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Spin rotation for ballistic electron transmission induced by spin-orbit interaction

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We study spin-dependent electron transmission through one- and two-dimensional curved waveguides and quantum dots with account of spin-orbit interaction. We prove that for a transmission through an arbitrary structure there is no spin polarization provided the electron transmits in an isolated energy subband and only two leads are attached to the structure. In particular there is no spin polarization in the one-dimensional wire, for which a spin-dependent solution is found analytically. The solution demonstrates the spin evolution as dependent on a length of wire. The numerical solution for transmission of electrons through the two-dimensional curved waveguides coincides with the solution for the one-dimensional wire if the energy of electron is within the first energy subband. In the vicinity of edges of the energy subbands there are sharp anomalies of spin flipping.

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I. INTRODUCTION

The electron-spin precession phenomena at zero magnetic field induced by a variable spin-orbit interaction (SOI) in two-dimensional electron gas, (2DEG) systems were first proposed by Datta and Das[1] as a way for the realization of the spin transistor. For this, the spin precession is controlled via the Razhba SOI associated with the interface electric field present in GaAs heterostructures that contains the 2DEG channel:[2]

$$V_{SO}^{\alpha}=\hbar\alpha[\hat{p}_x\sigma_y-\hat{p}_y\sigma_x]. \tag{1}$$

The reason for the spin precession is that the spin operators do not commutate with the SOI operator, which leads to spin evolution for the electron transport. In particular the SOI has a polarization effect on particle scattering processes,[3] and this effect was considered for different geometries of confinement of the 2DEG.[4–9]

The most simple case of the stripe geometry, with the x axis along the stripe and the z axis perpendicular to the stripe, gives the following transformation of a spin state after transmission:

$$\begin{pmatrix}1\\0\end{pmatrix}\Rightarrow\begin{pmatrix}\cos\theta/2\\ \sin\theta/2\end{pmatrix}, \tag{2}$$

where[1,9]

$$\theta=2m^*\alpha L, \tag{3}$$

and L is the length of the stripe. Therefore, the Razhba SOI induces a spin precession of the transmitted electrons. Note that the spin precession is energy independent. This result is valid if the confinement energy $\hbar^2/2m^*d^2$, where d is the width of the stripe, is much larger than the spin-splitting energy induced by the SOI; therefore, the intersubband mixing is negligible.[9] For a strong SOI the spin rotation angle θ comes to depend on the Fermi energy for ballistic transport of electrons in the quasi-one-dimensional wires and stripes.[6,9] The Razhba SOI leads to spin precession in the (x,z) plane. Here we consider similar phenomena for electron transmission through the curved waveguide and quantum dots. The main difference between straight waveguide and curved wave guides is that the spin rotation is given by two angles.

Next we find the conditions under which there is no spin polarization of transmitted electrons. We imply that a flow of incident electrons has no spin polarization. For the spin polarization we consider the mean spin $\langle\sigma_\alpha\rangle$, $\alpha=x,y,z$, averaged over the electron flow. In particular, for transmission through a quantum dot we show the principal role of the third lead for the spin polarization.

II. SPIN-ORBIT INTERACTION IN THE INHOMOGENEOUS TWO-DIMENSIONAL CASE

We write the total Hamiltonian of a confined 2DEG as

$$H=-\frac{\hbar^2}{2m^*}\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}\right)+V(x,y)+V_{SO}, \tag{4}$$

where $V(x,y)$ is the lateral confining potential. Following to Moroz and Barnes[6] we assume that the SOI operator V_{SO} is formed by three contributions:

$$V_{SO}=V_{SO}^{\alpha}+V_{SO}^{\gamma}+V_{SO}^{\alpha\alpha}.$$

The first V_{SO}^{α} is related to the Razhba SOI [Eq. (1)], in which the SOI constant α proportional to the macroscopic interface-induced electric field is considered as constant. The second contribution V_{SO}^{γ} to the SOI comes from the electric field $\mathbf{E}(x,y)$ related to the confining potential.

In order to derive the second contribution to the SOI we begin with a general description of the SOI,[10]

$$V_{SO}=-\frac{e}{4m^2c^2}\left\{\sigma(\mathbf{E}\times\hat{\mathbf{p}})+\frac{i\hbar}{2}\sigma(\nabla\times\mathbf{E})\right\}. \tag{5}$$

For a microscopic electric field $\mathbf{E}$ the second term in Eq. (5) is equal to zero. However, for model cases of the confining potential $V(x,y)$ the electric field can violate an equality ∇

$\times\mathbf{E}=0$. In this case the second term in Eq. (5) is necessary to provide the hermiticity of the total SOI operator.

For a 2DEG confined at a semiconductor heterostructure interface we can reduce the z-coordinate, performing an average over an electron wave function $\psi_0(z)$ strongly localized along the z direction:

$$V_{SO}\Rightarrow\int dz\psi_0(z)V_{SO}\psi_0(z). \tag{6}$$

As a result we obtain

$$V_{SO}^{\gamma}=-\gamma\left\{\sigma_z(E_x\hat{p}_y-E_y\hat{p}_x)-E_z(\sigma_x\hat{p}_y-\sigma_y\hat{p}_x)-\frac{i}{2}\hbar\sigma_z\left(\frac{\partial E_x}{\partial y}-\frac{\partial E_y}{\partial x}\right)-\frac{i}{2}\hbar\left(\sigma_y\frac{\partial E_z}{\partial x}-\sigma_x\frac{\partial E_z}{\partial y}\right)\right\}. \tag{7}$$

Here the electric-field components have the meaning of integral (6), and depend on x and y only.

For a particular case of a straight wire directed along the y axis with a lateral confining potential $U=U(x)$, from Eq. (7) we obtain the expression given by Moroz and Barnes [formula (5) in Ref. 6]. They used a parabolic approximation for the confining potential. Here we consider a popular hard wall approximation and imply the following confining potential

$$U(x)=\begin{cases}0 & \text{if } |x|<d/2\\ U_0 & \text{if } |x|\geqslant d/2.\end{cases}$$

Then, substituting the electric field $E_x=-U'(x)$ into Eq. (7), we have

$$V_{SO}^{\gamma}(x)=\hbar k\gamma\sigma_z sign(x)U_0\delta(x\mp d/2). \tag{8}$$

For $|x|>d/2$, from the Schrödinger equation we have the solution

$$\psi(x)=C\exp\left(-\frac{\sqrt{2m^*(U_0-E)}}{\hbar}x\right), \tag{9}$$

where C is the normalization constant. Using a property of the delta function that a difference between derivatives of the wave function at the right and left of the delta function obey $\Delta\psi'(\pm d/2)=\pm 2m^*k\sigma_z\gamma U_0\psi(\pm d/2)$, we have from Eq. (9) that

$$\Delta\psi'(\pm d/2)\rightarrow 0$$

for $U_0\rightarrow\infty$. Therefore, in the hard wall approximation an effect of the second contribution V_{SO}^{γ} is limited to zero.

Next, for a numerical computation of the transmission through the semiconductor heterostructure we assume a connection to at least two electrodes in which there is no SOI. Then we can specify the electron state by quantum numbers, the number of the energy subband n, and the spin projection $\sigma=\sigma_z$. This assumption implies that far from waveguides or quantum dots the SOI constant α is equal to zero in the electrodes. Neglecting the real-space behavior of the micro-

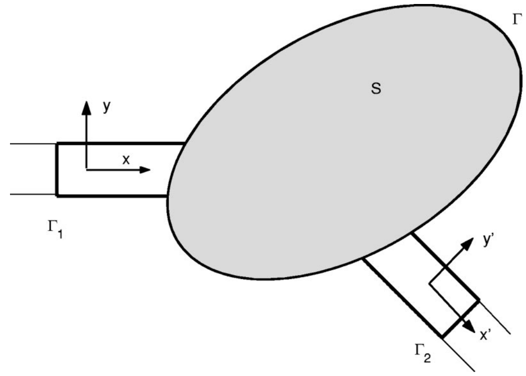

FIG. 1. Schematical view of a two-dimensional billiard with two attached leads. The dashed area shows a region with the SOI. Area S has a boundary Γ which crosses the input and output leads at Γ_1 and Γ_2, respectively.

scopic electric field at the edge of the heterestructure, we assume that the field is directed normal to the plane of the heterostructure everywhere, and has a stepwise behavior at the edges. As a result we obtain the stepwise behavior for the Razhba SOI constant α. Such a model was used by Hu and Matsuyama.[11] Similar to Eq. (7) we obtain that the third contribution to the SOI takes the following form:

$$V_{SO}^{\alpha\alpha}=-\hbar^2\frac{i}{2}\left(\sigma_y\frac{\partial\alpha}{\partial x}-\sigma_x\frac{\partial\alpha}{\partial y}\right). \tag{10}$$

III. TRANSMISSION THROUGH A BILLIARD WITH THE SOI

In this section we prove that the SOI gives no spin polarization for electron transmission through arbitrary billiards if the energy of the incident electron belongs to the first energy subband. In a dimensionless form the stationary Schrödinger equation has the following form:

$$\{-\nabla^2+v_{SO}\}\psi=\epsilon\psi,\quad \psi=\begin{pmatrix}u_1(x,y)\\ u_2(x,y)\end{pmatrix}. \tag{11}$$

Here $\epsilon=E/E_0$, $E_0=\hbar^2/2m^*L^2$, and

$$v_{SO}=\beta\left(i\sigma_x\frac{\partial}{\partial y}-i\sigma_y\frac{\partial}{\partial x}\right)-\frac{i}{2}\left(\sigma_y\frac{\partial\beta}{\partial x}-\sigma_x\frac{\partial\beta}{\partial y}\right). \tag{12}$$

L is a characteristic scale of the system, and $\beta=2m^*\alpha L$ is the dimensionless SOI constant. Correspondingly in Eqs. (11) and (12) coordinates x and y are also dimensionless.

Let S be an area of the structure under consideration which involves a billiard the SOI and leads, as shown in Fig. 1. Let Γ denote a boundary which crosses input and output leads at Γ_1 and Γ_2, respectively. We suppose that there is no spin-orbit interaction in the leads, i.e., $\beta=0$ at Γ_i, $i=1$ and 2. At boundary Γ we imply Dirichlet boundary conditions for a solution of Schrödinger equation (11) $\psi|_{\Gamma}=0$. As the scale L we take $L=d$.

Therefore we can write a solution in the electrodes as

$$|inc,n,\sigma\rangle=\sqrt{2}\sin(\pi ny)\exp(ik_nx)|\sigma\rangle,$$

$$|refl,n,\sigma\rangle=\sqrt{2}\sum_{m,\sigma'} r_{mn,\sigma,\sigma'}\sin(\pi my)\exp(-ik_m x)|\sigma'\rangle, \tag{13}$$

$$|tr,n,\sigma\rangle=\sqrt{2}\sum_{m,\sigma'} t_{mn,\sigma,\sigma'}\sin(\pi my')\exp(ik_m x')|\sigma'\rangle$$

where $|\sigma\rangle$ is a spin-state defined spin projections along some axis, say the z axis. The energy is

$$\epsilon=k_n^2+\pi^2n^2, \tag{14}$$

where $n=1,2,\ldots$ numerates a number of the energy subbands.

Introducing complex derivatives

$$\frac{\partial}{\partial z}=\frac{1}{2}\left(\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}\right), \tag{15}$$

we write the Schrödinger equation (11) as

$$\left(\frac{\partial^2}{\partial z\partial z^*}+\frac{1}{4}\epsilon\right)u_1+\frac{\beta}{2}\frac{\partial u_2}{\partial z}+\frac{u_2}{4}\frac{\partial \beta}{\partial z}=0,$$

$$\left(\frac{\partial^2}{\partial z\partial z^*}+\frac{1}{4}\epsilon\right)u_2-\frac{\beta}{2}\frac{\partial u_1}{\partial z^*}-\frac{u_1}{4}\frac{\partial \beta}{\partial z^*}=0, \tag{16}$$

where u_1 and u_2 are components of the spin state. We assume that there is an auxiliary degenerated state with components v_1 and v_2. In particular, it might be a Kramers degenerated state. Then for these two states the Green formula follows

$$\int_S(u\Delta v-v\Delta u)dS=\oint_\Gamma\left(u\frac{\partial v}{\partial n}-v\frac{\partial u}{\partial n}\right)dl, \tag{17}$$

where n is an exterior normal to the boundary Γ.

From the Schrödinger equation we have

$$v_2\left(\frac{\partial^2}{\partial z\partial z^*}+\frac{1}{4}\epsilon\right)u_1+\frac{\beta}{2}v_2\frac{\partial u_2}{\partial z}+\frac{1}{4}u_2v_2\frac{\partial \beta}{\partial z}=0,$$

$$u_2\left(\frac{\partial^2}{\partial z\partial z^*}+\frac{1}{4}\epsilon\right)v_1+\frac{\beta}{2}u_2\frac{\partial v_2}{\partial z}+\frac{1}{4}u_2v_2\frac{\partial \beta}{\partial z}=0, \tag{18}$$

$$u_1\left(\frac{\partial^2}{\partial z\partial z^*}+\frac{1}{4}\epsilon\right)v_2-\frac{\beta}{2}u_1\frac{\partial v_1}{\partial z^*}-\frac{1}{4}u_1v_1\frac{\partial \beta}{\partial z^*}=0,$$

$$v_1\left(\frac{\partial^2}{\partial z\partial z^*}+\frac{1}{4}\epsilon\right)u_2-\frac{\beta}{2}v_1\frac{\partial u_1}{\partial z^*}-\frac{1}{4}u_1v_1\frac{\partial \beta}{\partial z^*}=0.$$

Combining each pair of equations in Eqs. (18), we obtain

$$v_2\frac{\partial^2 u_1}{\partial z\partial z^*}+u_2\frac{\partial^2 v_1}{\partial z\partial z^*}+\frac{1}{4}\epsilon(u_1v_2+u_2v_1)+\frac{1}{2}\frac{\partial(\beta u_2v_2)}{\partial z}=0,$$

$$u_1\frac{\partial^2 v_2}{\partial z\partial z^*}+v_1\frac{\partial^2 u_2}{\partial z\partial z^*}+\frac{1}{4}\epsilon(u_1v_2+u_2v_1)-\frac{1}{2}\frac{\partial(\beta u_1v_1)}{\partial z^*}=0. \tag{19}$$

Extracting the second equation from the first one in Eq. (19), we obtain

$$v_2\frac{\partial^2 u_1}{\partial z\partial z^*}-u_1\frac{\partial^2 v_2}{\partial z\partial z^*}+u_2\frac{\partial^2 v_1}{\partial z\partial z^*}-v_1\frac{\partial^2 u_2}{\partial z\partial z^*}+\frac{1}{2}\frac{\partial(\beta u_1v_1)}{\partial z^*}+\frac{1}{2}\frac{\partial(\beta u_2v_2)}{\partial z}=0. \tag{20}$$

Integration of this equation over the billiard area S with use of Green formula (17) gives

$$\oint_\Gamma\left(v_2\frac{\partial u_1}{\partial n}-u_1\frac{\partial v_2}{\partial n}\right)dl+\oint_\Gamma\left(u_2\frac{\partial v_1}{\partial n}-v_1\frac{\partial u_2}{\partial n}\right)dl+\int_S\frac{\partial(2\beta u_1v_1)}{\partial z^*}dS+\int_S\frac{\partial(2\beta u_2v_2)}{\partial z}dS=0. \tag{21}$$

Since at Γ either $u_1=0$, $v_1=0$, or $\beta=0$, the last two integrals in Eq. (21) are equal to zero, and formula (21) can be rewritten as follows:

$$\sum_{i=1,2}\int_{\Gamma_i}\left(v_2\frac{\partial u_1}{\partial n}-u_1\frac{\partial v_2}{\partial n}\right)dlbf+\sum_{i=1,2}\int_{\Gamma_i}\left(u_2\frac{\partial v_1}{\partial n}-v_1\frac{\partial u_2}{\partial n}\right)dl=0. \tag{22}$$

This formula is sufficient to establish some symmetry rules between ingoing and outgoing states. In what follows the function u, describes the transmission through the waveguide, while the function v is auxiliary. Let us consider the first-channel transmission for $\epsilon<4\pi^2$. In order to ignore evanescent modes we will consider that boundaries Γ_i cross the leads far from the scattering region, as shown in Fig. 1. Electrons incident from the input lead are completely spin polarized up. This means that for the incident state [Eq. (13)] $|\sigma\rangle=\binom{1}{0}$. We denote the corresponding state interior the structure S as $\binom{u_{1\uparrow}(x,y)}{u_{2\uparrow}(x,y)}$ which is used as the u solution in Eq. (22). Correspondingly $\binom{u_{1\downarrow}(x,y)}{u_{2\downarrow}(x,y)}$ denotes the v solution in Eq. (22) for the case of electron spin polarized down. We suppose that boundaries Γ_1 and Γ_2 cross the leads normally the leads and that the x axis is parallel to the leads. Hence the normal n is parallel to the x axis. Then from Eq. (13) at the boundary Γ_2, which crosses the output lead, we obtain the relation

$$\frac{\partial f}{\partial n}=ik_1f, \tag{23}$$

where function f refers to all components $u_{1\uparrow}$, $u_{2\uparrow}$, $u_{1\downarrow}$, and $u_{2\downarrow}$.

These relations allow us to exclude the boundary Γ_2 from Eq. (22). At the boundary Γ_1 which crosses the input lead we have

$$\frac{\partial u_{1\uparrow}}{\partial n}=ik_1u_{1\uparrow}-2ik_1\sin(\pi y),$$
$$\frac{\partial u_{2\uparrow}}{\partial n}=ik_1u_{2\uparrow},$$
$$\frac{\partial u_{1\downarrow}}{\partial n}=ik_1u_{1\downarrow},\tag{24}$$
$$\frac{\partial u_{2\downarrow}}{\partial n}=ik_1u_{2\downarrow}-2ik_1\sin(\pi y).$$

We imply here that the origin of the x, y coordinate system is at the boundary Γ_1. Substituting relations (24) into Eq. (22), we obtain

$$\int_{\Gamma_1}(u_{1\uparrow}-u_{2\downarrow})\sin(\pi y)dy=0.\tag{25}$$

Since at the boundary Γ_1,

$$u_{1\uparrow}=\tilde{u}_{1\uparrow}(x)\sin(\pi y),\quad u_{2\downarrow}=\tilde{u}_{2\downarrow}(x)\sin(\pi y).$$

From Eq. (25) we obtain

$$u_{1\uparrow}=u_{2\downarrow}.$$

Thus from Eq. (13) it follows that the amplitudes of the reflection are

$$r_{\uparrow,\uparrow}=r_{\downarrow,\downarrow}.\tag{26}$$

Next, we take that the state $\binom{u_1}{u_2}=\binom{u_{1\uparrow}}{u_{2\uparrow}}$ coincides with the state $\binom{v_2}{v_2}$ in Eq. (22). Then Eq. (22) simplifies as follows:

$$\sum_{i=1,2}\int_{\Gamma_i}\left(u_2\frac{\partial u_1}{\partial n}-u_1\frac{\partial u_2}{\partial n}\right)dl=0.$$

Substituting relations (24) into this formula, we obtain

$$-2ik\int_{\Gamma_1}u_{2\uparrow}\sin(\pi y)dy=0.\tag{27}$$

This gives us that $u_{2\uparrow}=0$, or, according to Eq. (13), $r_{\uparrow\downarrow}=0$. Also, we similarly obtain that $u_{1\downarrow}=0$ at the boundary Γ_1. Thus we can write the second symmetry rule for reflection amplitudes:

$$r_{\uparrow,\downarrow}=r_{\downarrow,\uparrow}=0.\tag{28}$$

From symmetry rules (26) and (28) and from the current preservation, it follows that the transmission probabilities

$$T_\sigma=\sum_{\sigma'}|t_{\sigma,\sigma'}|^2=T\tag{29}$$

do not depend on the spin polarization of the incident electron.

Up to now we considered incident waves as spin polarized along the z axis at the boundary Γ_1. Let us now consider a flow of incident electrons which have no averaged spin polarization. In particular we can show that half of the electrons have an incident state with spin-up and half have an incident state with spin-down. Let us consider corresponding transmitted waves at the boundary Γ_2. We prove that for a transmission through a billiard with two attached leads there is no averaged spin polarization, i.e., $\langle\sigma_\alpha\rangle=0$, $\alpha=x,y,z$ if electrons are spin unpolarized in the first energy subband. As previously we take the incident state in the form of Eq. (13), and write the states in the leads as

$$|\psi_\uparrow\rangle=\begin{pmatrix}u_{1\uparrow}\\u_{2\uparrow}\end{pmatrix},\quad|\psi_\downarrow\rangle=\begin{pmatrix}u_{1\downarrow}\\u_{2\downarrow}\end{pmatrix}\tag{30}$$

where the up and down arrows indicate that electrons are incident with spins up and down. In the Green formulas (22) we take the first function u as $|\psi_\uparrow\rangle$ and the second function v as $\hat{\sigma}_y\hat{C}|\psi_\downarrow\rangle$, where $\hat{C}$ is a complex conjugation. This means that the second function is a Kramers degenerated state which responds to transmission from the output lead to the input lead. However as we noted below formula (22) the second state in Eq. (30) is auxiliary and does not reflect the real physical transmission described by the first state in Eq. (30). Hence

$$\begin{pmatrix}u_1\\u_2\end{pmatrix}=\begin{pmatrix}u_{1\uparrow}\\u_{2\uparrow}\end{pmatrix},\quad\begin{pmatrix}v_1\\v_2\end{pmatrix}=\begin{pmatrix}iu_{2\downarrow}^*\\-iu_{1\downarrow}^*\end{pmatrix}.\tag{31}$$

Let us calculate integral (22). As it was derived above for the first-channel transmission there is no reflection with spin flipping. Therefore at a boundary Γ_1 crossing the input lead $u_{2\uparrow}=0$, $u_{1\downarrow}=0$, therefore, since $u_2=0$ and $v_2=0$, integrals in Eq. (22) are equal to zero. The second contribution to integral (22) relates to the boundary Γ_2 crossing the output lead. Using the transmitted solution [Eq. (13)], one can write Eq. (22) as follows:

$$\int_{\Gamma_2}\left(v_2\frac{\partial u_1}{\partial n}-u_1\frac{\partial v_2}{\partial n}\right)dl+\int_{\Gamma_2}\left(u_2\frac{\partial v_1}{\partial n}-v_1\frac{\partial u_2}{\partial n}\right)dl$$
$$=2ik\oint_{\Gamma_2}(v_2u_1-u_2v_1)dy=0.$$

Therefore $u_1v_2=u_2v_1$, or in terms of notation (31),

$$u_{1\uparrow}u_{1\downarrow}^*=-u_{2\uparrow}u_{2\downarrow}^*.\tag{32}$$

From Eq. (32) it obviously follows that

$$|u_{1\uparrow}||u_{1\downarrow}|=|u_{2\uparrow}||u_{2\downarrow}|.$$

Moreover relation (29) implies that

$$|u_{1\uparrow}|^2+|u_{2\uparrow}|^2=|u_{1\downarrow}|^2+|u_{2\downarrow}|^2.$$

From these two relations one can obtain that

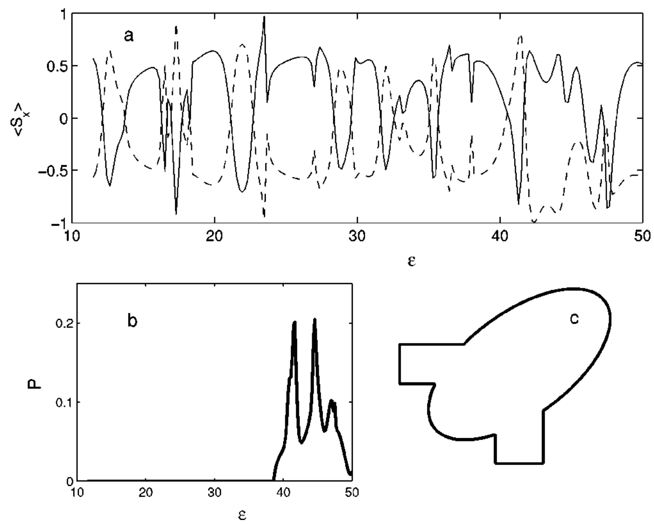

FIG. 2. Spin polarizations of electrons transmitted through the two-terminal quantum dot vs the energy of the electron in the first energy subband and partially in the second energy subband. (a) A solid line shows $\langle S_x \rangle$ if there are electron incidents in the spin-up polarized state (10). A dashed line shows $\langle S_x \rangle$ for an incoming electron in a spin-down polarized state (01). (b) The total spin polarization [Eq. (37)]. (c) Geometry of the structure of a quantum dot with two orthogonal leads.

$$|u_{2\uparrow}| = |u_{1\downarrow}|, \quad |u_{1\uparrow}| = |u_{2\downarrow}|. \tag{33}$$

Finally, relations (32) and (33) give

$$u_{1\uparrow} u_{2\uparrow}^{*} = -u_{1\downarrow} u_{2\downarrow}^{*}. \tag{34}$$

Mean values of the spin components in corresponding states [Eqs. (30)] are

$$\begin{aligned} \langle \sigma_x \rangle_{\uparrow} &= \mathrm{Re}(u_{1\uparrow} u_{2\uparrow}^{*}), \quad \langle \sigma_y \rangle_{\uparrow} = \mathrm{Im}(u_{1\uparrow} u_{2\uparrow}^{*}), \\ \langle \sigma_z \rangle_{\uparrow} &= |u_{1\uparrow}|^2 - |u_{2\uparrow}|^2, \\ \langle \sigma_x \rangle_{\downarrow} &= \mathrm{Re}(u_{1\downarrow} u_{2\downarrow}^{*}), \quad \langle \sigma_y \rangle_{\downarrow} = \mathrm{Im}(u_{1\downarrow} u_{2\downarrow}^{*}), \\ \langle \sigma_z \rangle_{\downarrow} &= |u_{1\downarrow}|^2 - |u_{2\downarrow}|. \end{aligned} \tag{35}$$

Equations (33)–(35) give rise to

$$\langle \sigma_\alpha \rangle_{\uparrow} = -\langle \sigma_\alpha \rangle_{\downarrow}, \quad \alpha = x,y,z, \tag{36}$$

i.e., the spin polarizations are exactly opposite in sign for the transmission of electrons incident in corresponding spin-polarized states.

Thus, for transmission through any billiard with a SOI with two attached leads, the spin polarization does not exist if the flow of electrons is incident in the first energy subband $\epsilon < 4\pi^2$ and has no spin polarization. This result is rather unexpected, since transmission through the four-terminal structure[5,7] gives rise to a positive/negative spin polarization of electrons flowing to the left/right outgoing electrodes. Hence we could expect a similar effect for electron transmission through a Γ -type structure. However, as shown in Fig. 2, a numerical computation of the electron transmission through a structure with two leads orthogonal to each other (see the inset of Fig. 2) in fact demonstrates the absence of spin polarization, in complete correspondence to relations (36). In Fig. 2(a) the outgoing mean spin component $\langle \sigma_x \rangle$ is shown versus energy provided, that the electron is incident in a spin-up polarized state [Eq. (10)] (solid line). Correspondingly, the dashed line shows the case of an incoming electron in a spin-down polarized state (01). The mean spin components $\langle \sigma_y \rangle$ and $\langle \sigma_z \rangle$ have similar but not the same energy dependencies, and are not shown in Fig. 2. The total spin polarization

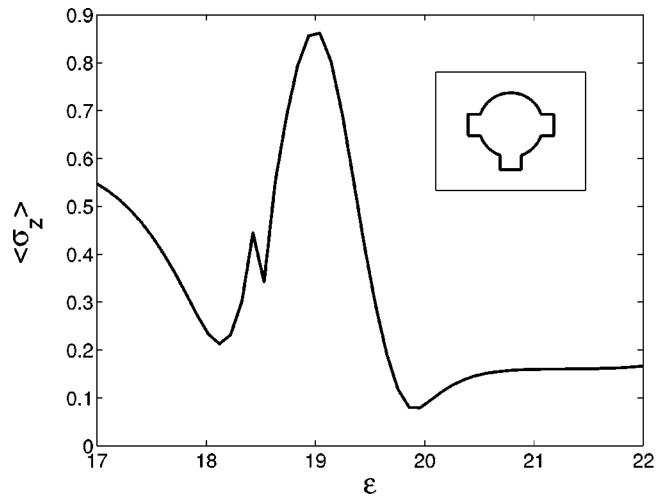

FIG. 3. Spin polarization of electrons transmitted through the three-terminal quantum dot vs the energy of an electron in the first energy subband. The upper inset shows the geometry of the structure.

$$P = \sum_{\alpha} [\langle \sigma_\alpha \rangle_{\uparrow} + \langle \sigma_\alpha \rangle_{\downarrow}] \tag{37}$$

is shown in Fig. 2(b). One can see that the spin polarization arises only if the energy exceeds the edge of the second subband 4π,[2] in complete agreement with analysis above.

Also, if there is no intersubband transmission $t_{mn,\sigma\sigma'} = 0$, $m \neq n$, the spin polarization is equal to zero for an arbitrary energy. It takes place approximately, for example, for adiabatic structures similar to curved waveguides (Sec. V). However, in the vicinity of edges of the energy subbands $\pi^2 n^2$ the SOI gives rise to intersubband mixing. As a result, in numerical calculations we obtain a strong spin polarization near the edges. Moreover, if the billiard is connected to three or more leads, the spin polarization of the transmitted electrons exists even for the transmission in the first energy subband. The effect of the third lead is demonstrated in Fig. 3. Hence this effect proposes a method of the spin transistor complementary to the way proposed by Datta and Das.[1] The spin polarization of transmitted electrons can be governed by a value of the connection of the third lead with the quantum dot. The most simple way is to apply a local electric field in the vicinity of the connection, which implies a potential barrier closing the connection of the dot with the third lead.

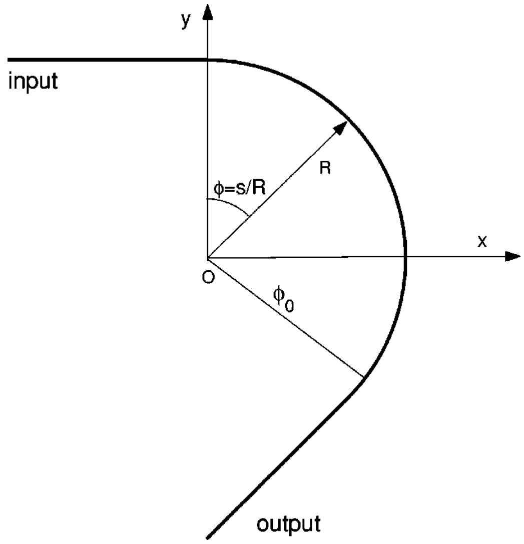

FIG. 4. Schematical view of a one-dimensional curved wire.

IV. ONE-DIMENSIONAL CURVED WIRE

A model in which only single-channel transmission takes place is a one-dimensional wire. Therefore, for a transmission through a one-dimensional wire of any form the SOI cannot give rise to the spin polarization. However, this model is interesting in that it allows one to find spin evolution analytically. A case of a straight wire was considered by in Refs. 1 and 9. Here we consider a curved wire consisting of a segment of a circle with a radius R attached to an infinite straight one-dimensional wires, as shown in Fig. 4.

We take a length of the segment to be $L=\phi_0 R$ and the position coordinate to be $s=\phi R$. The Hamiltonian of the wire has the form[12,13]

$$H=\frac{\hbar^2}{2m^*R^2}\tilde{H},$$
$$\tilde{H}=\left[\frac{\partial}{i\partial\phi}+\frac{\beta}{2}(\sigma_y\cos\phi+\sigma_x\sin\phi)\right]^2-\frac{\beta^2}{4}, \tag{38}$$

where $\beta=2m^*\alpha R$ is the dimensionless SOI constant. Since $[J_z,H]=0$ where $J_z=-i(\partial/\partial\phi)-\frac{1}{2}\sigma_z$, a particular solution of the stationary Schrödinger equation $\tilde{H}|\psi\rangle=\epsilon|\psi\rangle$ has the following form:[12–14]

$$|\psi\rangle=\begin{pmatrix} Ae^{i\mu\phi} \\ Be^{i(\mu-1)\phi}\end{pmatrix}. \tag{39}$$

The parameter μ defines the dimensionless wave number as $k=\mu/R$, and is arbitrary until boundary conditions are imposed. Substituting state (39) into the Schrödinger equation, one can obtain a relation between the energy of electron ϵ and the wave number μ,

$$(\epsilon-\mu^2)[\epsilon-(\mu-1)^2]-\beta^2(\mu-1/2)^2=0, \tag{40}$$

which gives

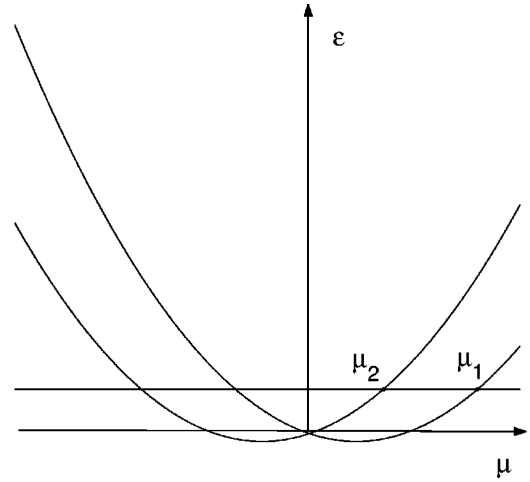

FIG. 5. The energy spectrum defined by formula (41) for $\alpha=1$. Values of μ_1 and μ_2 are shown by thick points, and correspond to a clockwise movement of the electron along the curved wire.

$$\epsilon_\nu=(\mu-1/2)^2+1/4+\nu|\mu-1/2|\sqrt{\beta^2+1}, \quad \nu=\pm 1. \tag{41}$$

Spectrum (41) is shown in Fig. 5.

For a fixed energy ϵ, Eq. (41) gives four solutions for the wave number μ. It is well known that[3] for electron transmission through a potential profile a reflection is negligibly small if the characteristic length of the inhomogeneity much exceeds the wavelength (the adiabatic regime). For our case we assume that the radius of curvature of the wire is much larger in comparison with the electron wavelength. Thus we can ignore the reflection for electron transmission through the quasi-one-dimensional waveguide.

Since there is no reflection for transmission through the one-dimensional waveguide, we need only those values of the wave number μ which correspond to a clockwise movement of the electron for the wire shown in Fig. 4. We denote its μ_1 and μ_2 as shown in Fig. 5. In what follows we use the following relation between μ_1 and μ_2:

$$2\lambda=\mu_1-\mu_2=\sqrt{1+\beta^2}. \tag{42}$$

For each μ_1 and μ_2 the normalized eigenstates follow from Eqs. (39) and (41):

$$\langle\phi|1\rangle=\frac{1}{\sqrt{1+\xi^2}}\begin{pmatrix} i\xi e^{i\mu_1\phi} \\ e^{i(\mu_1-1)\phi}\end{pmatrix}, \tag{43}$$

$$\langle\phi|2\rangle=\frac{1}{\sqrt{1+\xi^2}}\begin{pmatrix} e^{i\mu_2\phi} \\ i\xi e^{i(\mu_2-1)\phi}\end{pmatrix},$$

where

$$\xi=\frac{\beta}{1+\sqrt{1+\beta^2}}. \tag{44}$$

Therefore, a general solution of the Schrödinger equation for the electron transmission without reflection can be written as

$$|\psi(\phi)\rangle=\sum_{\nu=1,2} a_{\nu}\langle\phi|\nu\rangle. \tag{45}$$

where coefficients a_{ν} can be found in accordance with the incident state. To begin with we follow a case considered by Mireles and Kirczenow,[9] which took an incident electron with a spin state polarized along the z axis. An equality $\langle\psi(0)|=(1,0)$ yields

$$|\psi(\phi)\rangle=\frac{e^{i(\mu_1+\mu_2-1)\phi/2}}{1+\xi^2}\begin{pmatrix}\xi^2 e^{i(\lambda+1/2)\phi}+e^{-i(\lambda-1/2)\phi}\\ -i\xi e^{i(\lambda-1/2)\phi}+i\xi e^{-i(\lambda+1/2)\phi}\end{pmatrix}, \tag{46}$$

from which we obtain

$$|\langle(1,0)|\psi(\phi)\rangle|^2=1-\frac{4\xi^2}{(1+\xi^2)^2}\sin^2\frac{(\mu_1-\mu_2)\phi}{2}. \tag{47}$$

For the limit of a straight wire $R\to\infty$, the dimensionless SOI constant is $\beta\to\infty$. Therefore, in accordance with Eq. (44), $\xi\to1$. Then from Eq. (47) we obtain from the same result as in Ref. 9. It might be thought that there is no difference between the curved and straight wires considered in Ref. 9 except inessential factors. However, let us consider the initial spin polarization along the x axis as $(1,1)/\sqrt{2}$, which is not equivalent to the z axis in the geometry shown in Fig. 4. Then it follows from Eqs. (43) and (45) that

$$|\psi(\phi)\rangle=\frac{1}{\sqrt{2}}\frac{e^{i(\mu_1+\mu_2-1)\phi/2}}{1+i\xi}\begin{pmatrix}i\xi e^{i(\lambda+1/2)\phi}+e^{-i(\lambda-1/2)\phi}\\ e^{i(\lambda-1/2)\phi}+i\xi e^{-i(\lambda+1/2)\phi}\end{pmatrix}, \tag{48}$$

and, correspondingly,

$$\frac{1}{2}|\langle(1,1)|\psi(\phi)\rangle|^2=\frac{1}{(1+\xi^2)},$$

$$[\cos^2(\lambda-1/2)\phi+\xi^2\cos^2(\lambda+1/2)\phi]. \tag{49}$$

Substituting Eq. (42) and $\phi=L/R$ into Eqs. (49), we have the following expression for the probability of detection a spin polarized along the x axis:

$$|\langle(11)|\psi(\phi)\rangle|^2=\frac{1}{(1+\xi^2)}\left\{\cos^2\left(\frac{(k_1-k_2)L}{2}-\frac{L}{2R}\right)+\xi^2\cos^2\left(\frac{(k_1-k_2)L}{2}+\frac{L}{2R}\right)\right\}. \tag{50}$$

One can see that for $R\gg L$ probabilities (49) limit one to the case of the straight wire considered in Ref. 9, for which we have a simple spin precession. The spin evolution for state (48) is given by mean values of the spin components:

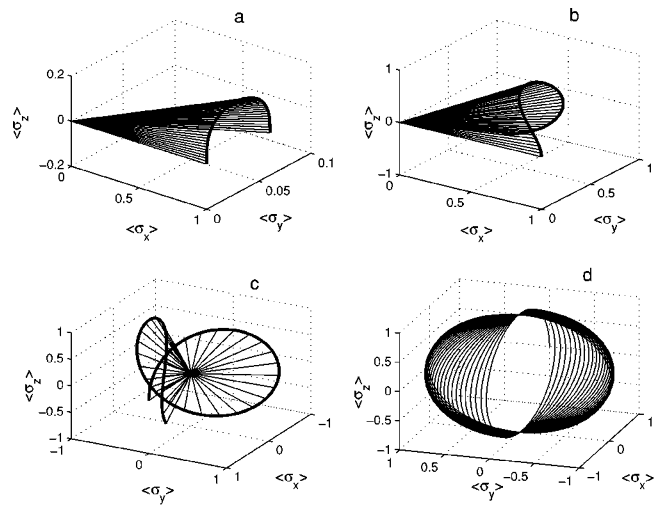

FIG. 6. Spin evolution given by formulas (51) for (a) $\beta=0.2$, (b) $\beta=1$, (c) $\beta=3$, and (d) $\beta=100$.

$$\langle\sigma_x\rangle=\frac{\cos(2\lambda-1)\phi+\xi^2\cos(2\lambda+1)\phi}{1+\xi^2},$$

$$\langle\sigma_y\rangle=\frac{\sin(2\lambda-1)\phi-\xi^2\sin(2\lambda+1)\phi}{1+\xi^2}, \tag{51}$$

$$\langle\sigma_z\rangle=\frac{2\xi\sin 2\lambda\phi}{1+\xi^2}.$$

For the straight wire ($R\gg L,\xi=1$) we again obtain a simple spin precession

$$\langle\sigma_x\rangle=\cos(2\lambda\phi),\ \langle\sigma_y\rangle=0,\ \langle\sigma_z\rangle=\sin(2\lambda\phi), \tag{52}$$

with the angular velocity of the precession equal to $2\lambda=\mu_1-\mu_2$. The spin evolution for the curved wire is shown in Fig. 6 for different β's (correspondingly different R's). One can see that the spin evolution is simple only for limiting cases of β, and correspondingly so is the radius of the curved wire [Figs. 6(a) and 6(d)]. However for intermediate values of β the spin evolution is not so simple as shown in Figs. 6(b) and 6(c).

In conclusion let us introduce unitary matrix of the spin evolution as

$$|\psi(\phi)\rangle=T(\phi)|\psi(0)\rangle, \tag{53}$$

and find to which rotation of the coordinate system it corresponds. Substituting Eqs. (45) and (43) into Eq. (53) we present the evolution as

$$|\psi(\phi)\rangle=\frac{1}{\sqrt{1+\xi^2}}e^{i(\mu_1+\mu_2-1)\phi/2}U(\phi)\left[a_1e^{i\lambda\phi/2}\begin{pmatrix}i\xi\\1\end{pmatrix}+a_2e^{-i\lambda\phi/2}\begin{pmatrix}1\\i\xi\end{pmatrix}\right], \tag{54}$$

where

$$U(\phi)=\begin{pmatrix} \exp(i\phi/2) & 0 \\ 0 & \exp(-i\phi/2) \end{pmatrix}. \tag{55}$$

Finally we can rewrite Eq. (54) as

$$|\psi(\phi)\rangle=\frac{1}{\sqrt{1+\xi^2}}e^{i(\mu_1+\mu_2-1)\phi/2}U(\phi)\Lambda U^{-2\lambda}(\phi)\begin{pmatrix} a_2 \\ a_1 \end{pmatrix}, \tag{56}$$

where

$$\Lambda=\begin{pmatrix} 1 & i\xi \\ i\xi & 1 \end{pmatrix}. \tag{57}$$

Since from Eq. (56) it follows that

$$|\psi(0)\rangle=\frac{1}{\sqrt{1+\xi^2}}\Lambda\begin{pmatrix} a_2 \\ a_1 \end{pmatrix}, \tag{58}$$

we find the matrix of the spin evolution comparing Eq. (56) to Eq. (53) as follows:

$$T(\phi)=e^{i(\mu_1+\mu_2-1)\phi/2}U(\phi)\Lambda U^{-2\lambda}(\phi)\Lambda^{-1}. \tag{59}$$

Substituting matrixes (55) and (57) in Eq. (59) we can write the spin evolution matrix as

$$T(\phi)=\frac{1}{1+\xi^2}\begin{pmatrix} e^{-i(\lambda-1/2)\phi}+\xi^2 e^{i(\lambda+1/2)\phi} & -2\xi\sin(\lambda\phi) \\ 2\xi\sin(\lambda\phi) & e^{i(\lambda-1/2)\phi}+\xi^2 e^{-i(\lambda+1/2)\phi} \end{pmatrix}. \tag{60}$$

Because transformation (53) is unitary it corresponds to a rotation of the coordinate system defined by the Eulerian angles angles (φ,γ,θ) (Ref. 15):

$$R(\varphi,\theta,\gamma)=e^{-i\varphi(1/2)\sigma_z}e^{-i\theta(1/2)\sigma_y}e^{-i\gamma(1/2)\sigma_z}.=\begin{pmatrix} \cos\left(\frac{1}{2}\theta\right)\exp\left[-i\frac{1}{2}(\varphi+\gamma)\right] & -\sin\left(\frac{1}{2}\theta\right)\exp\left[-\frac{1}{2}i(\varphi-\gamma)\right] \\ \sin\left(\frac{1}{2}\theta\right)\exp\left[i\frac{1}{2}(\varphi-\gamma)\right] & \cos\left(\frac{1}{2}\theta\right)\exp\left[i\frac{1}{2}(\varphi+\gamma)\right] \end{pmatrix}. \tag{61}$$

Therefore equation $R=T$ gives the following equations:

$$\gamma=\varphi+\phi,$$

$$\sin\left(\frac{1}{2}\theta\right)=\frac{2\xi}{1+\xi^2}\sin(\lambda\phi), \tag{62}$$

$$\tan\varphi=\frac{1-\xi^2}{1+\xi^2}\tan\lambda\phi.$$

For the limiting case $R\to\infty$ or $\xi\to 1$ from Eq. (62) we have, $\varphi=0$, $\theta=2\lambda\phi$, and $\gamma=\phi$. In accordance to the definition of the Eulerian angles [Eq. (61)] we therefore have a fast spin precession with an angular velocity 2λ with a consequent slow rotation of the precession plane by an angle $\gamma=\phi\ll\lambda\phi$ around the z axis. This result is in full coincidence with the evolution of the mean spin components shown in Fig. 6(d). Obviously, the slow rotation of the plane spin precession around the z axis, defined by the angle $\gamma=\phi$, directly corresponds to a rotation of the local coordinate system for electron transmission along a curved wire in the geometry shown in Fig. 4. However, for a finite radius R of the curved wire a spin state of the electron evolves in a rather complicated form, as seen from Eq. (62) and Fig. 6(b).

V. TWO-DIMENSIONAL CURVED WAVEGUIDE

For a consideration of the two-dimensional curved waveguide we introduce the curved coordinate system (s,u),[16,17] where s is the coordinate of central line along of the waveguide shown in Fig. 7.

We express the Hamiltonian of the waveguide in dimensionless form by

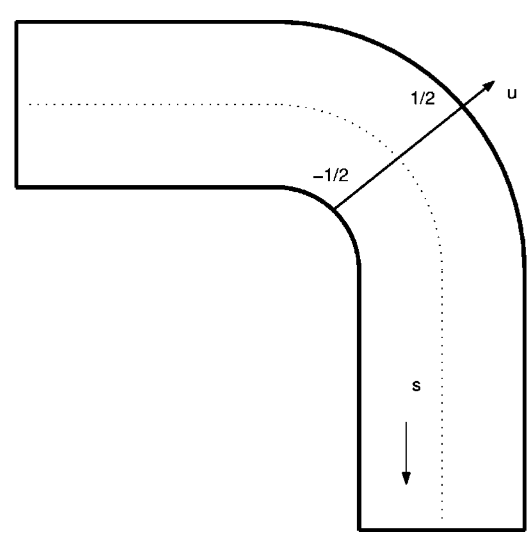

FIG. 7. A fragment of a two-dimensional curved wire with width $d=1$.

$$H=\frac{\hbar^2}{2m^*d^2}(\widetilde{H}_0+v_{SO}),$$

where

$$\widetilde{H}_0=-\Delta=-g^{-1/2}\frac{\partial}{\partial s}g^{-1/2}\frac{\partial}{\partial s}-g^{-1/2}\frac{\partial}{\partial u}g^{1/2}\frac{\partial}{\partial u}, \quad (63)$$

and d is the width of the waveguide. In what follows we consider a segment of the two-dimensional ring with a constant curvature $\gamma=1/R$ attached to straight leads with the same width as shown in Fig. 7. Therefore, for the segment we can write

$$x=a(s)-ub'(s),$$

$$y=b(s)+ua'(s), \quad (64)$$

$$a(s)=-R\cos(s/R),b(s)=R\sin(s/R),$$

$$g^{1/2}=1+u\gamma(s)=\frac{u+R}{R},$$

with $\gamma(s)$ the curvature of the curved waveguide which is taken below to be constant. The SOI takes the following form at the curved part of the waveguide:

$$v_{SL}=\beta\begin{pmatrix} 0 & e^{is/R}\left(\frac{\partial}{\partial u}+ig^{-1/2}\frac{\partial}{\partial s}\right)\\ -e^{-is/R}\left(\frac{\partial}{\partial u}-ig^{-1/2}\frac{\partial}{\partial s}\right) & 0\end{pmatrix}. \quad (65)$$

At the leads we assume that there is no the spin-orbital interaction ($\beta=0$) as well as $\gamma=0$, $g^{1/2}=1$.

The Schrödinger equation

$$\widetilde{H}\begin{pmatrix}\psi_\uparrow\\ \psi_\downarrow\end{pmatrix}=\epsilon\begin{pmatrix}\psi_\uparrow\\ \psi_\downarrow\end{pmatrix},$$

with the total Hamiltonian as $\widetilde{H}=\widetilde{H}_0+v_{SO}$, takes the following forms

$$\begin{aligned} &g^{-1/2}\frac{\partial}{\partial s}\left(g^{-1/2}\frac{\partial\psi_\uparrow}{\partial s}\right)+g^{-1/2}\frac{\partial}{\partial u}\left(g^{1/2}\frac{\partial\psi_\uparrow}{\partial u}\right)+\epsilon\psi_\uparrow\\ &\quad-\beta e^{is/R}\left(\frac{\partial\psi_\downarrow}{\partial u}+ig^{-1/2}\frac{\partial\psi_\downarrow}{\partial s}\right)=0,\\ &g^{-1/2}\frac{\partial}{\partial s}\left(g^{-1/2}\frac{\partial\psi_\downarrow}{\partial s}\right)+g^{-1/2}\frac{\partial}{\partial u}\left(g^{1/2}\frac{\partial\psi_\downarrow}{\partial u}\right)+\epsilon\psi_\downarrow\\ &\quad+\beta e^{-is/R}\left(\frac{\partial\psi_\uparrow}{\partial u}-ig^{-1/2}\frac{\partial\psi_\uparrow}{\partial s}\right)=0. \end{aligned} \quad (66)$$

The solutions of Eqs. (66) which satisfy to the Dirichlet boundary conditions ($u=\pm 1/2$) can be presented as[16,17]

$$\begin{aligned}\psi_\uparrow(u,s)&=\sum_{n=1}^{\infty}A_{\uparrow n}(s)\sin[\pi n(u+1/2)],\\ \psi_\downarrow(u,s)&=\sum_{n=1}^{\infty}A_{\downarrow n}(s)\sin[\pi n(u+1/2)].\end{aligned} \quad (67)$$

Substitution of Eq. (67) into Eqs. (66) gives

$$\begin{aligned}&\sum_{n=1}^{\infty}[L_{mn}A''_{\uparrow n}(s)+P_{mn}A_{\uparrow n}(s)-\beta e^{i\gamma s}Q_{mn}A_{\downarrow n}(s)\\ &\quad-i\beta e^{i\gamma s}R_{mn}A'_{\downarrow n}(s)]=[(\pi m)^2-\epsilon]A_{\uparrow m},\\ &\sum_{n=1}^{\infty}[L_{mn}A''_{\downarrow n}(s)+P_{mn}A_{\downarrow n}(s)+\beta e^{-i\gamma s}Q_{mn}A_{\uparrow n}(s)\\ &\quad-i\beta e^{-i\gamma s}R_{mn}A'_{\uparrow n}(s)]=[(\pi m)^2-\epsilon]A_{\downarrow m}.\end{aligned} \quad (68)$$

Here we introduced the following notations:

$$\begin{aligned}L_{mn}&=2\int_{-1/2}^{1/2}\frac{\sin[\pi m(u+1/2)]\sin[\pi n(u+1/2)]}{(1+u\gamma)^2}du,\\ P_{mn}&=2\pi n\int_{-1/2}^{1/2}\frac{\gamma\sin[\pi m(u+1/2)]\cos[\pi n(u+1/2)]}{1+u\gamma}du,\\ R_{mn}&=2\int_{-1/2}^{1/2}\frac{\sin[\pi m(u+1/2)]\sin[\pi n(u+1/2)]}{1+u\gamma}du,\\ Q_{mn}&=2\pi n\int_{-1/2}^{1/2}\sin[\pi m(u+1/2)]\cos[\pi n(u+1/2)]du.\end{aligned} \quad (69)$$

VI. NUMERICAL RESULTS

In numerical practice we solve the systems of equations (68) and (69) taking a finite number of waveguide modes. This number of modes was controlled by the normalization condition, and the sum of the total reflection probabilities and the total transmission ones was equal to a unit. The spin components $\langle\sigma_\alpha\rangle$ were calculated at an attached outgoing

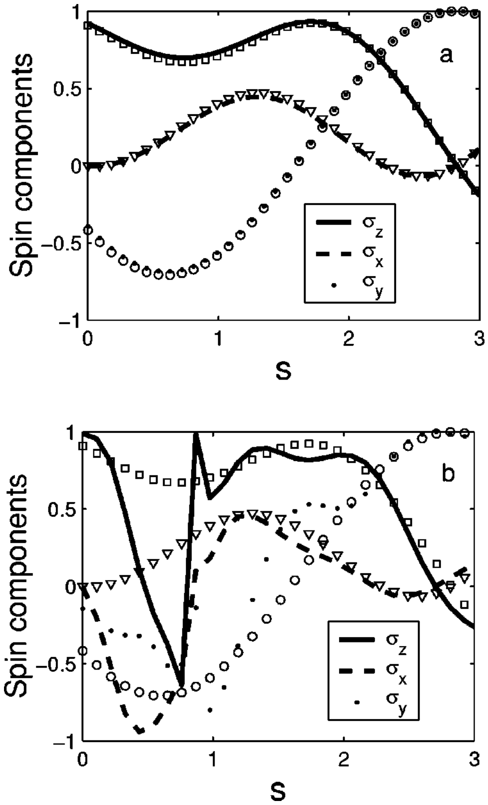

FIG. 8. The spin components dependent on the length s of a curved two-dimensional waveguide. The result of a calculation based on state (48) for the curved one-dimensional wire is shown by squares (σ_z), triangles (σ_x), and circles (σ_y). The radius of the wire is $R=d$, where d is the width of the waveguide. The dimensionless spin-orbit constant $\beta=2m^*\alpha d$ equals a unit. (a) The dimensionless energy $\epsilon=25$ (the first channel transmission) and (b) $\epsilon=39.25$ (near an edge of the second subband).

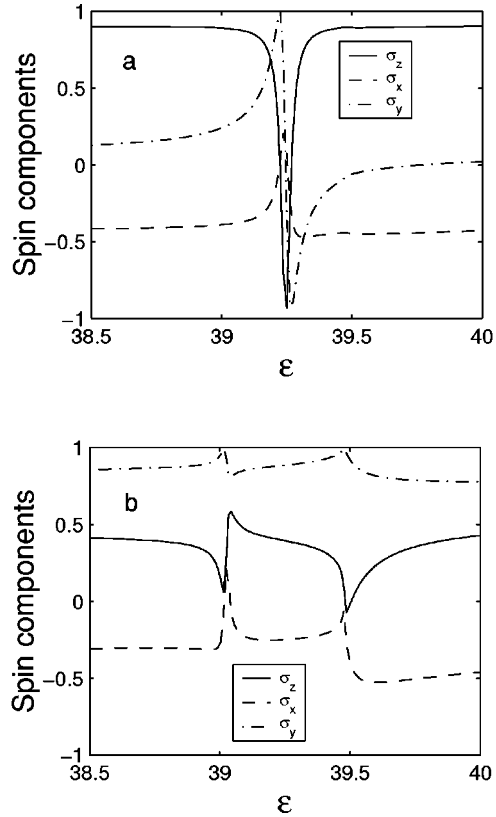

FIG. 9. The spin components dependent on the energy of incident electron for (a) $\phi_0=90°$ (curved waveguide) and (b) $\phi_0=180°$. $\beta=1$.

straight electrode, in which we assumed that there was no spin-orbit interaction, by the following formula:

$$\langle \sigma_\beta(s)\rangle = \frac{\int_{-1/2}^{1/2} du \langle \psi(u,s)|\hat{\sigma}_\beta|\psi(u.s)\rangle}{\int_{-1/2}^{1/2} du \langle \psi(u,s)|\psi(u.s)\rangle}. \qquad (70)$$

In Fig. 7 the outgoing electrode as well as the incoming one are not shown. Figure 8 shows the evolution of the spin components [Eq. (70)] versus the longitudinal coordinate s.

It is surprising that for an energy of incident electron far from the edge of the energy subband, the spin evolution almost coincides with the one-dimensional curved wire (shown in Fig. 8 by squares, triangles, and circles). In Fig. 9(a) the energy dependence of the spin components is shown, which demonstrates the remarkable phenomenon of spin flipping at the edge of the second energy subband $E_2=(2\pi)^2\approx 39.4$. It is interesting that increasing the region of the SOI by increasing the length of the curved waveguide or increasing the spin-orbit constant leads to a double flipping of the electron spin for transmission through the waveguide, as shown in Fig. 9(b) . This phenomenon is a consequence of the intersubband mixing by the SOI, as discussed in Sec. III.

Therefore, one can expect a strong deviation of the curved two-dimensional waveguides from the one-dimensional one for the spin evolution near edges of the subbands $\pi^2 n^2$. In fact, one can see from Fig. 9(b) that for an energy of the incident electron of $E\approx 4\pi^2$, the spin evolution with the length of the curved waveguide strongly deviates from the case of a one-dimensional curved wire.

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