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Spin Polarization in Quantum Dots by Radiation Field with Circular Polarization[1]

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For circular quantum dot (QD), taking into account the Razhba spin–orbit interaction (SOI), an exact energy spectrum is obtained. For a small SOI constant, the eigenfunctions of the QD are found. It is shown that the application of a radiation field with circular polarization removes the Kramers degeneracy of the QD eigenstates. Effective spin polarization of electrons transmitted through the QD owing to a radiation field with circular polarization is demonstrated. *© 2001 MAIK "Nauka/Interperiodica".*

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The spin dependence of the electronic properties of artificial nanostructures is one of today's leading problems in the physics of electronic devices. Of interest are both the improvement of actual devices, like the GaAs polarized electron source (GaAs-PES) [1], and a search for new devices like spin transistors [2]. The effects of the spin degree of freedom on the electron transport properties of semiconductor heterostructures in the presence of inhomogeneous magnetic fields have been intensively studied experimentally [3, 4]. Experiments focusing on fundamental issues used inhomogeneous magnetic fields created either by vortices in superconductors [5, 6] or by ferromagnetic layers [7–9]. Theoretically, the spin-dependent resonant tunneling through magnetic barriers was calculated [10, 11] and the dependence of the spin polarization of transmitted electrons

$$P = \sum_\sigma \sigma T_\sigma / \sum_\sigma T_\sigma \qquad (1)$$

on the magnetic configuration, applied bias, and incident electron energy was found [12].

The spin dependence of the electron transport across nonmagnetic semiconductor heterostructures at zero applied magnetic field arises due to the spin–orbit interaction (SOI). Basically, this phenomenon originates from the well-known phenomenon that the SOI has a polarizing effect on the particle scattering processes [13]; it was considered for different microdevices [14–17].

In this letter, we consider a possibility of resonant spin polarization of transmitted electrons by radiation field with circular polarization. It is well known in atomic spectroscopy that circularly polarized radiation field can transmit an electron from a multiplet state with a half-integer total angular momentum to a continuum with a definite spin polarization [18]. In this article, we consider similar phenomenon for the electron ballistic transport in quantum dots and in microelectronic devices with bound states.

At first stage, we consider a circular quantum dot with hard walls fabricated by metallic gates with applied negative electric potential. Because the standard technique of fabrication of microelectronic devices with depletion of 2DEG is based on the semiconductor GaAs/$Al_xGa_{1-x}As$, the SOI in the Razhba form [19]

$$V_{SL} = \hbar K[\sigma_x p_y - \sigma_y p_x] \qquad (2)$$

is important, where σ_x and σ_y are the Pauli spin matrices. The parameter of spin–orbit coupling K depends on the confining potential profile along the z direction, and, e.g., estimation for InAs structure with effective mass $m^* = 0.023m_0$ gives $\hbar^2 K \sim 6 \times 10^{-3}$ eV nm [20] and $\hbar^2 K \sim 10^{-3}$ eV nm for GaAs structure.

Using the natural energy scale of the QD $E_0 = \hbar^2/2m^*R^2$, where R is the QD radius, and the complex coordinates $z = x + iy$, we rewrite SOI (2) as

$$V_{SL} = 2\beta\begin{pmatrix} 0 & -\partial/\partial z \\ \partial/\partial z^* & 0 \end{pmatrix}, \qquad (3)$$

where the space variables x, y, z are normalized to the QD radius R, and

$$\beta = 2m^*KR. \qquad (4)$$

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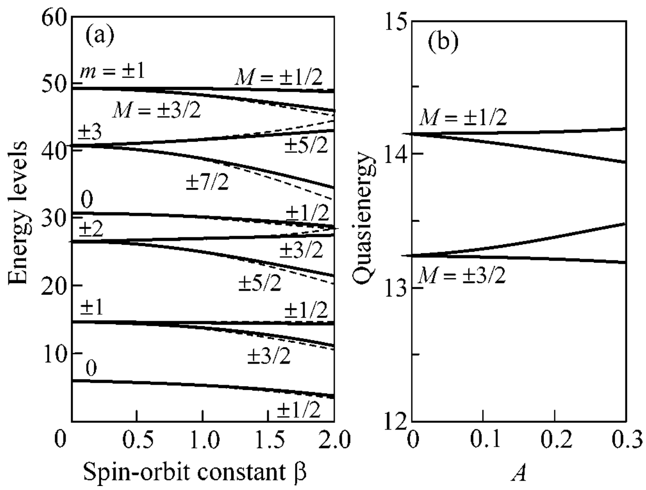

Fig. 1. (a) Energy levels of the QD with the SOI versus the spin–orbit constant β. The exact spectrum (17) is shown by dashed lines, while the approximated energy levels (18) are shown by solid lines. (b) The quasienergy levels of the QD effected by the radiation field with circular polarization versus the amplitude A of the radiation field for the spin–orbit coupling constant $\beta = 1$. In both cases, the QD radius equals unity.

The total Hamiltonian of the QD

$$H = -\nabla^2 + V(r) + V_{SL} \tag{5}$$

commutes with the z projection of the total angular momentum

$$\hat{J}_z = L_z + \frac{1}{2}\sigma_z \tag{6}$$

and with operator of time reversal

$$\hat{K} = -i\sigma_y C, \tag{7}$$

where C is the complex conjugation operator. The first integral of motion (6) allows one to represent eigenstates of Eq. (5) as

$$\psi_m = \begin{pmatrix} u(r)e^{im\phi} \\ v(r)e^{i(m+1)\phi} \end{pmatrix}, \tag{8}$$

because $\hat{J}_z\psi_m = (m+1/2)\psi_m$.

Substituting Eq. (2) into equation $H\psi_m = \epsilon\psi_m$, one can obtain the following systems of radial equations:

$$\begin{gathered} r^2u'' + ru' + (\epsilon r^2 - m^2)u \\ + \beta r^2\left(\frac{d}{dr} + \frac{(m+1)}{r}\right)v = 0, \\ r^2v'' + rv' + (\epsilon r^2 - (m+1)^2)v \\ - \beta r^2\left(\frac{d}{dr} - \frac{m}{r}\right)u = 0. \end{gathered} \tag{9}$$

Taking

$$u = aJ_m(\mu r), \quad v = bJ_{m+1}(\mu r)$$

and using properties of Bessel functions, we have from Eq. (9)

$$\begin{gathered} \left[r^2\frac{d^2}{dr^2} + r\frac{d}{dr} + \left(\epsilon + \beta\mu\frac{b}{a}\right)r^2 - m^2\right]J_m(\mu r) = 0, \\ \left[r^2\frac{d^2}{dr^2} + r\frac{d}{dr} + \left(\epsilon + \beta\mu\frac{a}{b}\right)r^2 - (m+1)^2\right]J_{m+1}(\mu r) = 0. \end{gathered} \tag{10}$$

This equations are compatible only if

$$\mu = (\epsilon + b\beta\mu/a)^{1/2}, \tag{11}$$

$$\mu = (\epsilon + a\beta\mu/b)^{1/2}. \tag{12}$$

Correspondingly, we obtain $a = b$ with

$$\mu_{1\pm} = \beta/2 \pm \sqrt{\epsilon + (\beta/2)^2} \tag{13}$$

or $a = -b$ with

$$\mu_{2\pm} = -\beta/2 \pm \sqrt{\epsilon + (\beta/2)^2}. \tag{14}$$

As a result, we obtain two pairs of linearly independent solutions for Eq. (2). The first one is

$$\Phi^{\pm}_{1,m}(r,\phi) = \begin{pmatrix} J_m(\mu_{1\pm}r)e^{im\phi} \\ J_{m+q}(\mu_{1\pm}r)e^{i(m+1)\phi} \end{pmatrix}. \tag{15}$$

In a similar way, the next pair can be written.

We imply the Dirichlet boundary condition at $r = R$ for a linear combination of solutions (15)

$$C\Phi^{+}_{1,m}(R,\phi) + D\Phi^{-}_{1,m}(R,\phi) = 0. \tag{16}$$

It gives us the following exact equation for the energy spectrum of the QD with the SOI:

$$\begin{gathered} J_m(\mu_{1+}R)J_{m+1}(\mu_{1-}R) \\ -J_m(\mu_{1-}R)J_{m+1}(\mu_{1+}R) = 0. \end{gathered} \tag{17}$$

A few lowest energy levels of the QD versus the SOI constant β are shown in Fig. 1a. It is easy to see that the next pair of equations leads to the same equation as Eq. (17).

Equation (17) can be solved approximately for small constant of the SOI $\beta \le \sqrt{\epsilon}$. If we substitute Eq. (4) into this inequality, we obtain for the GaAs dot that the approximation of small β is valid for $R < 10^{-4}$ cm and low eigenenergies. Expanding Eq. (13) and the Bessel functions over small β, one can obtain after lengthy but elementary calculations the following expressions for approximate energy levels:

$$\epsilon_{mn,1} \approx \frac{x_{nm}^2}{R^2} + \frac{\beta^2}{4}\left[-1 + \frac{2x_{nm}J'_{m+1}(x_{nm})}{J_{m+1}(x_{nm})} - \frac{x_{nm}J''_m(x_{nm})}{J'_m(x_{nm})}\right],$$
$$\epsilon_{mn,2} \approx \frac{x_{nm}^2}{R^2} + \frac{\beta^2}{4}\left[-1 + \frac{2x_{nm}J'_{m-1}(x_{nm})}{J_{m-1}(x_{nm})} - \frac{x_{nm}J''_m(x_{nm})}{J'_m(x_{nm})}\right], \quad (18)$$

where x_{nm} is the nth zero of the Bessel function $J_m(x)$. The approximated spectrum of energy levels (18) is shown in Fig. 1a by dashed lines as a function of the SOI constant β. One can see that for the lowest eigenenergies the approximation is valid even for β exceeding unity.

It is easy to obtain that the SOI gives rise to splitting of degenerate energy levels of the QD with $M = m \pm 1/2$, expect the level with $m = 0$, with value of splitting as

$$\Delta_{mn} = \epsilon_{mn,2} - \epsilon_{mn,1} = \frac{\beta^2}{2}x_{nm}\left[\frac{J'_{m-1}(x_{nm})}{J_{m-1}(x_{nm})} - \frac{J'_{m+1}(x_{nm})}{J_{m+1}(x_{nm})}\right]. \quad (19)$$

Again using smallness of the SOI constant β, one can obtain from Eq. (16)

$$C = 1, \quad D = (-1)^m\left[1 + \beta R\frac{J'_{m+1}(x_{nm})}{J_{m+1}(x_{nm})}\right] \quad (20)$$

and from Eq. (15) the eigenstates

$$\Psi_{M=m+1/2} = \begin{pmatrix} J_m(x_{nm}r/R)e^{im\phi} \\ \frac{\beta}{2}e^{i(m+1)\phi}\left(rJ'_{m+1}(x_{nm}r/R) - RJ'_{m+1}(x_{nm})\frac{J_{m+1}(x_{nm}r/R)}{J_{m+1}(x_{nm})}\right) \end{pmatrix},$$
$$\Psi_{M=m-1/2} = \begin{pmatrix} \frac{\beta}{2}e^{i(m-1)\phi}\left(rJ'_{m-1}(x_{nm}r/R) - RJ'_{m-1}(x_{nm})\frac{J_{m-1}(x_{nm}r/R)}{J_{m-1}(x_{nm})}\right) \\ J_m(x_{nm}r/R)e^{im\phi} \end{pmatrix}. \quad (21)$$

The next pair of degenerate states with $M = -(m \pm 1/2)$ can easily be obtained by applying the Kramers operator (7) to states (21).

Next consider application of the radiation field with circular polarization

$$\mathbf{A}(t) = A(\sin\omega t, \cos\omega t, 0). \quad (22)$$

Note that below we are using the dimensionless radiation field amplitude $A \longrightarrow edA/c\hbar$ [21], where d is the width of leads, attachment of which will be considered below. Similar to the two-level system, an effect of this radiation field can be considered exactly by transformation to the rotating coordinate system by the unitary operator $\exp(i\omega t\hat{J}_z)$ to give rise to the following effective Hamiltonian:

$$\tilde{H} = H - \omega\hat{J}_z + 2iA\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial z^*}\right), \quad (23)$$

where H is given by Eq. (5). Since

$$\frac{\partial}{\partial z}J_m(\mu r)e^{im\phi} = \frac{\mu}{2}J_{m-1}(\mu r)e^{i(m-1)\phi},$$
$$\frac{\partial}{\partial z^*}J_m(\mu r)e^{im\phi} = -\frac{\mu}{2}J_{m+1}(\mu r)e^{i(m+1)\phi}, \quad (24)$$

it obviously follows that the perturbation V can mix only states M and M' differing by $\Delta M = \pm 1$. One can see from Eq. (23) that the radiation field with circular polarization effects the QD like an external magnetic field, i.e., lifts the Kramers degeneracy. This phenomenon firstly was considered by Ritus for an atom affected by the radiation field with circular polarization [22]. Because of $[J_z, V] \neq 0$, we can not present exact eigenstates of the Hamiltonian (23). However, it is clear that the splitting of degenerated quasienergy states $\pm M$ can be found in the second order of perturbation theory to give rise to $\Delta E \sim A^2$. In fact, numerical calculation of eigenvalues of the effective Hamiltonian (23) clearly demonstrates the quadratic behavior of the quasienergy levels versus the amplitude of the radiation field, as is shown in Fig. 1b. Moreover the eigen states $\tilde{\psi}$ of the Hamiltonian (23) are spin polarized ones. In particular, we calculated numerically the spin polarization $\langle S_z\rangle = \langle\tilde{\psi}|\hat{S}_z|\tilde{\psi}\rangle$ for a few lowest states of the QD and found that $\langle S_z\rangle \approx \pm 0.9$ for the first doublet and very slightly depends on A.

Now let us attach leads with a width d to the QD and consider a transmission of electrons unpolarized by the spin through the QD. If coupling of leads with the QD is weak, we have resonant transmission of electrons.

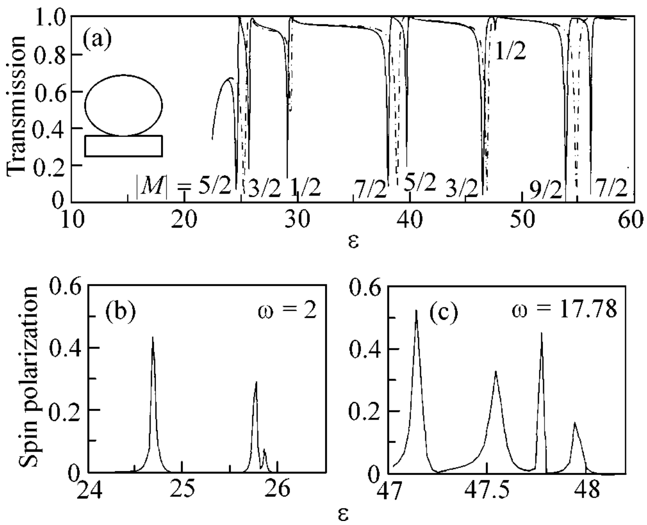

Fig. 2. (a) The transmission probability through the QD without (dashed line) and with the spin–orbit interaction ($\beta = 0.75$, solid line). The radiation field is turned off. (b) The spin polarization versus the energy of incident electrons for the case when the radiation field is not resonant ($\beta = 0.75$). (c) The spin polarization versus the energy of incident electrons for the case of the frequency resonant to transition between the state $|M| = 3/2$ and $|M| = 1/2$ ($\beta = 0.75$).

Because of strong spin polarization of the eigenstates of the QD effected by the radiation field, we can expect the resonant transmission with corresponding spin state while electrons with opposite spin state are reflecting. The aforesaid establishes the basic principle of the spin polarization via the resonant transmission through the QD effected by the radiation field with the circular polarization.

Here, we consider a case with tangential attachment as is shown in the inset of Fig. 2a. This case was considered in [23] and gives rise to resonant dips of the transmission probability. The computer calculations show that such kind of geometry is more effective for the spin polarization in comparison with the standard case of symmetrical attachment of leads to the QD. The only difference is that the electrons with the spin state coincided with that of the eigenstate of the QD are resonantly reflecting, while electrons with the opposite spin are transmitted, giving rise to the spin polarization of outgoing electron beam. Since tangential attachment of leads violates symmetry of the QD relative up to down, there should be spin polarization of transmitted electrons even without the radiation field [14–17]. However, this effect is very negligible in comparison with effect of the radiation field.

The process of electron transmission through the QD with application of a radiation field is complicated because of the appearance of new satellite channels in electron transmission specified by quasi energies [24] $E_n = E_F + n\hbar\omega$, $n = \pm1, \pm2, \ldots$, where

$$E_F = \hbar^2/2m^*d^2[k^2 + (\pi p/d)^2], \quad p = 1, 2, 3, \ldots . \tag{25}$$

A detailed computational procedure of the electron transmission with application of the radiation field is described in [21]. Here, we present only results of the computation shown in Fig. 2. Since Eq. (1) is obtained for the spin state of an incident electron up and down relative the z axis, it follows that $P = \langle S_z \rangle$. Therefore, it necessary to apply operator $\exp(i\hat{\mathbf{S}}\theta)$ to the incident spin state in order to obtain the spin polarization along the x and y axes. As a result, one can obtain the total spin polarization described by a value $P_{\rm tot} = (S_x^2 + S_y^2 + S_z^2)^{1/2}$ which is shown in Figs. 2b and 2c.

Figure 2b clearly demonstrates that for arbitrary frequency of the radiation field but *nonresonant* to transition between the eigenenergies E_M of the QD shown in Fig. 1a we have the resonant spin polarization for $E_F \approx E_M$. Moreover one can see that the energy dependence of the spin polarization is split in accordance with Fig. 1b with a value of the splitting of order A^2. Because of smallness of the radiation field amplitude, the first resonant peak of the spin polarization in Fig. 2b is not resolved.

However, if $\hbar\omega \approx E_{M'} - E_M$, a picture of the resonant spin polarization of the transmitted electrons changes crucially, as is shown in Fig. 2c. As a result, we have enhanced spin polarization for the case $E_F \approx E_M$, for the frequency of the radiation field is tuned to transition between the states $M = 1/2$, $m = 0$, $\epsilon_{1/2} = 29.33$ and $M = 3/2$, $m = 1$, $\epsilon_{3/2} = 47.12$ for the spin-orbit constant $\beta = 0.75$. Moreover, since the frequency of the radiation field is resonant to transition between the QD eigenstates, we observe strong splitting of peaks of the spin polarization because of the Raby splitting. The last is linear to the radiation field amplitude.

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