Correlation-induced coupling of wave fields in disordered media

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The interaction of two wave fields of different physical natures in a disordered medium with a zero-mean coupling between these fields is studied for the example of spin and elastic waves in zero-mean magnetostrictive media. It is shown that correlations between inhomogeneities of the coupling parameter and any other parameter of the medium lead to the appearance of an effective coupling parameter between the averaged waves, which is proportional to the intensity of the correlation and, correspondingly, to the possibility of the excitation of the averaged wave of one field by a force acting on the other wave field.

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I. INTRODUCTION

It is well known that the interaction between two wave fields of different physical natures is strongest in the vicinity of the crossing resonance frequency ω_r, at which the dispersion curves of the fields cross, if this crossing occurs. Compound oscillations of both wave fields appear, their degeneracy is removed, and a gap in the spectrum, proportional to the coupling parameter between the wave fields, occurs in the vicinity of this frequency in a homogeneous medium. Here the off-diagonal elements of the susceptibility matrix χ_{ij} have maxima, i.e., the most effective excitation of oscillations of one physical nature by a force applied to the field of the other nature takes place. These effects occur for interacting waves of any physical nature: spin and elastic,[1] elastic and electromagnetic,[2] etc.

In a number of papers[3–5] the interaction between two wave fields of different natures has been investigated in media with an inhomogeneous coupling parameter, whose mean value P is assumed to be equal to zero. This model was named the model of disorder-induced crossing resonance. In this situation the averaged wave of one nature interacts only with fluctuation waves (scattered by inhomogeneities) of the other nature. This interaction is proportional to the rms fluctuation λ_c of the coupling parameter. The interaction results in many resonance effects in the vicinity of the crossing resonance $\omega=\omega_r$, appearing both in the dispersion laws of the averaged waves, as well as in the diagonal elements of the susceptibility matrix. However, the averaged waves of each field are not coupled with each other: the off-diagonal elements of the averaged susceptibility matrix are equal to zero in all orders of the perturbation theory for the self-energy in the Green function.[4] An excitation of the averaged wave of one field by a force acting on the other field is impossible.

It is reasonable that this cross excitation becomes possible if a nonzero value of P exists in the medium along with the inhomogeneities of the coupling parameter;[6] in this case the off-diagonal elements of the susceptibility matrix are proportional to P.

In the present paper another physical effect that leads to nonzero off-diagonal elements of the susceptibility matrix, even in the case that $P=0$, is introduced. It is caused by the mutual correlations between the inhomogeneities of the coupling parameter and the inhomogeneities of any other parameter of the medium under consideration. As far as we know, such a mechanism for the formation of nonzero off-diagonal elements of the susceptibility matrix was not been considered earlier.

For the sake of definiteness we study this effect for the example of the magnetoelastic interaction, but the main qualitative results obtained in this work do not depend on the specific physical nature of the interacting wave fields, if the latter have a crossing resonance point $\omega=\omega_r\neq 0$.

II. MATRIX OF SUSCEPTIBILITY OF THE AVERAGED WAVES

Excitations in a magnetoelastic medium are governed by the system of Landau-Lifshitz equations for the magnetization and the equations for the elastic displacements

$$\dot{\mathbf{M}}=-g\left[\mathbf{M}\times\left(-\frac{\partial\mathcal{H}}{\partial\mathbf{M}}+\frac{\partial}{\partial\mathbf{x}}\frac{\partial\mathcal{H}}{\partial(\partial\mathbf{M}/\partial\mathbf{x})}\right)\right],$$

$$\mu\ddot{u}_i=\frac{\partial}{\partial x_j}\frac{\partial\mathcal{H}}{\partial u_{ij}}, \tag{1}$$

where $\mathbf{M}$ is the magnetization, $\mathbf{u}$ is the elastic displacement vector, $u_{ij}=1/2[(\partial u_i/\partial x_j)+(\partial u_j/\partial x_i)]$ is the elastic strain tensor, g is the gyromagnetic ratio, μ is the density of the medium, and summation over a repeated index is assumed here and in Eq. (2) below.

We assume that our system is an elastically isotropic ferromagnet with a single magnetic symmetry axis, so that the corresponding magnetoelastic potential energy $\mathcal{H}$ takes the form

$$\mathcal{H}=\frac{1}{2}\alpha(\nabla\mathbf{M})^2-\frac{1}{2}\beta(\mathbf{Mn})^2-\mathbf{HM}+\frac{1}{2}d_1u_{ii}^2+\frac{1}{2}d_2\times(u_{ij}u_{ij}+u_{ij}u_{ji})+\frac{1}{2}BM_iM_ju_{ij}. \tag{2}$$

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Here α is the exchange parameter, $\mathbf{n}$ is the direction of the magnetic anisotropy axis, β is the magnitude of this anisotropy, d_1 and d_2 are the elastic Lamé constants, $\mathbf{H}$ is the magnetic field, and B is the magnetoelastic parameter.

In the general case all parameters characterizing the system can depend on the coordinates in a disordered medium, but for the demonstration of the effect under consideration it is enough to assume that only the density μ, the magnitude of the anisotropy β, and the coupling parameter B fluctuate, and all other parameters are constants. As we see below, this assumption is not fundamental. Let us represent the fluctuating parameters in the form

$$\mu(\mathbf{x})=\mu+\Delta\mu\rho_u(\mathbf{x}),$$
$$\beta(\mathbf{x})=\beta+\Delta\beta\rho_s(\mathbf{x}), \quad (3)$$
$$B(\mathbf{x})=B+\Delta B\rho_c(\mathbf{x}),$$

where μ, β, B and $\Delta\mu$, $\Delta\beta$, ΔB are the mean values and rms fluctuations of the parameters, respectively, and $\rho_i(\mathbf{x})$ $(i=u,s,c)$ are centered $[\langle\rho_i\rangle=0]$ and normalized $[\langle\rho_i^2\rangle=1]$ random functions.

Let an external dc magnetic field and the anisotropy axis be directed along the z axis of the coordinate system. The equilibrium direction of the magnetization, then, also coincides with the z axis. We consider the excitation of the medium by bulk forces $\mathbf{p}$ and $\mathbf{h}$, with the first of them affecting the elastic subsystem and the second one influencing the magnetic subsystem. We assume that these forces are perpendicular to the z axis. Therefore, only their x and y components have nonzero values. Linearizing the system (1) with respect to the small deviation $\mathbf{m}(\mathbf{x},t)$ from the equilibrium magnetization $\mathbf{M}_0$, using the scalar approximation for the elastic waves ($v_t=v_l=v$, where v_t and v_l are the speeds of the transverse and longitudinal elastic waves, respectively), and neglecting the terms describing both the nonresonant interaction between the elastic and the left-polarized spin waves and the terms describing the interaction between the spin waves and the longitudinal elastic waves (u_z), we obtain the following integral equations for the Fourier transforms of the circular components $m=M_x+iM_y$ and $u=u_x+iu_y$:

$$[(\omega-i\Gamma_u)^2-\omega_u^2(\mathbf{k})]u(\mathbf{k})+\omega^2\frac{\Delta\mu}{\mu}\int\rho_u(\mathbf{k}-\mathbf{k}_1)u(\mathbf{k}_1)d\mathbf{k}_1 + \frac{iMk_z}{2\mu}\left[B_0m(\mathbf{k})+\Delta B\int\rho_c(\mathbf{k}-\mathbf{k}_1)m(\mathbf{k}_1)d\mathbf{k}_1\right] = \Omega_u^2p_k, \quad (4)$$

$$[\omega-\omega_s(\mathbf{k})-i\Gamma_s]m(\mathbf{k})-\omega_M\Delta\beta\int\rho_s(\mathbf{k}-\mathbf{k}_1)m(\mathbf{k}_1)d\mathbf{k}_1 - \frac{i\omega_M M}{2}\left[B_0k_zu(\mathbf{k})+\Delta B\int k_{1z}u(\mathbf{k}_1)\rho_c(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1\right] = \omega_M h_k.$$

In these equations ω_u and ω_s are the initial dispersion laws of the elastic and spin waves, respectively,

$$\omega_u=vk, \quad \omega_s=\omega_0+\alpha\omega_M k^2, \quad (5)$$

where $\omega_0=g(H+\beta M)$, $\omega_M=gM$, and Ω_u is a coefficient with the dimensions of frequency; we added the parameters Γ_u and Γ_s in order to model the initial damping of the corresponding waves.

We shall examine these equations in the vicinity of the crossing resonance point (ω_r,k_r), where ω_r and k_r are determined by the equations

$$\omega_u(k_r)=\omega_s(k_r)=\omega_r\equiv vk_r. \quad (6)$$

Introducing the dimensionless variables ϕ and ψ and the dimensionless forces Φ and Ψ by

$$\phi_k=(2\mu\omega_r\omega_M)^{1/2}\frac{u_k}{M}, \quad \psi_k=\frac{m_k}{M},$$
$$\Phi_k=\left(\frac{\mu\omega_M}{2\omega_r}\right)^{1/2}\Omega_u p_k, \quad \Psi_k=\frac{h_k}{M}, \quad (7)$$

we obtain the following system of equations:

$$\frac{1}{2\omega_r}[(\omega-i\Gamma_u)^2-\omega_u^2(\mathbf{k})]\phi(\mathbf{k})+\frac{\lambda_u}{2}\int\rho_u(\mathbf{k}-\mathbf{k}_1)\phi(\mathbf{k}_1)d\mathbf{k}_1 + \frac{iPk_z}{2k_r}\psi(\mathbf{k})+\frac{i\lambda_c k_z}{2k_r}\int\psi(\mathbf{k}_1)\rho_c(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1=\Omega_u\Phi_k, \quad (8)$$

$$[\omega-\omega_s(\mathbf{k})-i\Gamma_s]\psi(\mathbf{k})-\frac{\lambda_s}{2}\int\rho_s(\mathbf{k}-\mathbf{k}_1)\psi(\mathbf{k}_1)d\mathbf{k}_1 - \frac{iPk_z}{2k_r}\phi(\mathbf{k})-\frac{i\lambda_c}{2k_r}\int k_{1z}\phi(\mathbf{k}_1)\rho_c(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1 = \omega_M\Psi_k,$$

where $\lambda_u=\omega^2\Delta\mu/\mu\omega_r$, $\lambda_s=2\omega_M\Delta\beta$, $\lambda_c=\Delta BMk_r\sqrt{\omega_M/2\mu\omega_r}$ are the rms fluctuations of the density, anisotropy, and coupling parameter, respectively, while $P=B_0Mk_r\sqrt{\omega_M/2\mu\omega_r}$ is the mean value of the coupling parameter.

In order to obtain the averaged solution of Eqs. (8) it is convenient to introduce the matrix notations

$$G_{\mathbf{k}}^{-1}=\begin{pmatrix} \frac{1}{2\omega_r}[(\omega-i\Gamma_u)^2-\omega_u^2(\mathbf{k})] & i\frac{Pk_z}{2k_r} \\ -i\frac{Pk_z}{2k_r} & \omega-\omega_s(\mathbf{k})-i\Gamma_s \end{pmatrix},$$

$$R^{\parallel}_{\mathbf{k}-\mathbf{k}_1}=\begin{pmatrix} -\frac{\lambda_u}{2}\int \rho_u(\mathbf{k}-\mathbf{k}_1)\cdots d\mathbf{k}_1 & 0 \\ 0 & \frac{\lambda_s}{2}\int \rho_s(\mathbf{k}-\mathbf{k}_1)\cdots d\mathbf{k}_1 \end{pmatrix},$$

$$R^{\perp}_{\mathbf{k}-\mathbf{k}_1}=\frac{i\lambda_c}{2k_r}\begin{pmatrix} 0 & -k_z\int \rho_c(\mathbf{k}-\mathbf{k}_1)\cdots d\mathbf{k}_1 \\ \int k_{1z}\rho_c(\mathbf{k}-\mathbf{k}_1)\cdots d\mathbf{k}_1 & 0 \end{pmatrix},$$

$$f=\begin{pmatrix}\phi_{\mathbf{k}}\\ \psi_{\mathbf{k}}\end{pmatrix},\quad F=\begin{pmatrix}\Omega_u\Phi_{\mathbf{k}}\\ \omega_M\Psi_{\mathbf{k}}\end{pmatrix}. \tag{9}$$

Here $G_{\mathbf{k}}$ is the initial matrix Green function that describes only a uniform coupling between $\phi_{\mathbf{k}}$ and $\psi_{\mathbf{k}}$; $R^{\parallel}_{\mathbf{k}}$ and $R^{\perp}_{\mathbf{k}}$ are matrix linear integral operators with random kernels that take into account the inhomogeneities; and f and F are the vectors of the variables and forces of the system under consideration, respectively.

Using these notations we can rewrite the system of Eqs. (8) in the form of a matrix equation

$$G_{\mathbf{k}}^{-1}f_{\mathbf{k}}=R^{\parallel}_{\mathbf{k}-\mathbf{k}_1}f_{\mathbf{k}_1}+R^{\perp}_{\mathbf{k}-\mathbf{k}_1}f_{\mathbf{k}_1}+F_{\mathbf{k}}. \tag{10}$$

Let us formally determine $f_{\mathbf{k}}$ from this equation,

$$f_{\mathbf{k}}=[R^{\parallel}_{\mathbf{k}-\mathbf{k}_1}f_{\mathbf{k}_1}+R^{\perp}_{\mathbf{k}-\mathbf{k}_1}f_{\mathbf{k}_1}+F_{\mathbf{k}}]G_{\mathbf{k}} \tag{11}$$

and average Eq. (11) over the ensembles of all the random functions ρ_i:

$$\langle f\rangle=[\langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1}f_{\mathbf{k}_1}\rangle+\langle R^{\perp}_{\mathbf{k}-\mathbf{k}_1}f_{\mathbf{k}_1}\rangle+F_{\mathbf{k}}]G_{\mathbf{k}}. \tag{12}$$

We decouple products of random functions according to the general rule that is valid for any two random functions A and B:

$$\langle AB\rangle=\langle A\rangle\langle B\rangle+\langle[A]_c[B]_c\rangle, \tag{13}$$

where we denote the corresponding centered functions by square brackets with the index c. The application of this rule directly to Eq. (12) does not lead to the first-order decoupled terms in our case: the products $\langle A\rangle\langle B\rangle$ vanish because the components of the matrices $R^{\parallel}_{\mathbf{k}}$ and $R^{\perp}_{\mathbf{k}}$ are centered.

To obtain the terms of the second order we increase the indices in Eq. (11) by unity ($\mathbf{k}\rightarrow\mathbf{k}_1$, $\mathbf{k}_1\rightarrow\mathbf{k}_2$) and substitute them into the right-hand side of Eq. (12):

$$\begin{aligned}\langle f_{\mathbf{k}}\rangle=&[\langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1}R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2}f_{\mathbf{k}_2}\rangle+\langle R^{\perp}_{\mathbf{k}-\mathbf{k}_1}R^{\perp}_{\mathbf{k}_1-\mathbf{k}_2}f_{\mathbf{k}_2}\rangle\\ &+\langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1}R^{\perp}_{\mathbf{k}_1-\mathbf{k}_2}f_{\mathbf{k}_2}\rangle+\langle R^{\perp}_{\mathbf{k}-\mathbf{k}_1}R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2}f_{\mathbf{k}_2}\rangle]\\ &\times G_{\mathbf{k}_1}G_{\mathbf{k}}+F_{\mathbf{k}}G_{\mathbf{k}}.\end{aligned} \tag{14}$$

We apply the rule (13) for all the terms in the square brackets, putting $A=R_{\mathbf{k}-\mathbf{k}_1}R_{\mathbf{k}_1-\mathbf{k}_2}$, $B=f_{\mathbf{k}_2}$. For example, for the first term we have

$$\begin{aligned}\langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1}R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2}f_{\mathbf{k}_2}\rangle=&\langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1}R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2}\rangle\langle f_{\mathbf{k}_2}\rangle G_{\mathbf{k}_1}\\ &+\langle[R^{\parallel}_{\mathbf{k}-\mathbf{k}_1}R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2}]_c[f_{\mathbf{k}_2}]_c\rangle G_{\mathbf{k}_1}.\end{aligned} \tag{15}$$

Let us consider the explicit form of some matrix element $J_{\mathbf{k}}$ corresponding to the first term on the right-hand side of Eq. (15):

$$J_{\mathbf{k}}=\frac{\lambda_u^2}{4}\int\int\langle\rho_u(\mathbf{k}-\mathbf{k}_1)\rho_u(\mathbf{k}_1-\mathbf{k}_2)\rangle\langle\varphi_{\mathbf{k}_2}\rangle g_{\mathbf{k}_1}d\mathbf{k}_1d\mathbf{k}_2, \tag{16}$$

where we denote by $g_{\mathbf{k}_1}$ the corresponding component of the matrix $G_{\mathbf{k}_1}$.

The formula

$$\langle\rho(\mathbf{k})\rho(\mathbf{k}')\rangle=S(\mathbf{k})\delta(\mathbf{k}+\mathbf{k}') \tag{17}$$

is valid for any homogeneous random function, where $S(\mathbf{k})$ is the spectral density (Fourier component of the correlation function). Using this formula we calculate the integral with respect to $\mathbf{k}_2$ in Eq. (16):

$$J_{\mathbf{k}}=\langle\varphi_{\mathbf{k}}\rangle\frac{\lambda_u^2}{4}\int S_{uu}(\mathbf{k}-\mathbf{k}_1)g_{\mathbf{k}_1}d\mathbf{k}_1. \tag{18}$$

This is the final form of the matrix element considered. However, to avoid the introduction of new matrix notations, we can formally rewrite Eq. (18) again in the form of double integral, removing $\langle \varphi_{\mathbf{k}} \rangle$ from the integral:

$$J_{\mathbf{k}} = \langle \varphi_{\mathbf{k}} \rangle \frac{\lambda_u^2}{4} \int \int \langle \rho(\mathbf{k}-\mathbf{k}_1) \rho(\mathbf{k}_1 - \mathbf{k}_2) \rangle g_{\mathbf{k}_1} d\mathbf{k}_1 d\mathbf{k}_2 . \quad (19)$$

One can see from this result that the first term on the right-hand side of Eq. (15) can be represented as follows:

$$\langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1} R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2} \rangle \langle f_{\mathbf{k}_2} \rangle G_{\mathbf{k}_1} = \langle f_{\mathbf{k}} \rangle \langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1} R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2} \rangle G_{\mathbf{k}_1} . \quad (20)$$

In a similar manner, the other terms of the matrix equation (14) can be transformed. Neglecting the higher-order correlators contained in the terms corresponding to the second term on the right-hand side of Eq. (15), we obtain the solution of the matrix equation (10) in the second-order approximation in the form

$$\langle f_{\mathbf{k}} \rangle = \{ G_{\mathbf{k}}^{-1} - [\langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1} R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2} \rangle + \langle R^{\perp}_{\mathbf{k}-\mathbf{k}_1} R^{\perp}_{\mathbf{k}_1-\mathbf{k}_2} \rangle + \langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1} R^{\perp}_{\mathbf{k}_1-\mathbf{k}_2} \rangle + \langle R^{\perp}_{\mathbf{k}-\mathbf{k}_1} R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2} \rangle] G_{\mathbf{k}_1} \}^{-1} F_{\mathbf{k}} . \quad (21)$$

This is the matrix analog of the Bourret approximation.[7,8]

The term $\langle R^{\parallel}_{\mathbf{k}-\mathbf{k}_1} R^{\parallel}_{\mathbf{k}_1-\mathbf{k}_2} \rangle G_{\mathbf{k}_1}$ describes a contribution caused by the inhomogeneities of the parameters of each of the subsystems. The term $\langle R^{\perp}_{\mathbf{k}-\mathbf{k}_1} R^{\perp}_{\mathbf{k}_1-\mathbf{k}_2} \rangle G_{\mathbf{k}_1}$ describes a contribution caused by the inhomogeneities of the coupling parameter. The next two terms described the effects caused by the mutual correlations between the inhomogeneities of the parameters of the subsystems and of the coupling parameter.

To emphasize the effects caused by these correlations, let us assume that the mean value of the coupling parameter $P = 0$ and rewrite Eq. (21) in this case in an explicit form:

$$[D_u(\omega,k) - Q_u(\omega,k)] \langle \phi \rangle + \frac{ikP_{eff}(\omega,k)}{2k_r} \langle \psi \rangle = \Omega_u \Phi_k ,$$

$$[D_s(\omega,k) - Q_s(\omega,k)] \langle \psi \rangle - \frac{ikP_{eff}(\omega,k)}{2k_r} \langle \phi \rangle = \omega_M \Psi_k , \quad (22)$$

where

$$D_u(\omega,k) = \frac{1}{2\omega_r} [(\omega - i\Gamma_u)^2 - \omega_u^2(k)],$$

$$D_s(\omega,k) = \omega - i\Gamma_s - \omega_s(k), \quad (23)$$

$$Q_u(\omega,k) = \frac{\lambda_u^2}{4} \int \frac{S_{uu}(\mathbf{k}-\mathbf{k}_1)}{D_u(\omega,k_1)} d\mathbf{k}_1 + \frac{\lambda_c^2 k^2}{4k_r^2} \int \frac{S_{cc}(\mathbf{k}-\mathbf{k}_1)}{D_s(\omega,k_1)} d\mathbf{k}_1 ,$$

$$Q_s(\omega,k) = \frac{\lambda_s^2}{4} \int \frac{S_{ss}(\mathbf{k}-\mathbf{k}_1)}{D_s(\omega,k_1)} d\mathbf{k}_1 + \frac{\lambda_c^2}{4k_r^2} \int \frac{k_{1z}^2 S_{cc}(\mathbf{k}-\mathbf{k}_1)}{D_u(\omega,k_1)} d\mathbf{k}_1 ,$$

$$P_{eff}(\omega,k) = a_{sc} \frac{\lambda_s \lambda_c}{4} \int \frac{S_{sc}(\mathbf{k}-\mathbf{k}_1)}{D_s(\omega,k_1)} d\mathbf{k}_1 - a_{uc} \frac{\lambda_u \lambda_c k_r}{4kk_z} \int \frac{k_{1z} S_{uc}(\mathbf{k}-\mathbf{k}_1)}{D_u(\omega,k_1)} d\mathbf{k}_1 ,$$

and the functions S_{ij} are the elements of the matrix of the spectral densities [Fourier transforms of the corresponding elements of the matrix of the correlation functions $K_{ij}(\mathbf{r})$]:

$$S_{ij}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int K_{ij}(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} d^3 r . \quad (24)$$

The autocorrelation functions are defined by the expression

$$K_{ii}(\mathbf{r}) = \langle \rho_i(\mathbf{x}) \rho_i(\mathbf{x}+\mathbf{r}) \rangle \quad (25)$$

and, in accordance with the normalization of ρ_i, $K_{ii}(0) = 1$.

The mutual correlation functions K_{ij} $(i \neq j)$ are conveniently defined by

$$a_{ij} K_{ij}(\mathbf{r}) = \langle \rho_i(\mathbf{x}) \rho_j(\mathbf{x}+\mathbf{r}) \rangle, \quad (26)$$

where $K_{ij}(0) = 1$, and the intensities of the mutual correlations are defined by the numerical coefficients a_{ij}, which can have values ranging from -1 to 1.

The susceptibility matrix χ for the averaged waves is determined by the equation

$$\begin{pmatrix} \langle \phi_k \rangle \\ \langle \psi_k \rangle \end{pmatrix} = \hat{\chi} \begin{pmatrix} \Phi_k \\ \Psi_k \end{pmatrix} \quad (27)$$

and has the following elements:

$$\chi_{uu} = \frac{\Omega_u (D_s - Q_s)}{D}, \quad \chi_{ss} = \frac{\omega_M (D_u - Q_u)}{D},$$

$$\chi_{su} = \frac{ik\omega_M P_{eff}}{2k_r D}, \quad \chi_{us} = -\frac{ik\Omega_u P_{eff}}{2k_r D}, \quad (28)$$

where

$$D = (D_u - Q_u)(D_s - Q_s) - \left(\frac{kP_{eff}}{2k_r} \right)^2 . \quad (29)$$

As one can see, the mutual correlations lead to the appearance of the effective coupling parameter P_{eff}. The averaged wave equations (22), the expressions for the elements of susceptibility matrix (28), and the law of dispersion of averaged waves $D = 0$, which follows from Eq. (29), contain this parameter in the same way as the usual coupling paramter, if the latter has nonzero mean value and the mutual correlations are equal to zero (this situation is considered in Ref. 6). However, the effects that are due to the parametr P_{eff} are not the same that result from usual coupling parameter, because P_{eff} as distinct from P is a complex value and has a strong frequency dependence.

To estimate the coupling between the waves caused by the correlations let us consider the situation where all the inhomogeneities, including inhomogeneities of the coupling parameter, have the same origin. It is natural to assume in this case that all the correlation functions K_{ij} are equal to each other, and the correlation intensity is defined by the numerical coefficients a_{ij} only. Let all the correlation functions and spectral densities be defined by the expressions

$$K_{ij}=e^{-k_c r}, \quad S_{ij}(k)=\frac{1}{\pi^2}\frac{k_c}{(k^2+k_c^2)^2}, \tag{30}$$

respectively. Here k_c is the correlation wave number ($k_c \approx r_c^{-1}$, where r_c is the correlation radius of the inhomogeneities). The integrals in Eqs. (22) with the spectral density given by Eq. (30) have cumbersome forms, but we use here, as in Ref.6, their simplified forms that have been obtained in Ref. 4 for small values of Γ_u, Γ_s, and vk_c in comparison with ω_r. Then we obtain for Q_u, Q_s, and P_{eff} the following expressions:

$$Q_u=\frac{\lambda_u^2}{4}\frac{1}{D_u^*}+\frac{\lambda_c^2 k}{4k_r}\frac{1}{D_s^*},$$

$$Q_s=\frac{\lambda_s^2}{4}\frac{1}{D_s^*}+\frac{\lambda_c^2 k}{4k_r}\frac{1}{D_u^*}, \tag{31}$$

$$P_{eff}=\frac{\lambda_c}{4}\left[a_{sc}\frac{\lambda_s}{D_s^*}-a_{uc}\frac{\lambda_u}{D_u^*}\right],$$

where

$$D_u^*=\omega-\omega_u(k)-i\Gamma_u^*, \quad D_s^*=\omega-\omega_s(k)-i\Gamma_s^*. \tag{32}$$

The effective relaxation parameters Γ_u^* and Γ_s^* are the sums of the initial damping constants and the relaxations due to scattering

$$\Gamma_u^*\approx\Gamma_u+vk_c, \quad \Gamma_s^*\approx\Gamma_s+v_s k_c, \tag{33}$$

where $v_s\approx 2\alpha\omega_M k_r$ is the speed of spin waves at the crossing resonance point.

The terms corresponding to the correlation coefficients a_{uc} and a_{sc} have different signs. The physical reason for this will be discussed in the next section of this paper. We call attention now to the fact that it (for the same signs of a_{uc} and a_{sc}) can lead to the vanishing of P_{eff} followed by a change of its sign as the frequency varied, because $\lambda_u\sim\omega^2$ and λ_s does not depend on the frequency. A similar effect can also occur when the value of the correlation wave number k_c is changed, because the coefficients of k_c in Eq. (33) are quantities of different magnitudes ($v\gg v_c$).

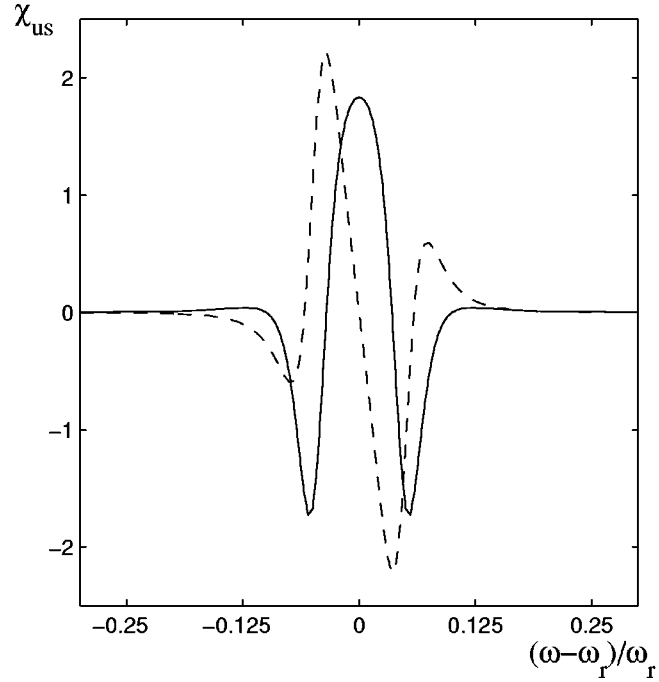

FIG. 1. Dependence of the off-diagonal elements of the susceptibility matrix χ'_{us} (solid curve) and χ''_{us} (dashed curve) on the frequency at $k=k_r$ for the correlation wave number: $k_c/k_r=0.01$. The values of the other parameters are $\Gamma_a/\omega_r=0.01$, $\Gamma_b/\omega_r=0.05$, $\lambda_u/\omega_r=0.03$, $\lambda_m/\omega_r=0.03$, $\lambda_c/\omega_r=0.1$, and $a_{uc}=a_{sc}=1$.

In Fig. 1 the dependence of $\chi'_{us}(\omega)$ and $\chi''_{us}(\omega)$ on the frequency ω is depicted at $k=k_r$ for the case of a small value of k_c when the term with the coefficient a_{uc} dominates. One can see that both curves have a complex resonance structure with positive and negative maxima. With the increase of k_c the term proportional to a_{uc} decreases and the amplitudes of all of the maxima decrease as well. Then the term proportional to a_{sc} becomes dominant, and all the amplitudes increase with increasing k_c but with the opposite sign. In Fig. 2 is shown how changing k_c changes the $|\chi_{us}|$. Curve *a* corresponds to a small value of k_c, curve *b*, which is almost indistinguishable from the horizontal axis on this scale, corresponds to the point of compensation of the terms proportional to a_{uc} and a_{sc}, curve *c* corresponds to a large value of k_c. The splitting of the maximum for small k_c corresponds to the case of small damping when the degeneracy at the crossing resonance points is removed and a gap $\Delta\omega$ in the spectrum proportional to P_{eff} is formed.

III. DISCUSSION OF THE RESULTS

First of all let us consider the situation when the mutual correlations are absent: $a_{uc}=a_{sc}=0$. In this case the dispersion law for the averaged waves $D=0$ splits into two independent dispersion laws, corresponding to the averaged elastic and spin wave,

$$D_u-Q_u=0, \quad D_s-Q_s=0, \tag{34}$$

the off-diagonal components of the susceptibility χ_{us} and χ_{su} are equal to zero, and the diagonal ones take the forms

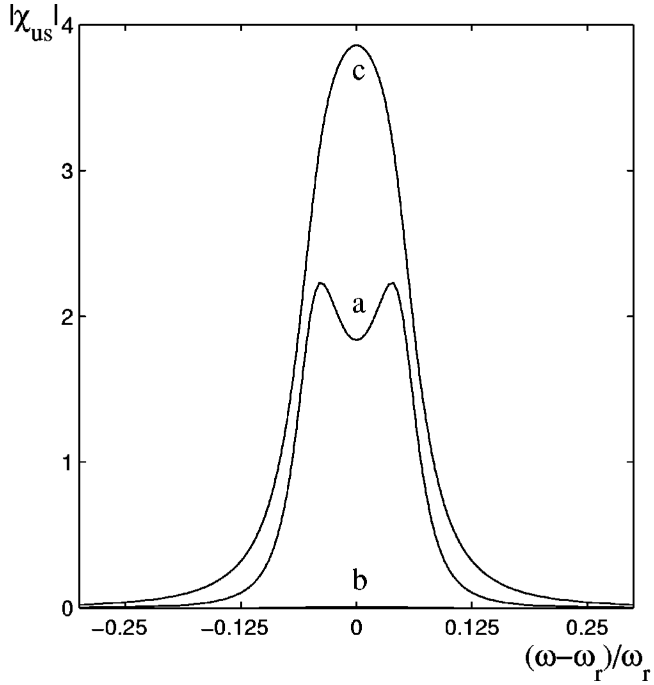

FIG. 2. Dependence of the modulus of the off-diagonal element $|\chi_{us}|$ of the susceptibility matrix on the frequency at $k=k_r$ for different values of the correlation wave number: $k_c/k_r=0.01$ (curve *a*), 0.0405 (curve *b* is almost indistinguishable from the horizontal axis on this scale), and 0.5 (curve *c*).

$$\chi_{uu}=\frac{\Omega_u}{D_u-Q_u}, \quad \chi_{ss}=\frac{\omega_M}{D_s-Q_s}. \tag{35}$$

One can see, that the dispersion law of the averaged elastic waves and, correspondingly, their susceptibility χ_{uu} in the vicinity of the magnetoelastic resonance, are strongly modified by the fluctuation spin waves: the expression D_u-Q_u depends on the parameters of both the elastic and magnetic systems. The same can be said about the dispersion law of the spin waves D_s-Q_s and their susceptibility χ_{ss}.

At the same time, a direct coupling between the averaged elastic and spin waves is absent: they are described by uncoupled dispersion laws, and the off-diagonal elements of the susceptibility matrix are equal to zero, that is, the excitation of the averaged wave of one field by a force acting on the other wave field is impossible. This rather nontrivial situation, which is characteristic of any coupled disordered media with the mean value of the coupling parameter equaling to zero, has been studied in detail in Refs. 3–5. It can be described in terms of the concept of two effective media in the same material introduced in Ref. 4. It is well known for the simpler situation of one wave field in a disordered material that plane waves are not eigenexcitations because of scattering from the inhomogeneities. However, an effective homogeneous medium can be introduced in which plane waves are eigenmodes, which correspond to averaged waves. These modes have a modified dispersion law and a finite lifetime as a result of interaction with scattered waves, which do not appear in this picture explicitly. So, the averaging of the stochastic integral wave equation is the equivalent of introducing an effective homogeneous dissipative medium with a modified dispersion law. It has been shown in Ref. 4 that the averaging of the wave equations of two fields of different nature interacting with each other when the mean value of the coupling parameter is zero is the equivalent of introducing two effective media in the same material. The dispersion laws and damping of each of these effective media depend on the parameters of both of these media. At the same time, the media are independent in the sense that each of the averaged wave fields propagates through its own effective medium without interacting with the partner wave field.

In Refs. 3–5 only inhomogeneities of the coupling parameter, characterized by the rms derivation λ_c, have been taken into account. One can see from Eqs. (22) that taking into account the internal inhomogeneities in the elastic (λ_u) and spin (λ_s) systems along with the inhomogeneities of the coupling parameter (λ_c) does not change anything radically in the situation described above, if $a_{uc}=a_{us}=0$: the effective media and, correspondingly, the averaged waves will continue to be uncoupled; only an additional modification of their dispersion laws occurs. Even the correlations between the inhomogeneities of the elastic and magnetic parameters ($a_{us}\neq 0$) do not change the situation, because the corresponding terms simply are absent in Eqs. (22) in the approximation considered. Only the availability of correlations between the inhomogeneities of the coupling parameter and some internal parameter of the elastic ($a_{uc}\neq 0$) or spin ($a_{sc}\neq 0$) system changes the whole picture qualitatively because the effective coupling parameter P_{eff} between the averaged waves appears and all phenomena connected with it develop: bound states of the averaged waves in the vicinity of the crossing resonance, off-diagonal elements of the susceptibility of the matrix and, correspondingly, the possibility of excitation of the averaged wave of one field by a force acting on the other wave field.

Let us take up first the physical nature of the phenomenon of the correlation-induced coupling between the averaged waves. The inhomogeneities of the coupling parameter in the medium with $P=0$ lead to the cross interaction between the coherent part of one wave (for example, the spin wave) with the scattered part of the other (elastic) wave. The inhomogeneities of some parameters of the elastic system lead to the interaction between the coherent and scattered parts of the same (elastic) waves. A correlation between the coupling parameter and an internal parameter of the elastic system induces a correlation between the elastic waves scattered from both of these parameters, and in turn between the coherent parts of the spin and elastic waves. The latter correlations arise as a result of averaging, as does the effective coupling parameter. This phenomenon does not depend on the nature of the waves, and is general for any two interacting fields.

Let us discuss the possibility of the experimental observation of this phenomenon. Note, first of all, that the existence of the mutual correlations of different parameters of the system are the rule rather than an exception. Thus, an amorphous state is characterized by the existence of disorder in the lengths and orientations of interatomic bonds (structural disorder), and an alloy is characterized by the disorder in the positions of atoms of the different components (compositional disorder). Inhomogeneities of other parameters of the system (density, anisotropy, magnetoelastic interaction, etc.) are a consequence, as a rule, of structural or compositional disorders. Therefore, they are correlated with the latter and, hence, with each other. For example, fluctuations of the composition of an alloy based on Fe and Ni, which has a zero-mean magnetostriction, lead to the correlation between inhomogeneities of the magnetoelastic parameter and the parameter of the magnetic anisotropy.

For the simplicity of the presentation we consider here the correlations of the coupling parameters only in the case of two parameters: one parameter of the elastic system (a_{uc}) and one parameter of the spin system (a_{sc}). The corresponding terms in the expression for P_{eff} (31) have opposite signs. This is due not to the different nature of these oscillations but to the different kinds of energy each represents: the term with a_{uc} describes in our case the correlation of the coupling parameter with the inhomogeneity of the kinetic energy (density of the material), and the term with a_{sc} describes the correlation of this parameter with the inhomogeneities of the potential energy (the value of the magnetic anisotropy). All parameters of the system are inhomogeneous in a real disordered material in some way or another, and their inhomogeneities can be correlated with the inhomogeneity of the coupling parameter. In this case the formula for P_{eff} contains the sum of all these correlations. The structure of Eq. (22) is that all correlators connecting the fluctuations of the parameters of the potential energy with the coupling parameter enter P_{eff} with a positive sign, and the parameters related to the kinetic energy enter with a negative sign. As each of these terms can have a different dependence on the frequency, the frequency dependence of P_{eff} in a real material can be considerably more complex than in the simple model described in the present paper.

The phenomenon of the correlation-induced coupling of wave fields should be most pronounced in materials in which the mean value of the usual coupling parameter P is equal to zero. If $P \neq 0$ the explicit form of the solution (21) becomes much more complicated than that given by Eqs. (22) and (23). Estimates show that the interaction of the averaged wave fields in the first approximation will be determined by the sum $P+P_{eff}$. The correlation-induced coupling will dominate if $a_{uc}\lambda_c\lambda_u/4/\Gamma_u^*$ or $a_{sc}\lambda_c\lambda_s/4/\Gamma_s^*$ is larger than P.

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[1] V. G. Baryakhtar and E. A. Turov, in *Spin Waves and Magnetic Excitations, Part 2*, edited by A. A. Borovik-Romanov and S. K. Sinha (North-Holland, Amsterdam, 1988), p. 333.
[2] K. Huang, Proc. R. Soc. London, Ser. A **208**, 352 (1951).
[3] V. A. Ignatchenko and L. I. Deich, Phys. Rev. B **50**, 16 364 (1994).
[4] V. A. Ignatchenko, M. V. Erementchouk, and L. I. Deych, Phys. Rev. B **57**, 521 (1998).
[5] L. I. Deych and A. A. Lisyansky, Phys. Lett. A **220**, 125 (1996).
[6] V. A. Ignatchenko, M. V. Erementchouk, A. A. Maradudin, and L. I. Deych, Phys. Rev. B **59**, 9185 (1999).
[7] R. C. Bourret, Physica (Amsterdam) **54**, 623 (1971).
[8] R. C. Bourret, U. Frisch, and A. Pouquet, Physica (Amsterdam) **65**, 303 (1973).