are initially in the ground state $|0\rangle$. The intermediate state $|1\rangle$ is one-photon resonant to both fields. Each field interacts only with the correspondent transition.

The equations that describe the spatial and temporal evolution of two pulses in a medium of three-level atoms of Λ-configuration are:

$$\frac{\partial b_0}{\partial \tau} = iG_1^* b_1 \exp(-ik_1 z), \quad \frac{\partial b_2}{\partial \tau} = iG_2^* b_1 \exp(-ik_2 z), \frac{\partial b_1}{\partial \tau} = iG_1 b_0 \exp(ik_1 z) + iG_2 b_2 \exp(ik_2 z), \tag{2}$$

$$\frac{\partial G_1}{\partial z} = iK_1 b_1 b_0^* \exp(ik_1 z), \quad \frac{\partial G_2}{\partial z} = iK_2 b_1 b_2^* \exp(ik_2 z). \tag{3}$$

Here $b_{0,1,2}$ – the probability amplitudes of states $|0>, |1>$ and $|2>$, respectively; $K_{1,2} = \pi\omega_1 |d_{10,12}|^2 N/c\hbar$ - the propagation coefficients; N – the atom concentration, d_{ij} – the dipole transition matrix elements; $k_{1,2}$ – the absolute values of wave vectors of interacting waves in vacuum; $\tau = t - z/c$ – the local time. All the dynamical variables are functions of both z and τ.

In terms $a_0 = b_0 \exp(ik_1 z)$, $a_2 = b_2 \exp(ik_2 z)$, $a_1 = ib_1$, the equations (2) and (3) can be written as:

$$\frac{\partial a_0}{\partial \tau} = G_1^* a_1, \quad \frac{\partial a_2}{\partial \tau} = G_2^* a_1, \quad \frac{\partial a_1}{\partial \tau} = -G_1 a_0 - G_2 a_2, \tag{4}$$

$$\frac{\partial G_1}{\partial z} = K_1 a_1 a_0^*, \quad \frac{\partial G_2}{\partial z} = K_2 a_1 a_2^*. \tag{5}$$

In adiabatic approximation the solution of the system (4) has the form:

$$a_0 \simeq \frac{G_2}{G}, \quad a_2 \simeq -\frac{G_1}{G}, \quad a_1 \simeq \frac{1}{G_1}\frac{\partial (G_2/G)}{\partial \tau} \simeq -\frac{1}{G_2}\frac{\partial (G_1/G)}{\partial \tau}, \tag{6}$$

where $G = \sqrt{G_1^2 + G_2^2}$. In the given approximation this solution does not depend on specific form of pulses.

Substituting the solution (6) in the field equations (5), we obtain a system of the connected nonlinear equations:

$$\frac{\partial G_1}{\partial z} = -(K_1/G)\frac{\partial (G_1/G)}{\partial \tau}, \qquad \frac{\partial G_2}{\partial z} = -(K_2/G)\frac{\partial (G_2/G)}{\partial \tau}. \tag{7}$$

If $K_1 = K_2 = K$, the equations (7) can be solved analytically, for example, by characteristic method. The exact solutions are:

$$G_1 = G(0,\tau)\frac{G_1(0,p)}{G(0,p)}, \qquad G_2 = G(0,\tau)\frac{G_2(0,p)}{G(0,p)}. \tag{8}$$

Here $p = Z^{-1}(Z(\tau) - z)$, $Z(\tau) = K^{-1}\int_{-\infty}^{\tau} d\tau' G^2(0,\tau')$, $Z^{-1}(z)$ - inverse function of $Z(\tau)$.

It is seen from (8), that the sum $G_1^2(\tau,z) + G_2^2(\tau,z)$ does not depend on coordinate z and is equal to $G^2(\tau,0) = G_1^2(\tau,z=0) + G_2^2(\tau,z=0)$. It is not difficult to show that $\sqrt{G_1^2(\tau,z) + G_2^2(\tau,z)}$ coincides with definition of dressed field pulses[7,8]: $G_- = a_0G_2 - a_2G_1$. Thus, in this case the pulses may be identified as dressed field states. Let's notice that the other combination $G_+ = a_0G_1 + a_2G_2 \equiv 0$ (see also[7]). When $K_1 \neq K_2$ it is not true. However in both cases the Manley-Raw relation take place, because its formulation is more general.

One can show from (7), that the value $K_2G_1^2(\tau,z) + K_1G_2^2(\tau,z)$ does not depend on coordinate z and is equal to $\tilde{G}^2(\tau) = K_2G_1^2(\tau,z=0) + K_1G_2^2(\tau,z=0)$. It is not difficult to show that $K_2G_1^2(\tau,z) + K_1G_2^2(\tau,z) = n_1(\tau,z) + n_2(\tau,z) = F(\tau)$ – does not depend on z, where $n_{1,2}(\tau,z)$ is the photons density. It is the Manley-Raw relation. Let's mark, that an integral $\int_{-\infty}^{\infty} d\tau[K_2G_1^2(\tau,z) + K_1G_2^2(\tau,z)] = constant$.

3. DISCUSSION OF RESULTS

3.1. The case of coherent population trapping

For demonstration of the main features of propagation of pulses under conditions of CPT we used gaussian envelope at the entrance of the medium: $G_{1,2}(\tau) = G_{1,2}^0 \exp(-\tau^2/2T_{1,2}^2)$, $T_1 > T_2$ (fig. 1,b). Here the coupling pulse with envelope $G_2(t)$ switches on earlier and switches off later than the probe pulse $G_1(t)$.

The expression for a_1 can be reduced in a form:

$$a_1 = (G_2\dot{G}_1 - G_1\dot{G}_2)/G^3. \tag{9}$$

It is easy to show, that in approximation (1) $|a_1| << 1$, i.e. the population of an intermediate state $|1\rangle$ is negligible during all time of interaction with pulses. The last physically means, that the resonant absorption of pulses is low. It is the electromagnetically induced transparency. Thus pulses will propagate over the distance essentially exceeding the length of linear absorption of a single weak probe radiation.

The population is distributed between initial $|0>$ and final $|2>$ states. So, there is an approximate equality:

$$|a_0|^2 + |a_2|^2 \simeq 1. \tag{10}$$

And solution for probability amplitudes $a_{0,2}$ is convenient to be presented as:

$$a_0 = \cos\theta, \qquad a_2 = -\sin\theta, \tag{11}$$

where $\cos\theta = \frac{G_2}{G}$, $\sin\theta = \frac{G_1}{G}$.

It is interesting to remark, that the equality (10) also reflects the fact, that atoms are trapped in a state of coherent population trapping with probability amplitude $a_- = (G_2/G)a_0 - (G_1/G)a_2 = \cos(\theta)a_0 - \sin(\theta)a_2 = 1$. Thus the reduction of resonance absorption of interacting pulses is due to coherent population trapping.

The atomic coherence of nonallowed transition $|0\rangle$-$|2\rangle$ $\rho_{20} = a_2a_0^*$ is

$$\rho_{20} = -\frac{G_1G_2}{G^2} = -\frac{1}{2}\sin 2\theta \tag{12}$$

If pulses have the same amplitude, then in the time moment $\tau = 0$ it reaches value $\rho_{20}(\tau = 0) = -\frac{1}{2}$ corresponding to the maximum coherence.

In Fig.2 and Fig.3 the level populations $\rho_{0,2} = |a_{0,2}|^2$ and the atomic coherence $|\rho_{20}| = |a_2a_0^*|$ versus time and length of medium are shown. It is seen that populations are varied nonmonotonously with the length. This spatial dependence is similar to the temporal dependence. Thus, we can speak about spatio-temporal analogy at propagation of such pulses. Fig.3 demonstrates that maximal atomic coherence is conserved over the length of a medium, considerably exceeding the length of linear absorption of a single probe pulse.

Fig.4 shows the normalizes Rabi frequencies $g_{1,2} = G_{1,2}T_1$ versus time and length of a medium calculated from (8). The dependencies illustrate that in resonant medium the pulses can propagate over a distance, which is several orders greater than the length of the linear absorption of weak probe pulse.

3.2. The case of adiabatic population transfer

Here we also used gaussian pulses, but with the counterintuitive sequence: $G_1(\tau) = G_1^0 \exp(-\tau^2/2T^2)$, $G_2(\tau) = G_2^0 \exp[-(\tau-\tau_0)^2/2T^2]$, τ_0 - the delay time between pulses. It follows from (6) that in the tail of the coupling pulse $|a_0|^2 \simeq 0$, and $|a_2|^2 \simeq 1$. In other words, the population of the ground state $|0\rangle$ is transferred into state $|2\rangle$.

In Fig.5 the normalized Rabi frequencies $g_{1,2} = G_{1,2}T$ versus time and depth of penetration of radiation in a medium calculated from formula (8) are shown. They demonstrate, that in a resonant medium the probe pulse can propagate over a distance, which is several orders greater than the length of linear absorption of weak probe pulse. However the energy of the leading edge of the probe pulse is partially absorbed, and the energy of the coupling pulse is amplified. The absorbed energy is used for the adiabatic transfer of the atomic system to an excited state and for amplification of the coupling pulse. Eventually the probe pulse is completely transferred into the coupling pulse.

In Fig.6 the populations $\rho_{0,2} = |a_{0,2}|^2$ versus time and length of medium are shown. APT thus makes it possible to achieve practically 100% inversion on dipole-forbidden transition in extensive media.

In both cases obtained analytic results coincide with the results of the numeric analysis of system of equations (4) and (5) (see also[17]).

4. CONCLUSION

We have presented the results of analytical calculation of spatial propagation of short laser pulses pairs in absorbing three-level media under conditions of coherent population trapping and adiabatic population transfer. The results show that in both cases a transparency can be maintained over several thousands one-photon absorption lengths. But complete transfer of energy from probe pulse to coupling pulse takes place at some pre-defined optical length. When the propagation coefficients are equal, i.e. $K_1 = K_2$, the process can be understood in terms of dressed fields pulses. For the case $K_1 \neq K_2$ it is not true, but the pulses comply with Manley-Raw relation which have more general character than conception of dressed fields pulses.

It is shown also how the population in initial and target states can evolve spatially, i.e. in the course of pulses propagation. APT leads to practically complete inversion at dipole-forbidden transition at a characteristic propagation distance of the probe pulse which can be over several thousands Beers lengths. In the case of CPT the maximal atomic coherence is also maintained on the large length. In this sense our results provide additional information on EIT propagation in case of short pulses with duration much less than atomic relaxation times.

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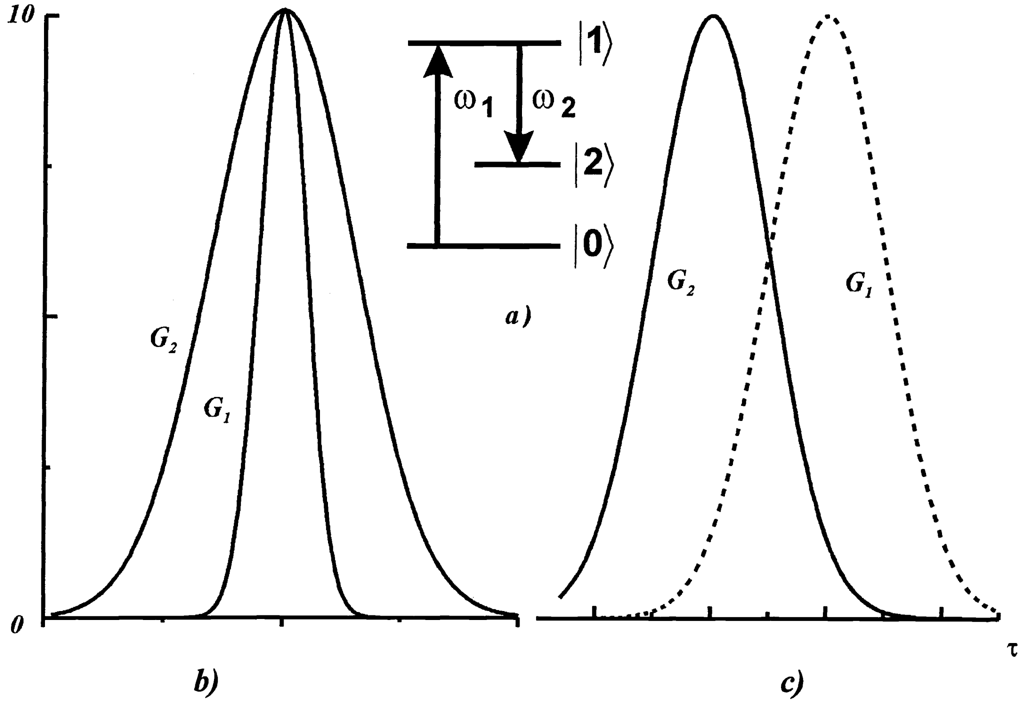

Fig.1. Configurations of energy levels in atom (a), Rabi frequencies at an input of a medium (b,c). $\omega_{1,2}$ — carrier frequencies of probe G_1 and coupling G_2 pulses, consequently: (b) — case of coherent population trapping (the duration of coupling pulse is more, than the duration of probe ($T_2 > T_1$)), (c) - case of adiabatic population transfer.

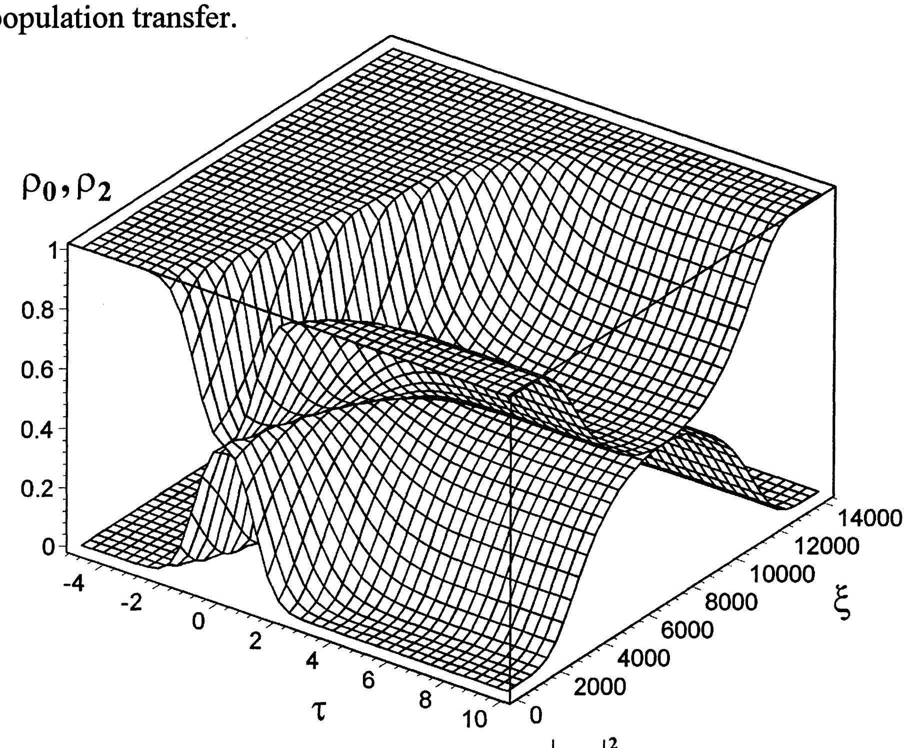

Fig.2. The dependencies of level populations $\rho_{0,2} = |a_{0,2}|^2$ from the time and the depth of penetration of radiation into a medium. The parameters are as follows: $T_2/T_1 = 3$, $G^0_{1,2}T_1 = 10$, $\Gamma_{10}T_1 = 0.1$, $\Gamma_{12}T_1 = 0.1$, $K_1 = K_2$.

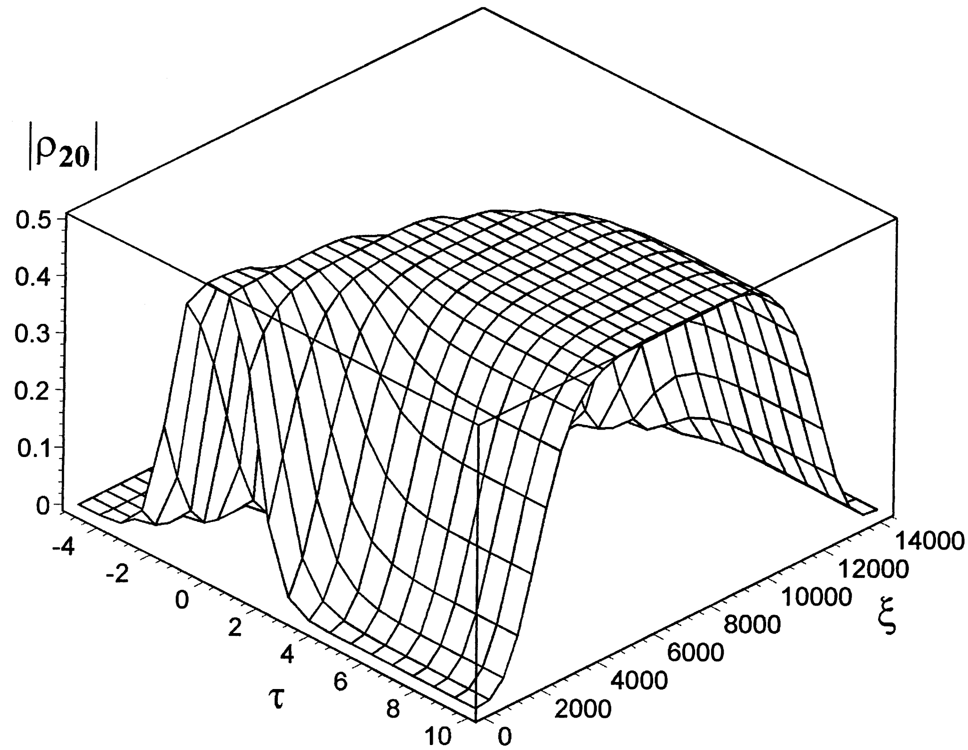

Fig.3. The dependencies of an atomic coherence $|\rho_{20}| = |a_2 a_0^*|$ from the time and the length of a medium. The parameters are the same as in fig.2

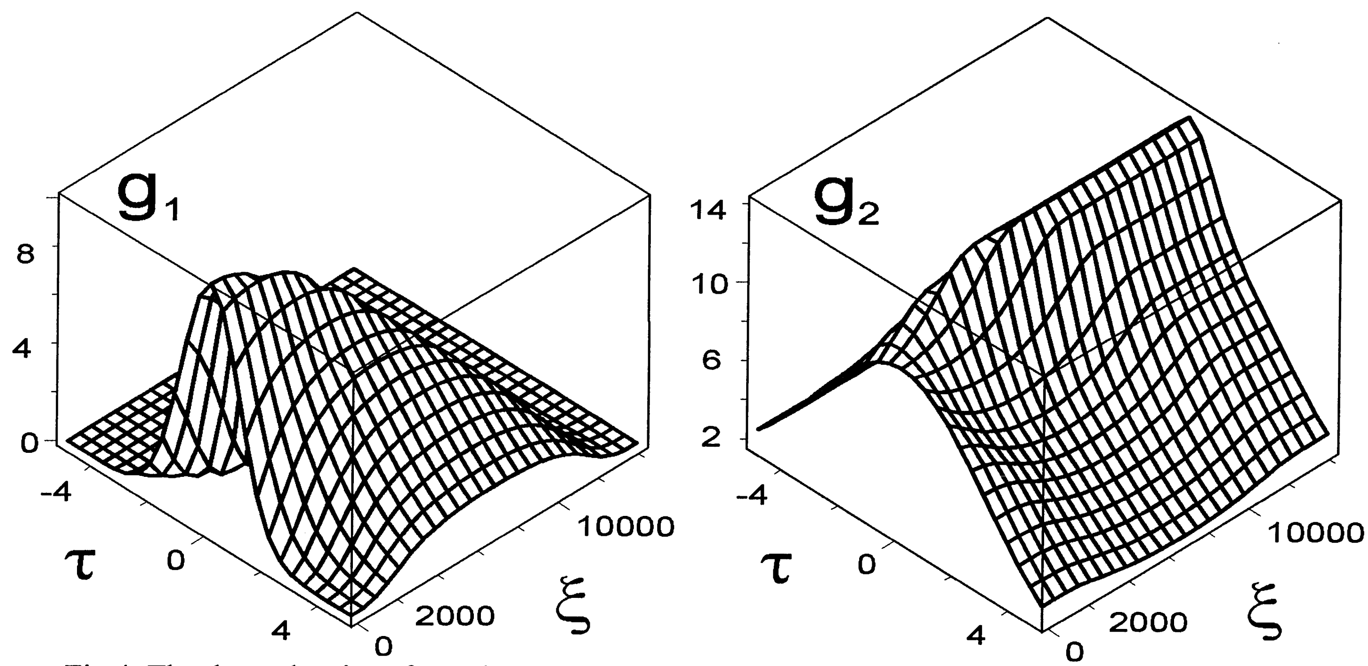

Fig.4. The dependencies of envelopes of Rabi frequencies $g_{1,2} = G_{1,2}T_1$ from the time and the depth of penetration of radiation into a medium. The parameters are the same as in fig.2

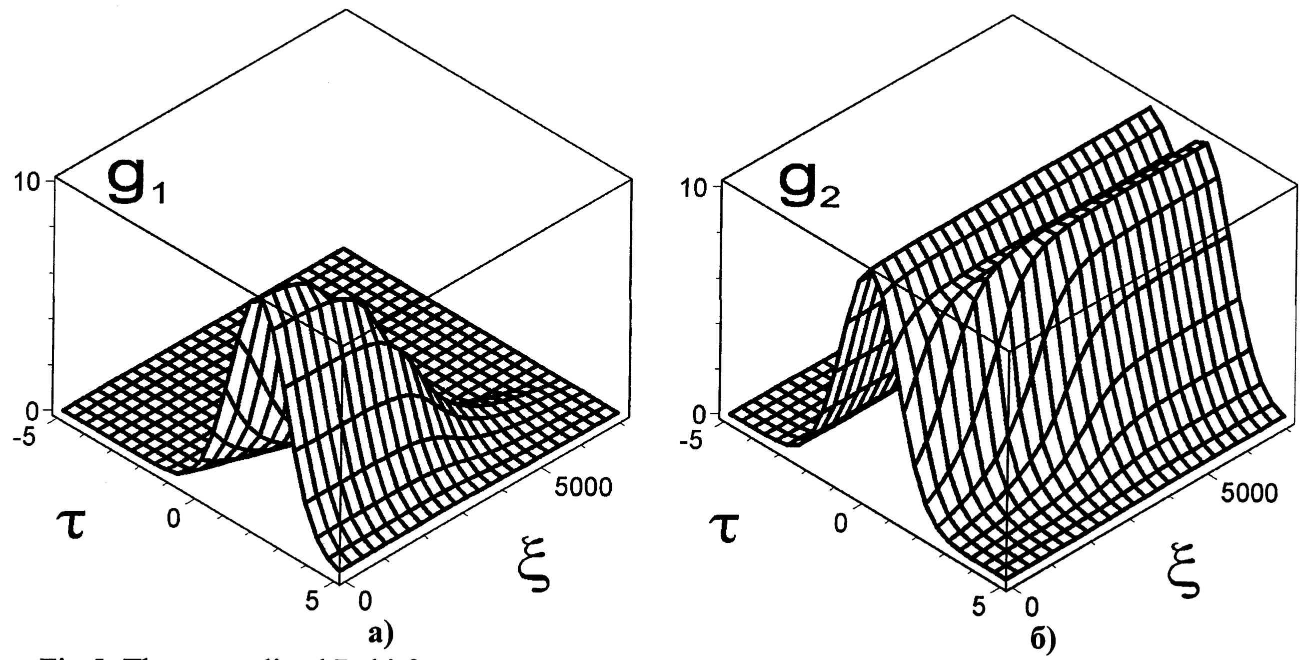

Fig.5. The normalized Rabi frequencies $g_1 = G_1T$ (a) and $g_2 = G_2T$ (b) versus time and length of a medium. The parameters are as follows: $t_0/T = 2$, $G_1^0T = 10$, $G_2^0T = 10$ ($G_{1,2}^0$— value of Rabi frequency $G_{1,2}$ in a maximum); $\Gamma_{10}T = 0.1$, $\Gamma_{12}T = 0.1$, $K_1 = K_2$.The time τ is measured in terms of pulse duration T, and length of a medium ξ — in terms of the length of linear absorption of a probe radiation with frequency ω_1

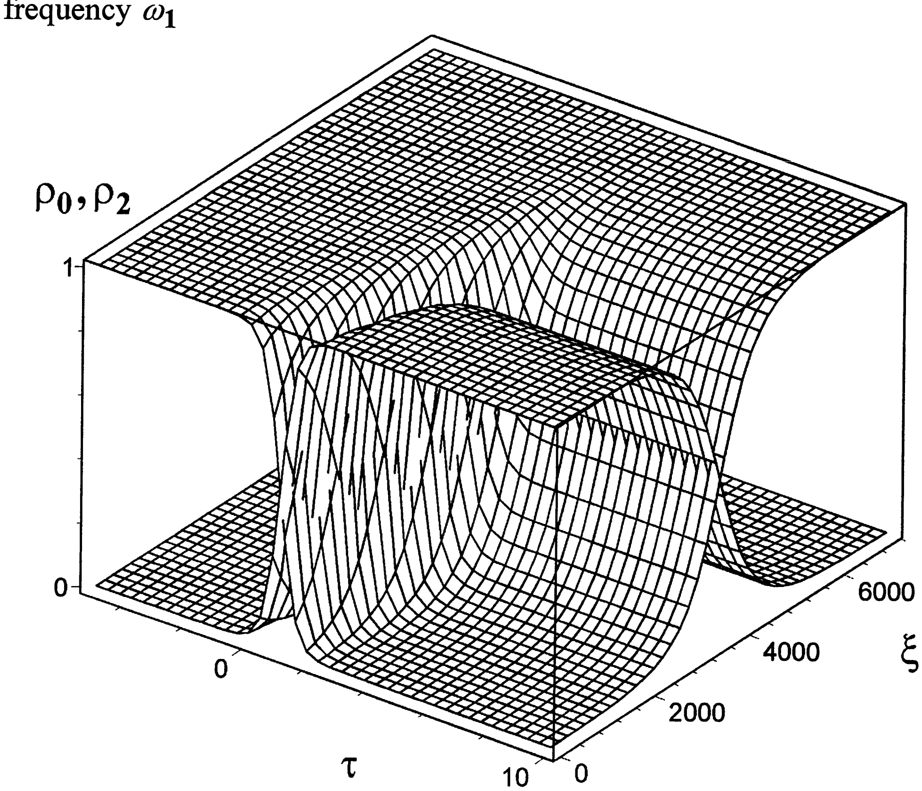

Fig.6. The level populations $\rho_{0,2} = |a_{0,2}|^2$ versus time and length of a medium. The parameters are the same as in Fig.5.