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RADIOPHYSICS

The Effect of Bound States in Microwave Waveguides on Electromagnetic Wave Propagation

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Abstract—The transmission of a *TE* microwave field with a frequency ω through Γ, *T*, and *X* waveguide junctions filled with a ferromagnetic is considered. These junctions are known to have bound states with below-cutoff frequencies. A probing microwave radiation with a frequency Ω applied to the scattering region generates magnetic oscillations with frequencies $\omega + n\Omega$ (where $n = 0, \pm 1, \pm 2, \ldots$), which resonantly combine with the bound waveguide states. This effect provides for a new method of studying bound waveguide states and efficiently controlling the transmission of microwave radiation. © *2001 MAIK "Nauka/Interperiodica".*

INTRODUCTION

It is well-known [1] that *TE* electromagnetic waves propagating through a planar waveguide of constant width may be represented in terms of scalar potential $\Psi(x, y)$,

$$\mathbf{E}(x, y) = ik\hat{z}\Psi(x, y), \quad \mathbf{B}(x, y) = -\hat{z}\nabla\Psi, \qquad (1)$$

which satisfies the Helmholtz equation

$$[\nabla^2 + \omega^2]\Psi = 0 \qquad (2)$$

(where ∇ is the Hamiltonian operator) with $\psi|_s = 0$ on the waveguide walls. The velocity of light is assumed to equal unity. Equation (2) coincides with the Schrödinger equation that describes ballistic electron transport in electron waveguides [2].

If the waveguide is curved (as shown in Fig. 1a), the Helmholtz equation has an additional bound solution (state) with a *TE* frequency below the cutoff frequency of the propagating *TE* waves: $\omega^2 < (\pi/d)^2$ [3], where d is the waveguide width. This bound state is localized near the waveguide bend. Such bound states, as particular solutions of Eq. (2), have been originally found for Γ waveguide junctions [4, 5] and *T* and *X* waveguide junctions [6, 7] (Figs. 1b–1d). These states are illustrated in Fig. 2. In [8, 9], the bound states were theoretically and experimentally studied as applied to *TE* modes in microwave Γ waveguides. When a probing radiation field was applied to the center of the waveguide bend, the reflected power of the microwave field propagating in the waveguide showed a resonant minimum from which the frequency of coupled electromagnetic oscillations was found [8]. The spatial field structure of localized bound states was also found. Different bound states have also been observed in dielectric waveguides artificially created in two-dimensional photon crystals [10].

However, these bound states by no means affect the propagation of *TE* waves in waveguides. Earlier [11, 12], we proposed a technique for combining bound states with ballistic transport of electrons in waveguides, using a radiation field with a frequency that is tuned to the resonance between the Fermi energy of electrons being transported and the energy of a

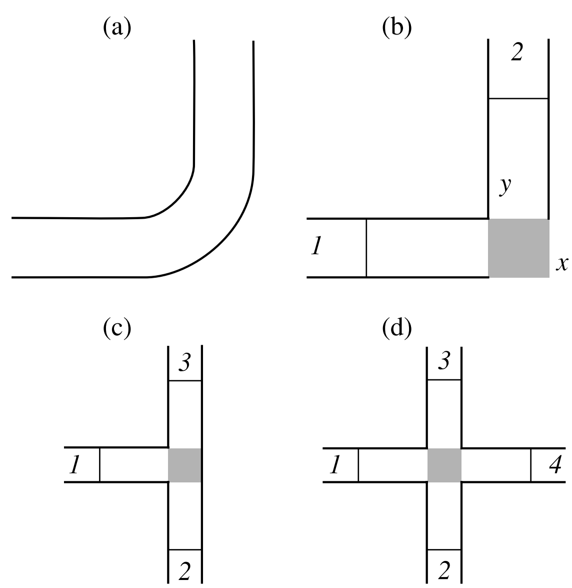

Fig. 1. Waveguide structures that support the bound states illustrated in Fig. 2. (a) Curved waveguide; (b) Γ, (c) *T*, and (d) *X* waveguide junctions. (*1*–*4*) Waveguide arms (shaded regions are those to which the probing field is applied).

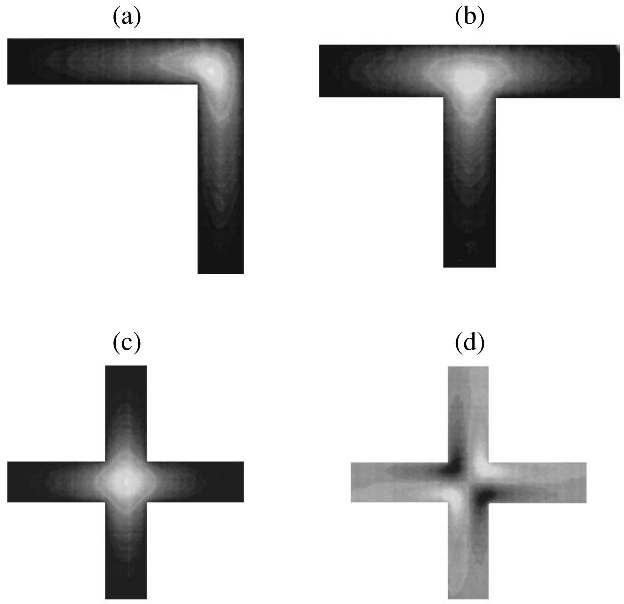

Fig. 2. Spatial structures of the electric field of the bound states in the waveguide junctions shown in Fig. 1.

bound state. As a result, a small-amplitude radiation field is capable of producing deep resonance dips in the waveguide transmittance. A similar approach is considered in this paper for *TE* waves propagating in Γ, *T*, and *X* waveguide junctions. However, in a linear medium, electromagnetic fields do not interact. Therefore, a medium is necessary that would cause such an interaction. In this paper, we consider a waveguide filled with a magnetically ordered material of magnetization *M*. Then, the interaction of the magnetic component of the probing field with the magnetic moment of the ferromagnetic will produce magnetic oscillations with frequencies $\omega + n\Omega$, where $n = 0, \pm1, \pm2, \dots$ and Ω is the frequency of the probing field. If the frequency of one of these harmonics coincides with that of the coupled oscillations, one can expect resonance interaction between the *TE* wave traveling through the junction and the bound state of the waveguide. Thus, by tuning Ω to the resonance between the propagating microwave radiation and the coupled electromagnetic oscillations, one may observe resonance anomalies in the transmission of microwave radiation through curved waveguides. This resonance technique for controlling the transmission of microwave radiation and studying the bound states may also be of applied value.

DYNAMICS OF THE MAGNETIZATION

The equation of motion for magnetic moment **M** in impressed electromagnetic fields [13] is written as

$$\frac{\partial \mathbf{M}}{\partial t} = g(\mathbf{M}\times(\mathbf{H}^{(m)} + \mathbf{H}^{(e)})), \qquad (3)$$

where $\mathbf{H}^{(e)}$ is the applied magnetic field consisting of the constant field H_0 and variable magnetic field of the probing radiation aligned with the *z* axis, and $\mathbf{H}^{(m)}$ is the magnetic component of the microwave field propagating in the waveguide ($\mathbf{H}^{(m)}$ lies in the *xy* plane).

It is convenient to introduce the complex amplitudes

$$M_\pm = M_x \pm iM_y, \quad H_\pm^{(m)} = H_x^{(m)} \pm iH_y^{(m)}, \qquad (4)$$

in terms of which Eq. (3) takes the form

$$\begin{aligned}\frac{\partial M_+}{\partial t} &= -igH^{(e)}M_+ + igM_zH_+^{(m)},\\ \frac{\partial M_z}{\partial t} &= -\frac{1}{2}ig[H_+^{(m)}M_- - H_-^{(m)}M_+].\end{aligned} \qquad (5)$$

For harmonic oscillations, $\xi_+ = M_+/M$ is small. Then,

$$M_z \approx M\left(1 - \frac{M_+M_-}{2M^2}\right) = M\left(1 - \frac{1}{2}|\xi_+|^2\right).$$

Accordingly, Eq. (5) can be written in the linear approximation as

$$\begin{aligned} i\frac{\partial \xi_+}{\partial t} &= gH^{(e)}\xi_+ - gH_+,\\ -\frac{1}{2}\frac{\partial|\xi_+|^2}{\partial t} &= g\,\mathrm{Im}(H_+^{(m)}\xi_-).\end{aligned} \qquad (6)$$

Let the probing microwave field have the simplest form, $H^{(e)}(\mathbf{r}, t) = H_0 + \lambda\cos\Omega t$, and be directed along the *z* axis. We assume that the alternating component of this field acts only in the waveguide region occupied by the bound state. Consider the amplitude $H_+^{(m)}(t) = H_+^{(0)}e^{i\omega t}$ that describes the field rotating counterclockwise with a circular frequency ω in the *xy* plane.

If the probing field is absent ($\lambda = 0$), the coupling between magnetization oscillations $\xi_+(t) = \xi_+^{(0)}e^{i\omega t}$ and the microwave field propagating in the waveguide is expressed in terms of the susceptibility [13] as $\xi_+^{(0)} = \chi(\omega)H_+^{(0)}$, where

$$\chi(\omega) = \frac{g}{\omega + gH_0}. \qquad (7)$$

In general, when $\lambda \neq 0$, we substitute $\xi_+(t) = \xi_+^{(1)}e^{i\omega t}$ into Eqs. (6) to obtain

$$i\frac{\partial \xi_+^{(1)}(t)}{\partial t} = (\omega + gH_0 + g\lambda\cos\Omega t)\xi_+^{(1)}(t) - gH_+^{(0)}. \qquad (8)$$

A solution to Eq. (8) can be sought in the form

$$\xi_+^{(1)}(t) = F(t)\exp\left[-i(\omega + gH_0)t - i\frac{g\lambda}{\Omega}\sin\Omega t\right].$$

Substituting it into Eq. (8) yields

$$F(t) = F_0 + igH_+^{(0)} \times \int \exp\left[i(\omega + gH_0)\tau + i\frac{g\lambda}{\Omega}\sin\Omega\tau\right]d\tau. \tag{9}$$

Since we consider forced oscillations, $F_0 = 0$. Therefore, the solution to Eq. (8) can again be represented in terms of susceptibility $\chi(t)$, which is now a periodic function of time:

$$\xi_+^{(1)}(t) = \chi_+(t)H_+^{(0)},$$

$$\chi_+(t) = ig\exp\left[-i(\omega + gH_0)t - i\frac{g\lambda}{\Omega}\sin\Omega t\right] \times \int \exp\left[i(\omega + gH_0)\tau + i\frac{g\lambda}{\Omega}\sin\Omega\tau\right]d\tau \tag{10}$$

$$= g\exp\left(-i\frac{g\lambda}{\Omega}\sin\Omega t\right)\sum_m \frac{J_m\left(\frac{g\lambda}{\Omega}\right)}{\omega + gH_0 + m\Omega}\exp(im\Omega t).$$

As follows from (10), the susceptibility $\chi_+(t)$ is a periodic function of time, with its period being equal to that of the applied probing field. Therefore, the solutions for the probing field and the magnetization have the form of a superposition of *TE* modes of different circular polarizations with frequencies $\omega + n\Omega$, where $n = 0, \pm1, \pm2, \ldots$:

$$H_+(t) = \sum_n \{h_n e^{i(\omega + n\Omega)t} + \tilde{h}_n e^{-i(\omega + n\Omega)t}\},$$
$$\xi_+(t) = \sum_n \{\xi_n(t)e^{i(\omega + n\Omega)t} + \tilde{\xi}_n(t)e^{-i(\omega + n\Omega)t}\}. \tag{11}$$

Then

$$\xi_n(t) = \xi_{(+)n}(t)h_n, \quad \tilde{\xi}_n(t) = \chi_{(-)n}(t)\tilde{h}_n,$$

where $\chi_{(\pm)n}(t)$ is the response of magnetic system (10) to the magnetic field rotating with a frequency $\omega + n\Omega$ counter- or clockwise (the plus and minus subscripts, respectively).

As follows from (10), the susceptibility $\chi_{(\pm)n}(t)$ has the period $2\pi/\Omega$. Therefore, we represent it as a Fourier series over this period:

$$\chi_{(+)n}(t)e^{in\Omega t} = \sum_m \chi_{(+)nm}e^{im\Omega t},$$
$$\chi_{(-)n}(t)e^{-in\Omega t} = \sum_m \chi_{(-)nm}e^{-im\Omega t}, \tag{12}$$

where

$$\chi_{(+)nm} = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} \chi_{(+)n}(t)e^{i(n-m)\Omega t},$$
$$\chi_{(-)nm} = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} \chi_{(-)n}(t)e^{-i(n-m)\Omega t}. \tag{13}$$

The best way to calculate $\chi_{(\pm)nm}$ is to make advantage of the fact that, as follows from Eqs. (8) and (10), $\chi_{+n}(t)$ satisfy the equations

$$i\frac{d\chi_{(+)n}}{dt} = (\omega + n\Omega + gH_0 + g\lambda\cos\Omega t)\chi_{(+)n} - g,$$
$$i\frac{d\chi_{(-)n}}{dt} = (gH_0 - \omega - n\Omega + g\lambda\cos\Omega t)\chi_{(-)n} - g. \tag{14}$$

Substituting expansions (12) into Eqs. (14), we arrive at the system of algebraic equations for $\chi_{(+)nm}$:

$$\frac{\lambda}{2}\chi_{(+)n,\,m-1} + (H_0 + \omega/g + m\Omega/g)\chi_{(+)nm} + \frac{\lambda}{2}\chi_{(+)n,\,m-1} = \delta_{nm},$$
$$\frac{\lambda}{2}\chi_{(-)n,\,m-1} + (H_0 - \omega/g - m\Omega/g)\chi_{(-)nm} + \frac{\lambda}{2}\chi_{(-)n,\,m+1} = \delta_{nm}. \tag{15}$$

EQUATIONS FOR ELECTROMAGNETIC FIELDS

Let us write the Maxwell equations in a magnetic medium:

$$\mathbf{B}^{(m)} = \mathbf{H}^{(m)} + 4\pi\mathbf{M}, \quad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}^{(m)}}{\partial t},$$
$$\nabla\times\mathbf{H}^{(m)} = 4\pi\mathbf{j} + \frac{\partial\mathbf{E}}{\partial t}, \quad \nabla\mathbf{B}^{(m)} = 0, \quad \nabla\mathbf{E} = 0. \tag{16}$$

Solutions to (16) in view of (11) will be sought such that the electric field is directed along the z axis, the magnetic field $\mathbf{H}^{(m)}$ lies in the xy plane, and both fields are functions of x and y. We represent E_z in terms of complex fields Ψ_n as

$$E_z(x, y, t) = \mathrm{Im}\left[\sum_n \Psi_n(x, y)e^{i(\omega + n\Omega)t}\right]. \tag{17}$$

It is convenient to introduce the Cauchy derivative

$$\frac{\partial}{\partial u} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right),$$

with which the following useful relationships can be written:

$$(\nabla\times\mathbf{M})_z = 2M\mathrm{Im}\left(\frac{\partial\xi_+}{\partial u}\right) = 2M\mathrm{Re}\left(-i\frac{\partial\xi_+}{\partial u}\right),$$
$$\nabla\mathbf{M} = 2\mathrm{Re}\left(\frac{\partial\xi_+}{\partial u}\right),$$
$$(\nabla\times\mathbf{H}^{(m)})_z = 2\mathrm{Im}\left(\frac{\partial H_+^{(m)}}{\partial u}\right) = 2\mathrm{Re}\left(-i\frac{\partial H_+^{(m)}}{\partial u}\right),$$
$$\nabla\mathbf{H}^{(m)} = \mathrm{Re}\left(\frac{\partial H_+^{(m)}}{\partial u}\right). \quad (18)$$

It is also useful to reduce Maxwell equations (16) to the second-order differential equations

$$\nabla\times\nabla\times\mathbf{E} = -\frac{\partial\nabla\times\mathbf{B}^{(m)}}{\partial t},$$
$$\nabla\times(\mathbf{B}^{(m)} - 4\pi\mathbf{M}) = 4\pi\mathbf{j} + \frac{\partial\mathbf{E}}{\partial t},$$

from which one more equation follows:

$$\nabla\times\nabla\times\mathbf{E} = -4\pi\frac{\partial\mathbf{j}}{\partial t} - 4\pi\nabla\times\frac{\partial\mathbf{M}}{\partial t} - \frac{\partial^2\mathbf{E}}{\partial t^2}.$$

Since the impressed current is zero ($\mathbf{j} = 0$), we have

$$\nabla^2\mathbf{E} - \frac{\partial^2\mathbf{E}}{\partial t^2} - 4\pi\nabla\times\frac{\partial\mathbf{M}}{\partial t} = 0. \quad (19)$$

Next, we combine the equation $\nabla(\mathbf{H}^{(m)} + 4\pi\mathbf{M}) = 0$ and (18) to obtain

$$\mathrm{Re}\left(\frac{\partial H_+^{(m)}}{\partial u} + 4\pi M\frac{\partial\xi_+}{\partial u}\right) = 0.$$

Substituting representation (11) for H_+ and ξ_+ into this formula, we come to

$$\mathrm{Re}\sum_n\left\{\mu_{(+)n}(t)\frac{\partial h_n}{\partial u}e^{i(\omega+n\Omega)t} + \mu_{(-)n}(t)\frac{\partial\tilde{h}_n}{\partial u}e^{-i(\omega+n\Omega)t}\right\} = 0, \quad (20)$$

where

$$\mu_{(\pm)n}(t) = 1 + 4\pi M\chi_{(\pm)n}(t),$$
$$\mu_{(\pm)nm} = \delta_{nm} + 4\pi M\chi_{(\pm)nm} \quad (21)$$

is the permeability.

From expansions (12),

$$\mathrm{Re}\left\{\sum_{nm}\left[\frac{\partial h_n}{\partial u}e^{i(\omega+n\Omega)t}\mu_{(+)nm} + \frac{\partial\tilde{h}_n}{\partial u}e^{-i(\omega+m\Omega)t}\mu_{(-)nm}\right]\right\} = 0,$$

hence,

$$\sum_n\frac{\partial h_n}{\partial u}\mu_{(+)nm} = -\sum_n\left(\frac{\partial\tilde{h}_n}{\partial u}\right)^*\mu^*_{(-)nm}. \quad (22)$$

Now consider the second Maxwell equation

$$\frac{\partial E_z}{\partial t} = 2\mathrm{Im}\left(\frac{\partial H_+}{\partial u}\right).$$

With Eqs. (11) and (17), we obtain

$$\sum_n\mathrm{Im}\left\{e^{i(\omega+n\Omega)t}\left[i(\omega+n\Omega)\Psi_n - 2\left(\frac{\partial h_n}{\partial u}\right)\right] - 2\left(\frac{\partial\tilde{h}_n}{\partial u}\right)e^{-i(\omega+n\Omega)t}\right\} = 0$$

or

$$i(\omega+n\Omega)\Psi_n - 2\left(\frac{\partial h_n}{\partial u}\right) = -2\left(\frac{\partial\tilde{h}_n}{\partial u}\right)^*. \quad (23)$$

Substituting this equation into (22), we have

$$\frac{1}{2}\sum_n\left(\frac{\partial h_n}{\partial u}\right)[\mu_{(+)nm} + \mu^*_{(-)nm}] = \frac{i}{4}\sum_n\mu^*_{(-)nm}(\omega+n\Omega)\Psi_n. \quad (24)$$

Formula (24) is a system of linear algebraic equations that relates $\partial h_n/\partial u$ and Ψ_n:

$$\frac{\partial\mathbf{h}}{\partial u} = \hat{A}\mathbf{\Psi}, \quad (25)$$

where $\mathbf{h} = (\ldots h_1, h_0, h_{-1}, \ldots)$ and $\mathbf{\Psi} = (\ldots\Psi_1, \Psi_0, \Psi_{-1}, \ldots)$.

Relationship (24) allows us to write a closed equation for Ψ_n. In fact, in view of Eqs. (18), Eq. (19) takes the form

$$\nabla^2 E_z - \frac{\partial^2 E_z}{\partial t^2} - 8\pi M\frac{\partial}{\partial t}\mathrm{Im}\frac{\partial\xi_+}{\partial u} = 0. \quad (26)$$

As follows from (11),

$$\frac{\partial \xi_+}{\partial u} = \sum_{nm} \left\{ e^{i(\omega + n\Omega)t} \chi_{(+)nm} \left(\frac{\partial h_n}{\partial u} \right) + e^{-i(\omega + n\Omega)t} \chi_{(-)nm} \left(\frac{\partial \tilde{h}_n}{\partial u} \right) \right\}.$$

Substituting this relationship into (26), we obtain, in view of (17),

$$\mathrm{Im} \sum_{nm} \left\{ e^{i(\omega + n\Omega)t} \left[\delta_{nm} (\nabla^2 \Psi_m + (\omega + n\Omega)^2 \Psi_m) - 8\pi M i (\omega + m\Omega) \chi_{(+)nm} \left(\frac{\partial h_n}{\partial u} \right) \right] + 8\pi M i e^{-i(\omega + n\Omega)t} (\omega + m\Omega) \chi_{(-)nm} \left(\frac{\partial \tilde{h}_n}{\partial u} \right) \right\} = 0$$

or

$$\nabla^2 \Psi_m + (\omega + m\Omega)^2 \Psi_m + 8\pi M i \sum_n (\omega + m\Omega) \times \left[\chi^*_{(-)nm} \left(\frac{\partial \tilde{h}_n}{\partial u} \right)^* - \chi_{(+)nm} \left(\frac{\partial h_n}{\partial u} \right) \right] = 0.$$

With the expression for $(\partial \tilde{h}_n / \partial u)^*$ from (23), we eventually derive the closed equation for Ψ_n:

$$\nabla^2 \Psi_m + (\omega + m\Omega)^2 \Psi_m + 4\pi M \sum_n (\omega + m\Omega) \times (\omega + n\Omega) \chi^*_{(-)nm} \Psi_n - 8\pi M i \sum_n (\omega + m\Omega) \times \left(\frac{\partial h_n}{\partial u} \right) (\chi_{(+)nm} - \chi^*_{(-)nm}) = 0, \tag{27}$$

where the derivatives $(\partial h_n / \partial u)$ are defined through Ψ_n by Eq. (25).

Let us check that, in the limit $\lambda \longrightarrow 0$, Eq. (27) describes the well-known problem of microwave propagation through a magnetic medium [14]. As follows from (10) and (13), the susceptibility matrix is diagonal:

$$\chi_{(+)nn} = \frac{g}{gH_0 + \omega + n\Omega}, \quad \chi_{(-)nm} = \frac{g}{gH_0 - \omega - n\Omega}.$$

Accordingly, Eqs. (24) and (27) take the form

$$\frac{1}{2} \left(\frac{\partial h_m}{\partial u} \right) [\mu_{(+)m} + \mu^*_{(-)m}] = \frac{i}{4} \mu^*_{(-)m} (\omega + m\Omega) \Psi_m,$$

$$\nabla^2 \Psi_m + (\omega + m\Omega)^2 \mu_{(-)m} \Psi_m - 2i(\omega + m\Omega)(\mu_{(+)m} - \mu_{(-)m}) \left(\frac{\partial h_m}{\partial u} \right) = 0,$$

where

$$\mu_{(\pm)m} = 1 + 4\pi M \chi_{(\pm)mm}.$$

Combining the last two equations, we obtain the equation given in [14]:

$$\nabla^2 \Psi_m + (\omega + m\Omega)^2 \frac{\mu^2 - \mu_a^2}{\mu} \Psi_m = 0, \tag{28}$$

where $\mu_{(\pm)m} = \mu \pm \mu_a$.

Let us turn back to the case when a waveguide is filled with a magnetic and is subjected to a local radiation field with a frequency Ω. Consider the simplest spatial inhomogeneity in the probing field when it is uniform in the region of the bound state and vanishes outside. In order to derive the boundary conditions at the interface, we invoke one more Maxwell equation

$$-\frac{\partial B_x}{\partial t} = \frac{\partial E_z}{\partial y}, \quad \frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x}, \tag{29}$$

where the magnetic induction can also be represented as a series in h_m and $\tilde{h}_m$ by virtue of (11).

Substituting these series for the magnetic induction B_+ and expansion (17) of the electric field E_z in $\Psi_n(x, y)$ into Eqs. (29) yields the equations

$$\frac{\partial \Psi_n}{\partial y} = (\omega + n\Omega) \sum_m (\mu_{(+)mn} h_m + \mu^*_{(-)mn} \tilde{h}^*_m), \tag{30}$$

$$\frac{\partial \Psi_n}{\partial x} = i(\omega + n\Omega) \sum_m (\mu_{(+)mn} h_m + \mu^*_{(-)mn} \tilde{h}^*_m), \tag{31}$$

where the matrix μ is defined by Eqs. (21).

From these two equations, one readily deduces the expressions for Cauchy's derivatives:

$$\frac{\partial \Psi_n}{\partial u} = -i(\omega + n\Omega) \sum_m \mu^*_{(-)mn} \tilde{h}^*_m, \qquad \frac{\partial \Psi_n}{\partial u^*} = i(\omega + n\Omega) \sum_m \mu_{(+)mn} h_m. \tag{32}$$

Equation (22), which follows from the equation $\nabla \mathbf{B} = 0$ (where B is the scalar product), can be extended to the nonuniform case as follows:

$$\sum_n \left\{ \frac{\partial h_n}{\partial u} \mu_{(+)nm} + \frac{\partial \mu_{(+)nm}}{\partial u} h_n \right\} = -\sum_n \left\{ \left(\frac{\partial \tilde{h}_n}{\partial u} \right)^* \mu^*_{(-)nm} + \frac{\partial \mu^*_{(-)nm}}{\partial u^*} \tilde{h}^*_n \right\}. \tag{33}$$

If the medium is inhomogeneous along the x axis, Eq. (33) can be used to show that the jumps at the interface satisfy the equation

$$\Delta\left\{\sum_n h_n \mu_{(+)nm}\right\} = -\Delta\left\{\sum_n \tilde{h}_n^* \mu^*_{(-)nm}\right\}$$

or, by virtue of Eqs. (32),

$$\Delta\left\{\frac{\partial\Psi_n}{\partial u}\right\} = \Delta\left\{\frac{\partial\Psi_n}{\partial u^*}\right\}. \tag{34}$$

Equation (23), which follows from the third Maxwell equation in (16), is satisfied everywhere:

$$\left(\frac{\partial\tilde{h}_n}{\partial u}\right)^* = \frac{\partial h_n}{\partial u} - \frac{i}{2}(\omega + n\Omega)\Psi_n. \tag{35}$$

The electric field must be continuous at the interface. Otherwise, as follows from (32), the fields h_m would be singular. Hence, Eq. (35) gives one more boundary condition:

$$\Delta\tilde{h}_m^* = \Delta h_m. \tag{36}$$

If the interface is perpendicular to the y axis, Eqs. (32), (33), and (35) yield the similar boundary conditions

$$\Delta\left\{\frac{\partial\Psi_n}{\partial u}\right\} = -\Delta\left\{\frac{\partial\Psi_n}{\partial u^*}\right\}, \qquad \Delta\tilde{h}_n^* = -\Delta h_m. \tag{37}$$

It is convenient to represent the equation for the generalized vector $\mathbf{\Psi}$ in the matrix form:

$$\frac{\partial\mathbf{\Psi}}{\partial u} = \hat{L}_-\tilde{\mathbf{h}}^*, \quad \frac{\partial\mathbf{\Psi}}{\partial u^*} = \hat{L}_+\mathbf{h}, \tag{38}$$

where

$$\hat{L}_{(-)nm} = -(\omega + n\Omega)\mu^*_{(-)mn}, \qquad \hat{L}_{(+)nm} = i(\omega + n\Omega)\mu_{(+)mn}. \tag{39}$$

Then, Eqs. (24) and (27) take the more compact form

$$(\hat{\mathbf{L}}_+ - \hat{\mathbf{L}}_-)\frac{d\mathbf{h}}{\partial u} = -\frac{1}{2}\hat{\mathbf{L}}_-\hat{\mathbf{P}}\mathbf{\Psi}, \tag{40}$$

$$\nabla^2\mathbf{\Psi} + \hat{\mathbf{L}}_-\hat{\mathbf{P}}\mathbf{\Psi} - 2(\hat{\mathbf{L}}_+ + \hat{\mathbf{L}}_-)\frac{d\mathbf{h}}{\partial u} = 0, \tag{41}$$

respectively, where

$$\hat{\mathbf{P}} = i\,\mathrm{diag}(\omega + n\Omega).$$

Combining Eqs. (40) and (41) gives the closed equation for Ψ:

$$\nabla^2\mathbf{\Psi} + \hat{\mathbf{L}}_-\hat{\mathbf{P}}\mathbf{\Psi} + (\hat{\mathbf{L}}_+ + \hat{\mathbf{L}}_-)(\hat{\mathbf{L}}_+ - \hat{\mathbf{L}}_-)^{-1}\hat{\mathbf{L}}_-\hat{\mathbf{P}}\mathbf{\Psi} = 0. \tag{42}$$

Using the matrix

$$\hat{\mathbf{D}} = (\hat{\mathbf{L}}_-^{-1} - \hat{\mathbf{L}}_+^{-1})\{(\hat{\mathbf{L}}_-\hat{\mathbf{P}} + (\hat{\mathbf{L}}_+ + \hat{\mathbf{L}}_-)(\hat{\mathbf{L}}_+ - \hat{\mathbf{L}}_-)^{-1}\hat{\mathbf{L}}_-)\hat{\mathbf{P}}\}$$

and the equality

$$(\hat{\mathbf{L}}_-^{-1} - \hat{\mathbf{L}}_+^{-1})\hat{\Delta} = 4\frac{\partial}{\partial u^*}\hat{\mathbf{L}}_-^{-1}\frac{\partial\mathbf{\Psi}}{\partial u} - 4\frac{\partial}{\partial u}\hat{\mathbf{L}}_+^{-1}\frac{\partial\mathbf{\Psi}}{\partial u^*},$$

we rewrite Eq. (42) as

$$4\frac{\partial}{\partial u^*}\hat{\mathbf{L}}_-^{-1}\frac{\partial\mathbf{\Psi}}{\partial u} - 4\frac{\partial}{\partial u}\hat{\mathbf{L}}_+^{-1}\frac{\partial\mathbf{\Psi}}{\partial u^*} + \hat{\mathbf{D}}\mathbf{\Psi} = 0. \tag{43}$$

In this case, boundary conditions (34) and (36) take the form

$$\Delta\left\{\hat{\mathbf{L}}_-^{-1}\frac{\partial\mathbf{\Psi}}{\partial u} - \hat{\mathbf{L}}_+^{-1}\frac{\partial\mathbf{\Psi}}{\partial u^*}\right\} = 0, \qquad \Delta\frac{\partial\mathbf{\Psi}}{\partial u} = \Delta\frac{\partial\mathbf{\Psi}}{\partial u^*}. \tag{44}$$

For the interface orthogonal to the x axis, the latter boundary condition means that $\partial\Psi/\partial y$ is continuous at the interface; hence, it is satisfied automatically. The former boundary condition in (44) is easily taken into account in Eq. (43), which also includes the interface where $\hat{\mathbf{L}}_\pm$ exhibits a jump. It can easily be checked that Eq. (43) is also consistent with boundary conditions (37) for the interface orthogonal to the y axis. This conclusion is important, because it allows us to solve Eq. (43) everywhere, including the interfaces at which $\hat{\mathbf{L}}_\pm$ is discontinuous.

Γ, *T*, AND *X* WAVEGUIDE JUNCTIONS

Consider Γ, *T*, and *X* waveguide junctions with the interfaces shown in Fig. 1. The shaded regions are those to which the probing field $\lambda\cos\Omega t$ is applied. We assume that the probing radiation is weak and its magnetic field crosses the two-dimensional waveguide orthogonally, i.e., along the z axis. In our calculations, we will use the following values typical of ferromagnetic materials: $M = 1700$ G and $g = 2 \times 10^7$ CGS. Let us introduce the dimensionless quantities

$$\tilde{\mathbf{r}} = \mathbf{r}/d, \quad \tilde{\omega} = d\omega/c, \quad \tilde{\Omega} = d\Omega/c, \qquad \tilde{H}_0 = gH_0 d/c, \quad \tilde{\lambda} = g\lambda d/c, \quad m = gMd/c, \tag{45}$$

where c is the velocity of light.

When the probing field is absent, the equation for the microwave field in terms of the dimensionless quantities takes the form

$$\nabla^2\Psi + k^2\Psi = 0, \qquad k^2 = \tilde{\omega}^2\frac{2\mu_+\mu_-}{\mu_+ + \mu_-}, \tag{46}$$

$$\mu_{\pm} = 1 + 4\pi\frac{m}{\tilde{H}_0 \pm \tilde{\omega}}.$$

For a typical centimeter-wave waveguide with the width $d = 1$ cm subjected to a constant magnetic field $H_0 = 1000$, formulas (45) yield $m \approx 1$, $\tilde{H}_0 \sim 1$, $\omega = 3 \times 10^{10}$ s^{-1}, and $\tilde{\omega} \sim 1$.

NUMERICAL SOLUTIONS

Our numerical analysis of the scattering of an incident microwave radiation in a magnetic-filled waveguide relies on system (43) of second-order linear differential equations for amplitudes $\{\Psi_n\}$. The boundary conditions specify the asymptotic behavior of the electromagnetic waves far away from the scattering region (waveguide diffraction conditions). A typical scattering scenario considers an incident wave only in one arm of the waveguide junction that will referred to as the first one. The scattered field penetrates into all the waveguide arms.

System (43) was numerically solved on a square mesh, which is natural for the waveguide junctions under study (Fig. 1). Since the probing radiation field is applied only to the region of scattering (the shaded region in Fig. 1), the operator $\hat{L}_{\pm}$ is no longer diagonal in this region, and the propagating microwave field mixes with coupled electromagnetic oscillations in the waveguide structure. In the waveguide arms (i.e., far from the scattering region), Eq. (43) is the Helmholtz equation and describes *TE* waves with frequencies $\omega + n\Omega$ in a perfect ferromagnetic [see Eq. (28)]. The waveguide diffraction problem is solved by joining a solution to Eq. (43) in the scattering region and all *TE* waves outside it. The problem is almost the same as that of ballistic electron transport in electron waveguides. The basis for the numerical solution of this problem in the steady-state case was set by Ando [15]. A generalization to the dynamic problem of electron scattering was given in our previous paper [12].

Let us define the transmission coefficient T_{ij} as the ratio of the output power in the ith waveguide arm to the input power in the jth waveguide arm, where i and j are the respective numbers of the input and output arms. Clearly, if there is no probing field,

$$1 + G = \sum_j T_{ij} = 1,$$

as follows from the energy conservation law.

However, when the probing field is applied, the electromagnetic power may be derived (or transmitted) as the microwave field passes through the junction. Thus, G defines the absorbed power of the probing field.

The power can be calculated in terms of the Poynting vector

$$\mathbf{\Pi} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{H}.$$

With representations (11) and (17), this equation yields after time averaging

$$\bar{\Pi}_x = \frac{c}{8\pi}\mathrm{Re}\sum_m\{\Psi_m\tilde{h}_m - \Psi_m^* h_m\},$$
$$\bar{\Pi}_y = \frac{c}{8\pi}\mathrm{Im}\sum_m\{\Psi_m\tilde{h}_m - \Psi_m^* h_m\}. \quad (47)$$

Figure 3 shows the resonant distribution of the powers $\bar{\Pi}$ for all the waveguide junctions considered above. It is clearly seen that the resonant mixing of the localized bound states causes the vortex structure of the power fluxes. The lower right panel in Fig. 3 demonstrates how the nodal lines of the second bound state of the X junction affect the power-flux pattern. These effects were also considered for ballistic electron transport through electron waveguides [12].

In order to find the power flux of the microwave field in the waveguide arms (outside the scattering region), we represent h_m in terms of Ψ_m using Eq. (38). Consider waveguide arm *1* (Fig. 1) to be the input and integrating over its cross section of unit width, we obtain the time-averaged power from (47) as

$$W_1 = -\frac{icL}{16\pi}\sum_m\left\{([\hat{L}_+]_m^{-1} - [\hat{L}_-]_m^{-1})\mathrm{Im}\left(\Psi_m^*\frac{\partial\Psi_m}{\partial x}\right)\right\}. \quad (48)$$

The power flux in the other waveguide arms are calculated likewise. As a result, we find the transmission coefficients

$$T_{ij} = W_j/W_i.$$

Figures 4a and 4c plot the frequency dependences of the transmission coefficient for the *TE* wave passing through the Γ waveguide junction when the frequency of the probing field is $\tilde{\Omega} = 0.4$. The thin lines refer to the zero probing field. These figures clearly show the resonant transmission of the *TE* wave at $\tilde{\omega} = 1.204$. Accordingly, the frequency of the bound electromagnetic oscillations in the Γ junction is $\tilde{\omega}_b = \tilde{\omega} - \tilde{\Omega} = 0.804$. At $\tilde{H}_0 = 1$, $m = 1$, and the frequency of the bound oscillations $\tilde{\omega}_b = 0.804$, formulas (45) yield $k_b^2 = 0.92$, which is close to the theoretical value $0.9291\pi^2$ for a Γ junction [4]. The bandwidth of the resonance dip in the transmission coefficient T_{12} is proportional to the probing field amplitude squared, as naturally follows from the dynamic perturbation theory.

Figure 4b plots the absorbed power of the microwave field versus frequency. It is seen that absorbed power (48) exhibits the resonant behavior due to the dynamic addition of the bound state. Note that, at certain frequencies of the probing field, the absorbed power exceeds unity. This means that, when passing through the scattering region, the microwave field takes

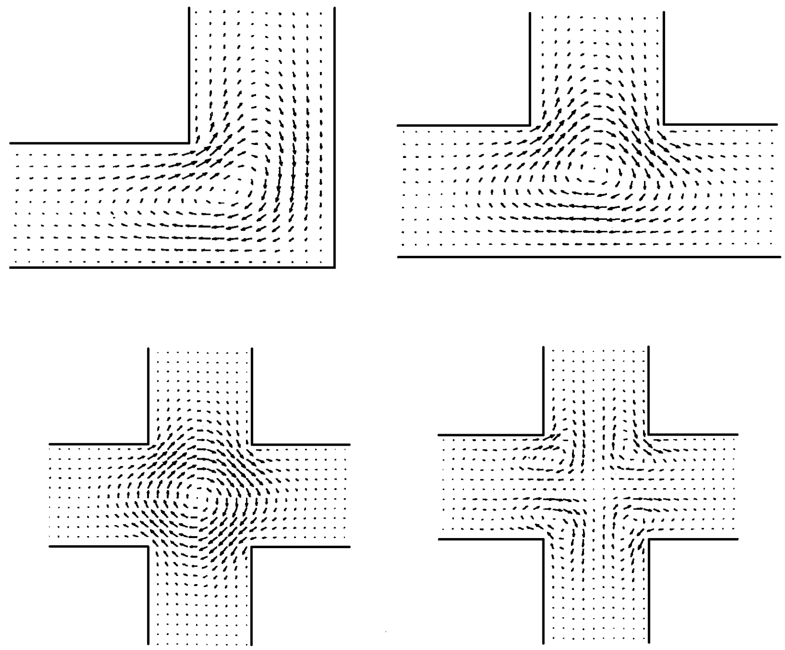

Fig. 3. Fluxes of the electromagnetic field power in the case of the resonance addition of the localized states. The lower right panel illustrates the addition of the second bound state to the *X* junction, which is antisymmetric with respect to the $x \longrightarrow -x$ and $y \longrightarrow -y$ reversals.

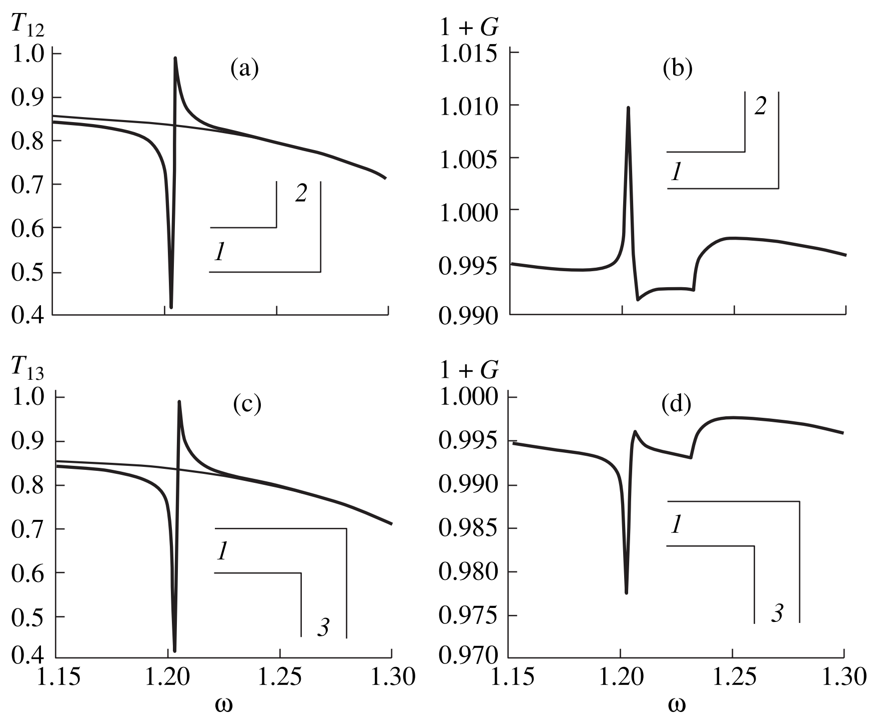

Fig. 4. Coefficient of the *TE* mode ($n = 1$) transmission through the Γ junction and the absorbed power *G* versus frequency at $\tilde{H}_0 = 1$ and $m = 1$. Thick line, resonance probing field with $\lambda = 0.1$; thin line, no probing field.

a small portion of the power of the probing radiation. Figures 4c and 4d refer to a Γ junction that guides the microwave field in the direction opposite to that considered in Figs. 4a and 4b. The Poynting vector is not invariant under the $y \longrightarrow -y$ reversal. Therefore, as follows from Figs. 4b and 4d, the frequency behavior of the absorbed power depends on the direction the output arm of the Γ junction. Accordingly, the transmission

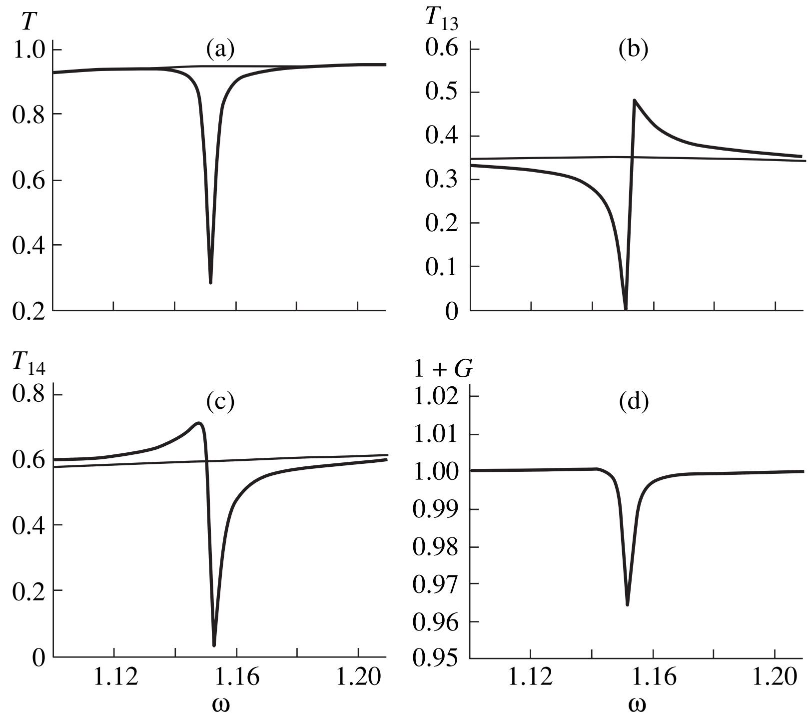

Fig. 5. Same as in Fig. 4 for the T junction at $\tilde{H}_0 = 1$, $m = 1$, $\tilde{\Omega} = 0.4$, and λ = (thin line) 0 and (thick line) 0.2. (a) Total transmission coefficient; (b, c) coefficients of transmission to waveguide arms *3* and *4*, respectively; and (d) coefficient of probing radiation absorption.

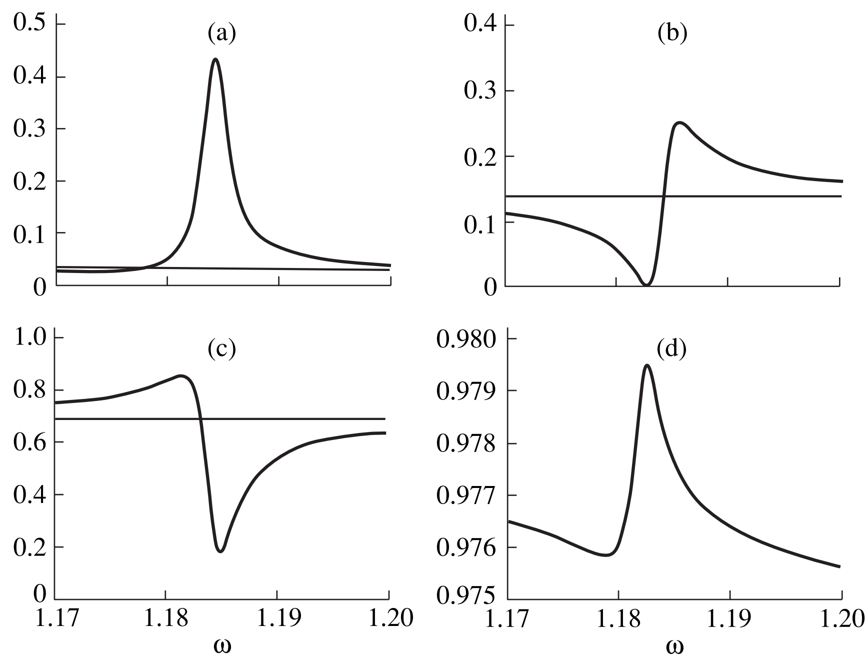

Fig. 6. Same as in Fig. 4 for the X junction at $\tilde{H}_0 = 1$, $m = 1$, $\tilde{\Omega} = 0.5$, and λ = (thin line) 0 and (thick line) 0.2. (a) Reflection coefficient; (b, c) coefficients of transmission to waveguide arms *2*(*3*) and *4*, respectively; and (d) coefficient of probing radiation absorption. The probing field is tuned to excite the fundamental bound state of the electromagnetic field in the structure.

coefficients, which are defined as the output-to-input microwave power ratio, are also not invariant. The difference in the absorbed power is, however, small, about 1%.

Figures 5–7 show similar effects for the T and X junctions. Figure 7 illustrates the possibility of resonantly controlling the transmission of the microwave field through the X junction via the resonance addition

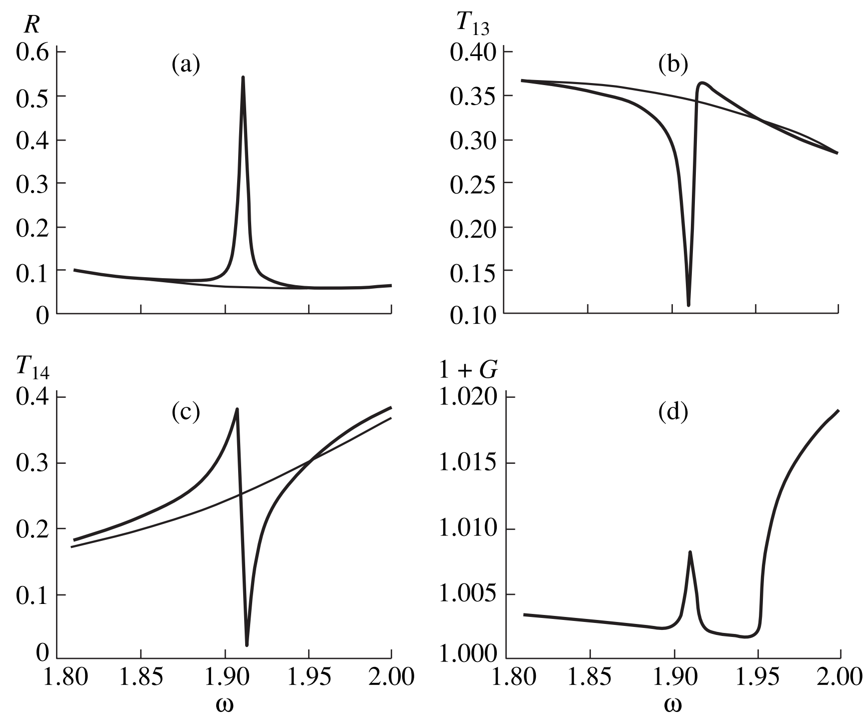

Fig. 7. Same as in Fig. 6 at $\tilde{H}_0 = 1$, $m = 1$, $\tilde{\Omega} = 0.4$, and λ = (thin line) 0 and (thick line) 0.15. The probing field tuned to excite the first bound state with the frequency above the cutoff frequency of the waveguide arms.

of the second bound state with the eigenfrequency ω_b = 1.91. For a hollow waveguide *X* junction, the frequency of the second bound state is ω_b = 606.91. The range of $w \geq 1.95$ includes the second passband for the electromagnetic waves, hence, the specific behavior of the absorbed microwave power at these frequencies. Recall that, because of the high permeability of ferromagnetics, all the frequency responses (the frequencies of the bound states and frequency transmission thresholds) of the waveguide junctions are significantly shifted towards the low-frequency region.

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