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Inversion in an Extended Three-Level Medium Produced by Adiabatic Population Transfer

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Abstract—Features of the adiabatic population transfer are studied with the spatial evolution of interacting pulses propagating in an optically dense medium of three-level Λ-atoms taken into account. A self-consistent analytical solution describing the spatial–temporal dynamics of interacting short pulses under the conditions of adiabatic population transfer is constructed in the adiabatic approximation with consideration for the first nonadiabatic correction. Practically complete inversion on a forbidden transition determined by coherent (adiabatic) population transfer is shown to take place over a length of the medium, which may exceed the absorption length of a weak probing pulse in the absence of control radiation on the adjacent transition by several orders of magnitude. *© 2001 MAIK "Nauka/Interperiodica".*

INTRODUCTION

Great interest in the study of the interaction of laser radiation with three-level atoms possessing Λ-configuration with simultaneous one- and two-photon resonance has been observed over the last decade. The effects of atomic coherence and quantum interference, which lead to significant variations in the optical properties of a medium, for instance, to induced transparency [1–3], attract particular attention. Interesting and sometimes unexpected effects appear in the case of pulsed laser radiation. Certain aspects of laser pulse propagation under the conditions of electromagnetically induced transparency were studied, for instance, in [4–10]. Situations when pulses have the same shape and their duration is longer than the relaxation time of the intermediate resonance state (long pulses) are usually considered: matched pulses [4] and dressed-field pulses [5, 6]. Adiabatons [8, 9] represent another typical case when the length of the control radiation considerably exceeds the length of the probing pulse. Exotic pulse shapes leading to matched soliton waves are also analyzed (see, e.g., [10]).

The time evolution of adiabatic population transfer (APT) in three-level atoms is understood well enough (see, e.g., [11–15]), but, as far as we know, the spatial evolution of interacting pulses in optically dense media and the spatial dynamics of the population of the final state excited in the process of coherent transfer were not studied. In this work, the influence of the spatial propagation of interacting pulses in an optically dense medium on the APT process when the length of interacting pulses is shorter than all relaxation times of the atomic subsystem is studied.

The APT effect is observed for pulses whose envelopes vary rather slowly and satisfy the adiabaticity criterion [14]

$$\sqrt{|G_1|^2 + |G_2|^2}\,T \gg 1, \qquad (1)$$

where $G_{1,2}$ are the Rabi frequencies ($G_1 \simeq G_2$) and T is the length of the interacting pulses.

Condition (1) physically means that the pulse envelope should vary slower than the effective Rabi frequency $G = \sqrt{|G_1|^2 + |G_2|^2}$. This condition can be met for short, but rather powerful, pulses of a counterintuitive sequence whose length is significantly shorter than all relaxation times of the atomic subsystem [6, 14]. This is the case that we shall examine.

The theoretical model consists of a system of coupled Schrödinger equations and reduced wave equations for the Rabi frequency, which self-consistently describe the temporal and the spatial dynamics of the atomic system and the radiation field. In approximation (1), the self-consistent analytical solution describing the spatial and temporal evolution of two identical partially overlapping (in time) short pulses of a counterintuitive sequence in an optically dense three-level medium under APT conditions is constructed and the spatial dynamics of the population of levels participating in the interaction process is analyzed. It is shown that a probing pulse interacting with the ground state can propagate over a distance considerably exceeding (by several orders of magnitude) the linear absorption length. However, it is completely transferred into the control pulse on a finite length. The latter significantly varies its shape and becomes double-peaked. APT leads to the complete inversion of population on a dipole-forbidden transition within the characteristic length of the

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propagation of a probing pulse, which also can be significantly longer than the linear absorption length of a weak probing pulse.

BASIC EQUATIONS AND THEIR SOLUTION

Consider the propagation of two partially overlapping pulses in a medium of three-level Λ-atoms (Fig. 1). The intermediate state $|1\rangle$ is in one-photon resonance with both fields, each of them interacting only with its own transition. Below we shall call the pulse with the frequency ω_1 the probing pulse, and the other will be called the control pulse. The Rabi frequencies of the pulses may be the same. The pulses follow in a definite temporal sequence, the counterintuitive sequence [14]. First, the control pulse $G_2(t)$ interacts with atoms on the transition $|2\rangle$–$|1\rangle$, and then the probing pulse, which is switched on somewhat later, interacts on the transition $|0\rangle$–$|1\rangle$ (Fig. 1). The pulses propagate collinearly along the z direction and have the same shape and duration.

The system of equations for the probability amplitudes $b_{0,1,2}$ and the Rabi frequencies $G_{1,2}$ of the interacting pulses in the system of coordinates with local time $\tau = t - z/c$ has the standard form

$$\frac{\partial b_0}{\partial \tau} = iG_1^* b_1 \exp(-ik_1 z),$$

$$\frac{\partial b_2}{\partial \tau} = iG_2^* b_1 \exp(-ik_2 z), \tag{2}$$

$$\frac{\partial b_1}{\partial \tau} = iG_1 b_0 \exp(ik_1 z) + iG_2 b_2 \exp(ik_2 z),$$

$$\frac{\partial G_1}{\partial z} = iK_1 b_1 b_0^* \exp(-ik_1 z),$$

$$\frac{\partial G_2}{\partial z} = iK_2 b_1 b_2^* \exp(-ik_2 z), \tag{3}$$

where $G_{1,2} = d_{01,21}E_{1,2}(\tau)/2\hbar$; $K_1 = \pi\omega_1|d_{10}|^2N/c\hbar = \alpha_1\Gamma_{10}/4$, $K_2 = \pi\omega_2|d_{21}|^2N/c\hbar = \alpha_2\Gamma_{12}/4$ are the propagation coefficients; $\alpha_{1,2}$ is the linear absorption coefficient of probing or control radiation when all atoms are in the states $|0\rangle$ or $|2\rangle$, respectively; Γ_{ij} are the transition half-widths; N is the atomic concentration; d_{ij} are the dipole matrix elements of the transitions; $k_{1,2}$ is the absolute value of the wave vector of the interacting waves in vacuum; and the asterisk * denotes complex conjugation. Assume that all atoms are in the ground state $|0\rangle$ at the instant when the fields are switched on ($\tau = -\infty$) and both pulses have the envelope $E_{1,2}(\tau, 0)$ at the entrance to the medium. We consider the envelope as being Gaussian for particular calculations: $E_1(\tau) = E_1^0\exp(-\tau^2/2T^2)$, $E_2(\tau) = E_2^0\exp[-(\tau - \tau_0)^2/2T^2]$, where T is the pulse length, which is considered to be shorter than all relaxation times of the atomic subsystem

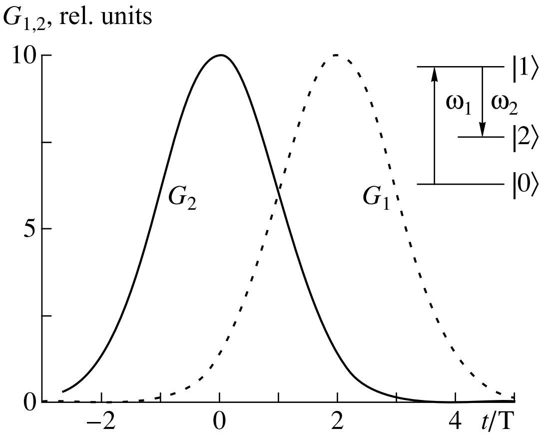

Fig. 1 The configuration of energy levels in atoms and the envelopes of the Rabi frequencies of pulses at the entrance to the medium. $\omega_{1,2}$ are the carrier frequencies of the probing $G_1(t)$ and the control $G_2(t)$ pulses, respectively.

[therefore, relaxation is not taken into account in Eqs. (2)] and τ_0 is the time delay of the probing pulse relative to the control pulse. We assume that the pulse amplitudes $E_{1,2}^0$ are real quantities. Parameters of the pulses are initially selected so that the adiabaticity condition (1) is fulfilled for $z = 0$.

It is convenient to change to new variables $a_0 = b_0\exp(ik_1z)$, $a_2 = b_2\exp(ik_2z)$, and $a_1 = ib_1$. In these variables, the system of equations has the form

$$\frac{\partial a_0}{\partial \tau} = G_1^* a_1, \quad \frac{\partial a_2}{\partial \tau} = G_2^* a_1,$$

$$\frac{\partial a_1}{\partial \tau} = -G_1 a_0 - G_2 a_2, \tag{4}$$

$$\frac{\partial G_1}{\partial z} = -K_1 a_1 a_0^*, \quad \frac{\partial G_2}{\partial z} = -K_2 a_1 a_2^*. \tag{5}$$

The solution of system (4) in the adiabatic approximation (with the first nonadiabatic correction taken into account) can be represented in the form

$$a_0 \simeq G_2/G = \cos\theta, \quad a_2 \simeq -G_1/G = -\sin\theta,$$

$$a_1 \simeq -\frac{\dot{\theta}}{G}, \tag{6}$$

where $G = \sqrt{G_1^2 + G_2^2}$ and $\tan\theta = G_1/G_2$.

It follows from (6) that $|a_0|^2 \simeq 0$ and $|a_2|^2 \simeq 1$ at the trailing edge of the control pulse ($\theta \longrightarrow \pi/2$), i.e., the population of the ground state is transferred to the state $|2\rangle$. Generally speaking, as will be shown below, the efficiency of the transition from the state $|0\rangle$ to the state $|2\rangle$ depends on the spatial coordinate z, and, additionaly, complete population transfer occurs only for a finite length of the medium.

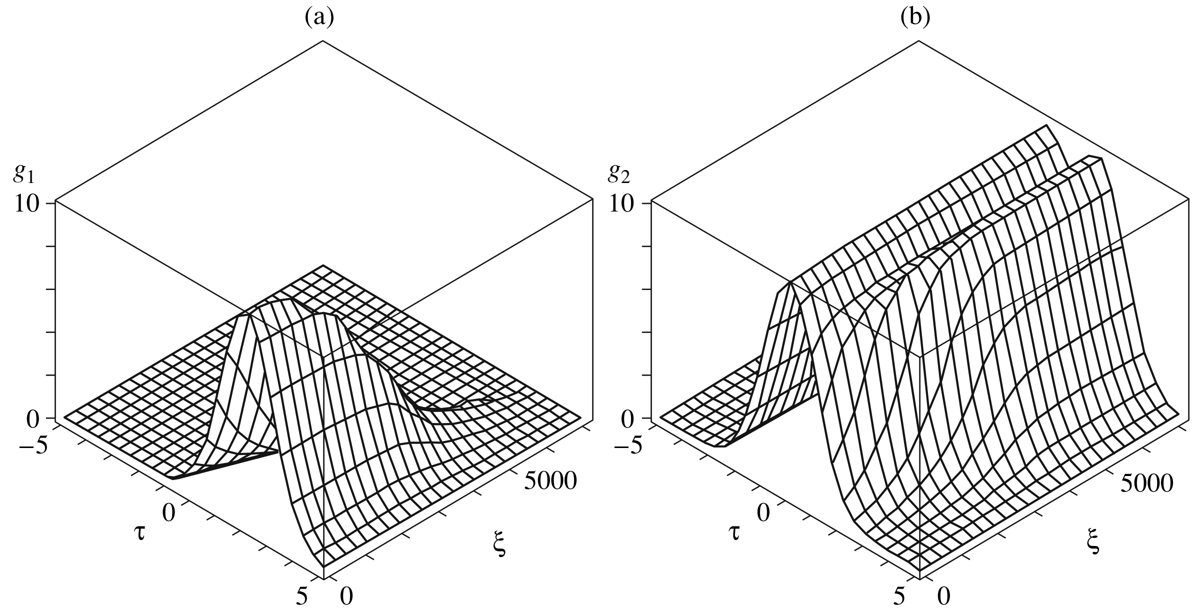

Fig. 2. The normalized envelopes of the Rabi frequencies (a) $g_1 = G_1T$ and (b) $g_2 = G_2T$ as functions of time and penetration depth of radiation into the medium. The time τ is measured in units of pulse duration T and the length of pulse propagation in the medium ξ is measured in units of the linear absorption length of probing radiation with a frequency ω_1.

Expression for the probability amplitude of the intermediate state a_1 can be brought to the form

$$a_1 = (G_2\dot{G}_1 - G_1\dot{G}_2)/G^3. \quad (7)$$

It is easy to show that we have $|a_1(\tau)| \ll 1$ in approximation (1), i.e., the population of the intermediate state $|1\rangle$ is practically zero during the entire time of interaction with the pulses. The latter physically means that the resonance absorption of the pulses is small (induced transparency). Therefore, pulses can propagate over distance much greater than the linear absorption length of weak probing radiation in the absence of control radiation.

It is interesting to note that $|a_0|^2 + |a_2|^2 = 1$. This reflects the fact that atoms are trapped into the state of coherent population trapping [16] whose probability amplitude is determined as $a_- = (G_2/G)a_0 - (G_1/G)a_2 = a_0\cos\theta - a_2\sin\theta = 1$. Thus, APT may be considered as a particular case of coherent population trapping.

With (6) taken into account, Eqs. (5) can be represented as

$$\frac{\partial G_1}{\partial z} = -(K_1/G)\frac{\partial(G_1/G)}{\partial\tau}, \quad \frac{\partial G_2}{\partial z} = -(K_2/G)\frac{\partial(G_2/G)}{\partial\tau}. \quad (8)$$

It is easy to show from (8) that the sum $K_2G_1^2(\tau, z) + K_1G_2^2(\tau, z)$ is independent of the coordinate z and is equal to $\tilde{G}^2(\tau, 0) = K_2G_1^2(\tau, z = 0) + K_1G_2^2(\tau, z = 0)$. This relation can be rewritten by using the photon number density $n_{1,2} = E^2/\hbar\omega_{1,2} = \pi NG_{1,2}^2/cK_{1,2}$: $n_1(\tau, z) + n_2(\tau, z) = \mathrm{const}(\tau)$, which represents the law of conservation of the total number of photons in the process of propagation of pulses under APT conditions (Manley–Raw relation).

Generally, the solution of system (8) cannot be written in quadratures. But for $K_1 = K_2 = K$ it can be found, for instance, by the method of characteristics, and can be represented in the form

$$G_1 = G(0, \tau)\frac{G_1(0, p)}{G(0, p)}, \quad G_2 = G(0, \tau)\frac{G_2(0, p)}{G(0, p)}, \quad (9)$$

where $p = Z^{-1}(Z(\tau) - z)$, $Z(\tau) = K^{-1}\int_{-\infty}^{\tau} d\tau' G^2(0, \tau')$ and $Z^{-1}(z)$ is the function inverse to $Z(\tau)$.

It is seen from (9) that $G_1^2(\tau, z) + G_2^2(\tau, z) = G_1^2(\tau, 0) + G_2^2(\tau, 0)$, i.e., it does not depend on the spatial coordinate z along the direction of pulse propagation. It is easily shown that $\sqrt{G_1^2(\tau, z) + G_2^2(\tau, z)}$ coincides with the definition of dressed-field pulses [5, 6]: $G_- = a_0G_2 - a_2G_1$. But in contrast to [5, 6], where long pulses were considered, pulses in our case are short, their duration being shorter than all relaxation times of the atomic system. Therefore, the latter should be considered as an extension of the concept of dressed pulses to the case of short pulses.

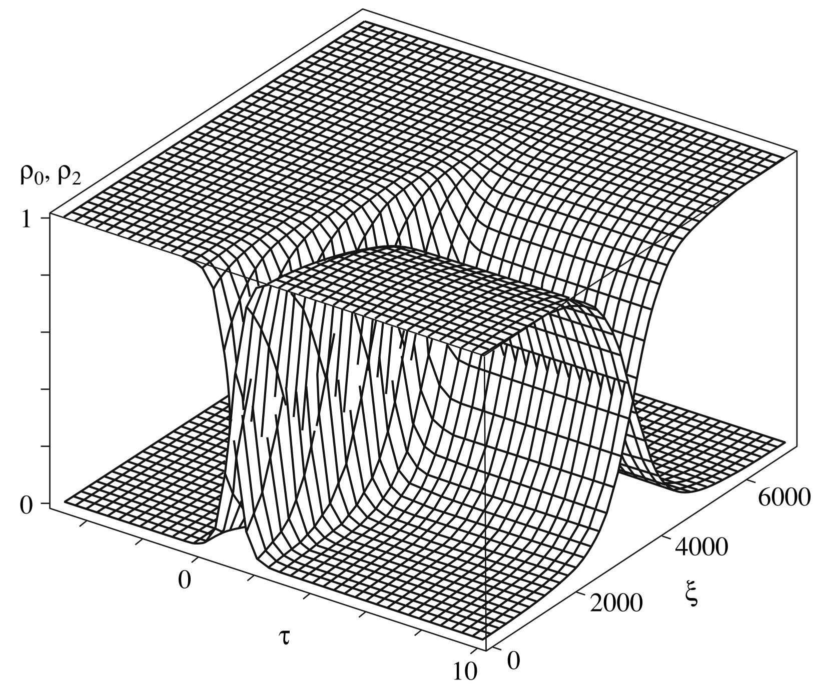

Fig. 3. The populations $\rho_{0,2} = |a_{0,2}|^2$ as functions of time and penetration depth of radiation into the medium. The values of the parameters are the same as in Fig. 2.

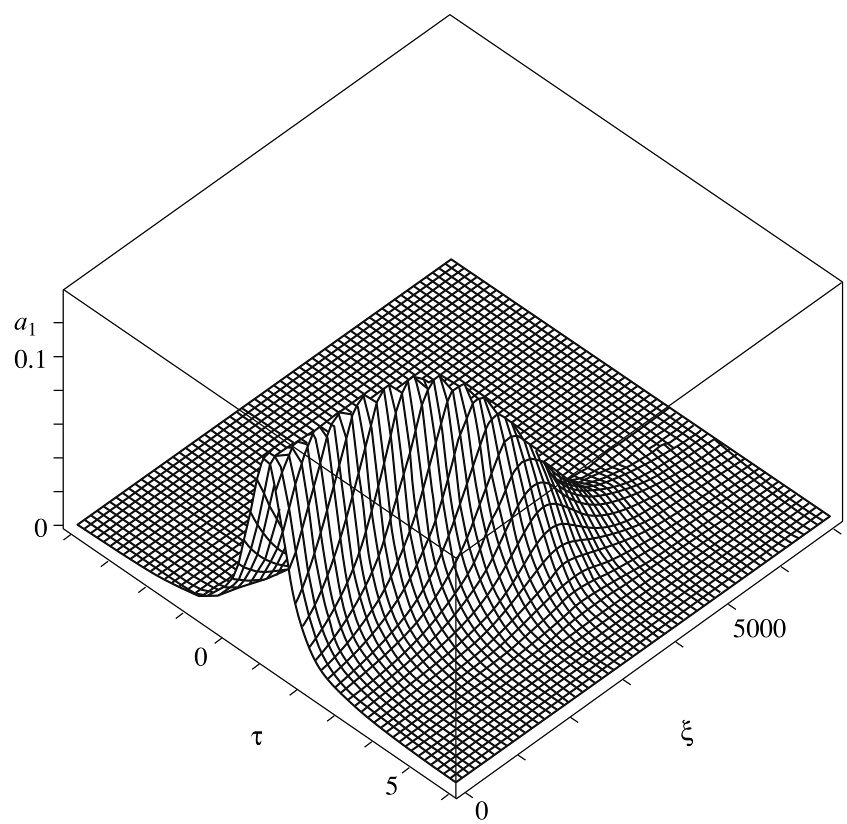

Fig. 4. The probability amplitude a_1 as a function of time and penetration depth of radiation into the medium. The values of the parameters are the same as in Fig. 2.

Figure 2 shows the normalized Rabi frequencies $g_{1,2} = G_{1,2}T$ as functions of time and the penetration depth of radiation into the medium calculated from formulas (9) for the following values of the parameters: $t_0/T = 2$, $G_1^0T = 10$, $G_2^0T = 10$, ($G_{1,2}^0$ are the values of the Rabi frequencies at the maximum), $\Gamma_{10}T = 0.1$, $\Gamma_{12}T = 0.1$, and $K_1 = K_2$. The plots demonstrate that a probing pulse in a resonant medium can propagate over a distance several orders of magnitude greater than the linear absorption length. In this case, the energy of the leading edge of the probing pulse is partially absorbed and the energy of the trailing edge of the control pulse increases. The energy absorbed goes into the adiabatic transfer of atoms to the excited final state and amplification of the control pulse. In the process of propagation, the amplitude of the probing pulse slowly decreases and the control pulse varies its shape and becomes double-peaked. Eventually, the probing pulse is completely transferred into the control pulse leading to an increase in the area of the latter. This transfer occurs within a finite length of the medium.

Figure 3 shows the populations $\rho_{0,2} = |a_{0,2}|^2$ as functions of time and the length of the medium. Population transfer is seen to occur within the length of the medium, which cannot exceed a certain value. The higher the intensities of the interacting pulses, the greater the length. In this case, practically complete inversion takes place on the forbidden transition. The dependence of the probability value of intermediate state $|1\rangle$ on the time and spatial coordinate in the medium is shown in Fig. 4. It can be seen that this value is much less than unity; i.e., the adiabacity condition is still satisfied under propagation of pulses.

The analytical results obtained coincide with the results of numerical analysis of the system of equations (4) and (5) (see also [17]). These conclusions are independent of the pulse shape if the adiabaticity condition is fulfilled.

CONCLUSIONS

The APT effect is studied in this work with the propagation of interacting counterintuitive pulses in an optically dense three-level medium with a Λ-configuration of atomic levels taken into consideration. The APT effect is shown to lead to the practically complete inversion of population on a dipole-forbidden transition in an extended medium whose length may be several orders of magnitude greater than the linear absorption length of a single probing pulse in the absence of control radiation. The medium thus prepared can be used for frequency conversion of picosecond and femtosecond lasers to anti-Stokes radiation with a tunable wavelength and, probably, for observing cooperative anti-Stokes scattering of light.

It was found that pulses propagating under APT conditions may be identified as dressed-field pulses only when the propagation constants are equal ($K_1 = K_2$). The results obtained extend the concept of dressed-field pulses to the case of short pulses and provide additional information on electromagnetically induced transparency. Generally, when the propagation constants $K_{1,2}$ are unequal, the Manley–Raw relation is valid, which cannot be reduced to the concept of dressed-field pulses. Moreover, the dynamics of pulse propagation changes in this case, particularly for $K_1 > K_2$. These results will be published elsewhere.

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