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Confinement of Atoms with Nondegenerate Ground States in a Three-Dimensional Dissipative Optical Superlattice

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Based on the developed kinetic theory of rectified radiative forces, we found sufficient conditions for purely optical (nonmagnetic) three-dimensional confinement and cooling of atoms with the $J = 0 \longrightarrow J = 1$ quantum transition in a weak field of mutually orthogonal bichromatic standing waves. We show that a deep stable atom localization of atoms in the cells of an effective light superlattice (with a spacing much larger than the light wavelength) can be achieved by controlling the phase shifts (time-difference phase) of the temporal oscillations in orthogonally polarized field components and by specially choosing the field parameters. The proposed scheme of purely optical confinement can be directly used for a large group of atoms like Yb isotopes and alkali-earth elements with even–even nuclei. *© 2002 MAIK "Nauka/Interperiodica".*

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Because of its unique physical applications, the optical localization of atoms is an extremely important trend in current studies of resonant light pressure [1–3].

One of the most successful and fruitful solutions to this problem is the confinement of atoms in a magneto-optical trap (MOT)[1] [4]. A nonuniform magnetic field is an integral MOT element, because it allows the Earnshaw optical theorem (EOT) [5] to be circumvented. This theorem states that a stable localization of atoms by spontaneous light pressure forces in a weak (unsaturating the quantum transition) resonant field is not possible. Bouyer *et al.* [6] showed how the EOT could be circumvented by purely optical (nonmagnetic) methods using optical pumping of atoms with degenerate ground states. This scheme does not work for atoms with the ground-state angular momentum $J_g = 0$.

Meanwhile, the EOT was proved [5] (see also [6] for a discussion) precisely for atoms with scalar linear polarizability, which the atoms with the $J_g = 0 \longrightarrow J_e = 1$ quantum transition are. The principal possibility of overcoming the fundamental EOT constraints (without applying a magnetic field) for such atoms using the so-called rectified radiative forces (RRFs) in weak[2] bichromatic fields was pointed out in [7] (2D localization) and [8] (3D localization). However, the final solution of the problem and the elucidation of specific practical conditions for the achievement of dissipative optical confinement by this method requires a mandatory allowance for quantum RRF fluctuations and for the effect of field phases on the RRF spatial structure.

Here, these factors are simultaneously taken into account in the Wigner atomic density matrix formalism for a simple field model in the form of mutually orthogonal bichromatic standing waves. We found sufficient conditions (imposed on the relative initial phase shifts and on the wave parameters) that provide a deep stable 3D localization of atoms and, thereby, ensure that the EOT constraints (i.e., suppression of the vortex RRF component and long-term particle confinement in ultradeep light-induced potential wells) are overcome.

The problem under study also has an interesting research-and-application aspect, because it is directly related to the purely optical confinement of a large group of atoms like odd–odd Yb isotopes and alkali-earth elements with the $J_g = 0 \longrightarrow J_e = 1$ quantum transition (strong singlet, ${}^1S_0$–${}^1P_1$, and intercombination, ${}^1S_0$–${}^3P_1$, transitions of this type were effectively used in MOT experiments [10–12]). These atoms are believed to be very promising objects for carrying out new fundamental cold-particle experiments (see [10–13] and references therein). We emphasize that the presence of a magnetic field (as in MOTs) is undesirable for several important physical applications of the optical confinement of atoms [6, 9].

Consider an ensemble of atoms in a bichromatic field with a complex amplitude,

$$\mathbf{E} = \mathbf{E}_0(\mathbf{r})e^{-i\Delta_0 t} + \mathbf{E}_1(\mathbf{r})e^{-i\Delta_1 t}, \qquad (1)$$

where Δ_0 and Δ_1 are the frequency detunings of the fields $\mathbf{E}_0$ and $\mathbf{E}_1$ from the frequency $\omega_0 \gg |\Delta_0|, |\Delta_1|$ of

[1] By its nature, a MOT is a dissipative optical trap, because the particles are simultaneously cooled and confined in it.

[2] Here, we do not consider the use of rectified gradient (dipole) forces in strong bichromatic fields [7, 9] for the localization of atoms.

the quantum transition between the ground (with angular momentum $J_g = 0$) and excited (with angular momentum $J_e = 1$) atomic states.

As was shown in [7, 8], zero total radiation flux densities for each field mode frequency are a necessary condition for stable confinement of atoms with the type of transition under consideration in weak biharmonic fields (it predetermines the suppression of the principal, quadratic (in field) vortex RRF component):

$$\sum_j \langle \mathbf{J}_{j\alpha} \rangle = 0, \quad j = x, y, z, \quad \alpha = 0, 1,$$

where $\mathbf{J}_{j\alpha}$ is the energy flux density of the field components in superposition (1) that are polarized along the unit vector $\mathbf{e}_j$ of a Cartesian coordinate system and that have the frequency detuning Δ_α; the angular brackets denote averaging over microscopic spatial oscillations with a period of the order of the light wavelength. The field model in the form of a superposition of mutually orthogonal standing waves satisfies this condition:

$$\begin{aligned} V_{x\alpha}(\mathbf{r}) &= V_\alpha e^{i\xi_{z\alpha}} \cos(k_\alpha z + \varphi_z), \\ V_{y\alpha}(\mathbf{r}) &= V_\alpha e^{i\xi_{x\alpha}} \cos(k_\alpha x + \varphi_x), \\ V_{z\alpha}(\mathbf{r}) &= V_\alpha e^{i\xi_{y\alpha}} \cos(k_\alpha y + \varphi_y), \end{aligned} \tag{2}$$

where $V_{j\alpha}(\mathbf{r}) = d(\mathbf{e}_j \mathbf{E}_\alpha(\mathbf{r}))/\hbar$ are the local Rabi frequencies, $d = \|d\|/\sqrt{3}$, $\|d\|$ is the reduced matrix element of the transition dipole moment, $k_\alpha = (\omega_0 + \Delta_\alpha)/c$ are the wave numbers, $\xi_{j\alpha}$ and φ_j are the phases of the temporal and spatial oscillations in the field components, and V_α are their real amplitudes. Note that the phase shifts of the spatial oscillations in the complex field amplitudes that have identical polarizations but belong to different frequency modes can always be made equal by appropriately choosing the coordinate system. Therefore, the phases φ_j in expression (2) do not depend on α.

We describe the state of the atoms interacting with a resonant optical field by using the Wigner matrix of density $\rho(\mathbf{r}, \mathbf{v}, t)$ [1, 2]. In the quasi-classical limit $\hbar k_\alpha \ll m v$ (v and m are the characteristic atomic velocity and mass, respectively) and in interaction representation, this matrix satisfies the kinetic equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \hat{\gamma}\right)\hat{\rho} = -i[\hat{V}\hat{\rho}] + \frac{1}{2m}\left\{\frac{\partial \hat{V}}{\partial \mathbf{r}}\frac{\partial \hat{\rho}}{\partial \mathbf{v}}\right\}, \tag{3}$$

where $\hbar\hat{V}$ is the dipole atom-field interaction operator; $\hat{\gamma}$ is the relaxation operator that includes the recoil effect during spontaneous transitions [1, 2]; and the square brackets and braces denote the commutator and anticommutator, respectively. Below, it is convenient for our analysis to consider $\hat{\rho}$ in Cartesian representation [8], i.e., in the representation of basis wave functions (intra-atomic motion) for the ground (φ^g) and excited (φ_j^e) states, in which the matrix elements of the transition dipole moment $\hat{\mathbf{d}}$ are directed along the unit vectors of the Cartesian coordinate system:

$$\langle \varphi_j^e | \hat{\mathbf{d}} | \varphi^g \rangle = \mathbf{e}_j d.$$

In the resonance approximation, the system of equations (3) can then be represented as

$$\begin{aligned} i\left(\frac{d}{dt} + \gamma_\perp\right)\rho_i &= \sum_j q_{ij} V_j \\ &- \frac{\hbar i}{2m}\sum_{j \neq i}\frac{\partial q_{ij}}{\partial \mathbf{v}}\frac{\partial V_j}{\partial \mathbf{r}} - \frac{\hbar i}{4m}\frac{\partial Q_i}{\partial \mathbf{v}}\frac{\partial V_i}{\partial \mathbf{r}}, \end{aligned} \tag{4}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}, \quad Q_i = f + q_{ii} - \sum_{l \neq i} q_{ll},$$

$$\begin{aligned} i\left(\frac{d}{dt} + \gamma\right)q_{ij} &= -i\gamma f \delta_{ij} + (\rho_i V_j^* - V_i \rho_j^*) \\ -\delta_{ij}\left(\sum_{l = x, y, z} \rho_l^* V_l - \text{c.c.}\right) &+ \delta_{ij}\frac{\hbar i}{2m}\sum_l\left(\frac{\partial \rho_l}{\partial \mathbf{v}}\frac{\partial V_l^*}{\partial \mathbf{r}} + \text{c.c.}\right) \\ &- \frac{\hbar i}{2m}\left(\frac{\partial \rho_i}{\partial \mathbf{v}}\frac{\partial V_j^*}{\partial \mathbf{r}} + \frac{\partial V_i}{\partial \mathbf{r}}\frac{\partial \rho_j^*}{\partial \mathbf{v}}\right), \\ \frac{df}{dt} + \left(\frac{\hbar}{m}\right)\sum_j &\left(\frac{\partial \rho_j}{\partial \mathbf{v}}\frac{\partial V_j^*}{\partial \mathbf{r}} + \text{c.c.}\right) = D(\hat{\rho}), \end{aligned} \tag{5}$$

where γ is the decay rate of the excited state, $\gamma_\perp = \gamma/2$, $f(\mathbf{r}, \mathbf{v}, t) = \mathrm{Sp}(\hat{\rho})$ is the Wigner particle distribution function in phase space $(\mathbf{r}, \mathbf{v})$, $q_{ii}(\mathbf{r}, \mathbf{v}, t)$ and $\rho_i(\mathbf{r}, \mathbf{v}, t)$ mean the densities of the distributions of the population difference and the projections of the complex amplitude of the induced dipole moment onto the axes of the Cartesian coordinate system, the functions $q_{ij}(\mathbf{r}, \mathbf{v}, t)$ for $i \neq j$ describe the coherence between the excited atomic states, and the term $\hat{D}(\rho)$ on the right-hand side of Eq. (5) describes the recoil effect during spontaneous transitions in the quasi-classical limit:

$$\hat{D}(\hat{\rho}) = \frac{\hbar^2 k^2}{5m^2}\gamma\sum_{i,j}\left(\delta_{ij}\frac{\partial^2}{\partial \mathbf{v}^2} - \frac{1}{2}\frac{\partial^2}{\partial \mathbf{v}_i \partial \mathbf{v}_j}\right)\rho_{ij},$$

$$\rho_{ij} = q_{ij} + (\delta_{ij}/4)\left(f - \sum_l q_{ll}\right).$$

Let the resonant fields be weak and the frequency detunings Δ_0 and Δ_1 be not very close to each other:

$$\left|\frac{V_{j\alpha}}{\nu_\alpha}\right|^2, \quad \left|\frac{V_{j\alpha}^2}{\nu_1\nu_0}\right|, \quad \left|\frac{V_{j\alpha}^2}{\nu_\alpha\gamma}\right| \le g \ll 1, \qquad \delta = |\Delta_1 - \Delta_0| \gg g|\nu_\alpha|, \tag{6}$$

where $\nu_\alpha = \Delta_\alpha + i\gamma_\perp$.

In that case, the excited-state populations and the Stark energy-level shifts are small [8] and the distribution function (DF) can be represented as the sum of a slowly varying [on time scales $t > \tau = (\omega_R g)^{-1}$, $\omega_R = \hbar k^2/2m$, $k = \omega_0/c$] principal component $\bar{f}$ and a small rapidly oscillating (with characteristic frequencies $\Omega_1 \gg \tau^{-1}$) addition to it $\tilde{f}$ (cf. [14]):

$$f = \bar{f} + \tilde{f}, \quad |\tilde{f}/\bar{f}| \ll \frac{\hbar k}{m v} \lesssim \sqrt{\frac{\omega_R}{\gamma}} \ll 1. \tag{7}$$

The density matrix elements that describe the light-induced internal motions in the atom can be eliminated from the system of equations (4) and (5) by the expansion of the field in powers (actually in the parameter $g \ll 1$) of the following structure:

$$\begin{aligned}\rho_j &= \rho_j^{(1)}(\mathbf{r}, \mathbf{v}, t|\bar{f}(\mathbf{r}, \mathbf{v}, t)) \\ &+ \rho^{(3)}(\mathbf{r}, \mathbf{v}, t|\bar{f}(\mathbf{r}, \mathbf{v}, t)) + \ldots, \\ q_{ij} &= -\delta_{ij}\bar{f} + \bar{q}_{ij}^{(2)}(\mathbf{r}, \mathbf{v}, t|\bar{f}(\mathbf{r}, \mathbf{v}, t)) + \ldots,\end{aligned} \tag{8}$$

where $\rho_j^{(\sigma)}$ and $q_{ij}^{(\sigma)}$ are the linear differential operators acting on $\bar{f}$ and the superscript denotes the order of smallness of the corresponding terms in $g \ll 1$.

Below, we restrict our analysis to slow atoms ($kv \ll \gamma$) and take into account the fact that in our problem (as we will see), the atomic temperature T (in energy units) that corresponds to the Doppler cooling limit is always much higher than the depth of the microscopic potential wells produced by rapidly oscillating (with a period of $\sim 1/k$) gradient forces,

$$T \gg U_g \sim \hbar\gamma g, \tag{9}$$

because $T \gtrsim \hbar\gamma/2$. Using the expansion (8) and additional averaging of the DF over small-scale spatial oscillations with a period of the order of the light wavelength [valid under the condition (9)], we obtain the following Fokker–Planck equation for the DF (for the averaged DF, we retain the original designation):

$$\frac{df}{dt} + \frac{1}{m}\frac{\partial}{\partial \mathbf{v}}(\mathbf{F}_{1R} + \mathbf{F}_R)f = \sum_{ij} D_{ij}\frac{\partial^2 f}{\partial v_i \partial v_j}, \tag{10}$$

where the linear (in velocity) force $\mathbf{F}_{1R}$ and the RRF $\mathbf{F}_R$, respectively, match the general formulas (11) and (12) from [8] derived in a simple model of preset motion and the velocity diffusion tensor D_{ij}, in the second order of smallness in the field, is given by the formula ($r_i = \mathbf{r}\mathbf{e}_i$)

$$D_{ij} = \frac{\hbar^2\gamma}{5}\left\langle \sum_{\alpha=0}^{1}\left\{\frac{5}{2}\sum_l \frac{1}{|\nu_\alpha|^2}\frac{\partial V_{l\alpha}^*}{\partial r_i}\frac{\partial V_{l\alpha}}{\partial r_j} - \frac{k^2}{2}\frac{V_{i\alpha}V_{j\alpha}^*}{|\nu_\alpha|^2} + \frac{k^2}{2}\delta_{ij}\left(\frac{|V_{i\alpha}|^2}{|\nu_\alpha|^2} + 2\sum_l \frac{|V_{l\alpha}|^2}{|\nu_\alpha|^2}\right)\right\}\right\rangle.$$

For our case of the fields (2), we have

$$\mathbf{F}_{1R} = -m\kappa\mathbf{v}, \quad \kappa = -\frac{\hbar k^2\gamma}{m}\left[\frac{V_0^2\Delta_0}{|\nu_0|^4} + \frac{V_1^2\Delta_1}{|\nu_1|^4}\right], \tag{11}$$

$$\mathbf{F}_R = -\nabla U + \operatorname{curl}\mathbf{A}, \tag{12}$$

$$D_{ij} = D\delta_{ij}, \quad D = \left(\frac{\hbar k}{m}\right)^2\frac{\gamma}{2}\left(\frac{V_0^2}{|\nu_0|^2} + \frac{V_1^2}{|\nu_1|^2}\right), \tag{13}$$

where the scalar, $U(\mathbf{r})$, and vector, $\mathbf{A}(\mathbf{r})$, RRF potentials are defined by the expressions

$$\begin{aligned}U &= -\frac{\hbar k\Gamma_1 V_0^2 V_1^2}{4\delta k|\nu_0|^2|\nu_1|^2}\left\{\sum_j \cos(2\delta k\mathbf{e}_j\mathbf{r})\right. \\ &\left.+\frac{1}{2}\sum_{i\ne j}\cos\Psi_{ij}(\cos[\delta k(\mathbf{e}_i - \mathbf{e}_j)\mathbf{r}] + \cos[\delta k(\mathbf{e}_i + \mathbf{e}_j)\mathbf{r}])\right\}, \\ \mathbf{A} &= -\frac{\hbar k\Gamma V_0^2 V_1^2}{2\delta k|\nu_0|^2|\nu_1|^2} \\ &\times\{\mathbf{e}_y\sin\Psi_{zx}(\cos[\delta k(z-x)] - \cos[\delta k(z+x)]) \\ &+ \mathbf{e}_x\sin\Psi_{yz}(\cos[\delta k(y-z)] - \cos[\delta k(y+z)]) \\ &+ \mathbf{e}_z\sin\Psi_{xy}(\cos[\delta k(x-y)] - \cos[\delta k(x+y)])\}, \\ \delta k &= k_1 - k_0, \quad \Psi_{ji} = (\xi_{j1} - \xi_{i1}) - (\xi_{j0} - \xi_{i0}), \\ \Gamma_1 &= \gamma\gamma_\perp(\Delta_1 - \Delta_0)\left(\frac{1}{|\nu_1|^2} + \frac{1}{|\nu_0|^2}\right), \\ \Gamma &= (\Delta_1\Delta_0 + \gamma_\perp^2)\left(\frac{\gamma}{|\nu_1|^2} + \frac{\gamma}{|\nu_0|^2}\right).\end{aligned} \tag{14}$$

Thus, the quadratic (in field) force $\mathbf{F}_{1R}$ is the friction force and the RRF $\mathbf{F}_R$, which arises in the fourth order of smallness in the field, is generally a potential-vortex force in nature. The latter is attributable to interference effects in the resonant light pressure [7, 8], which, in particular, shows up in the dependence of its spatial structure on the relative phase shifts of the standing waves: $(\xi_{j\alpha} - \xi_{i\alpha})$. In this case, the vortex RRF component is determined by the correlators (of the fourth order of smallness in the field) of the mixed products of the projections of the field amplitudes and their derivatives that refer to standing waves of different frequency modes and different polarizations (in the notation of [8], the terms in the expression for the RRF $\propto \langle \mathbf{J}_\alpha^{jl} I_{\alpha'}^{lj} \rangle$, $\alpha \neq \alpha'$, $j \neq l$).

Even for overdamped motion, where $\Omega^2/\kappa^2 = \varepsilon \sim \gamma\delta k/\omega_R k \ll 1$ ($\Omega^2 \sim F\delta k/m$), the vortex RRF component curl$\mathbf{A}$ can result in unstable motion (the EOT manifestation mechanism!) and hamper particle localization [7, 8]. Let us show that by controlling the relative phase shifts $\xi_{j\alpha} - \xi_{i\alpha}$ of bichromatic fields of the form (2), we can successfully solve this problem. Note that for two intersecting monochromatic standing waves (polarized along the same direction), control of the spatial radiative-force structure by varying the relative phase shifts of the waves was convincingly demonstrated in experiments [15].

Let the phase shifts of the bichromatic field components satisfy the condition (n_1 and n_2 are arbitrary integers)

$$\Psi_{zx} = 2\pi n_1, \quad \Psi_{yz} = 2\pi n_2. \tag{15}$$

In particular, this condition is always satisfied if the phase differences between orthogonally polarized waves, $\xi_{j\alpha} - \xi_{i\alpha}$, are multiples of π: $\xi_{j\alpha} - \xi_{i\alpha} = \pi m_{ij}$, where m_{ij} are arbitrary integers of the same parity. In that case, $\sin\Psi_{ij} = 0$, $\cos\Psi_{ij} = 1$, and, as follows from (12) and (14), the RRF is a purely potential ($\mathbf{A} = 0$) force. For $\kappa > 0$, it can generate a body-centered cubic superlattice (with spacing $L = \pi/\delta k \gg \lambda = 2\pi/k$) of atoms localized in potential wells with the characteristic depth

$$U_0 \cong \frac{\hbar\omega_0}{2}\left(\frac{\gamma^2}{|\nu_1|^2} + \frac{\gamma^2}{|\nu_0|^2}\right)\frac{V_1^2 V_0^2}{|\nu_1|^2|\nu_0|^2}. \tag{16}$$

Indeed, in this case, the Fokker–Planck equation (10) for $\kappa > 0$ admits a steady-state solution of the Boltzmann form:

$$f(\mathbf{r}, \mathbf{v}) \propto e^{-E(\mathbf{r})/T}, \quad E(\mathbf{r}) = \frac{m\mathbf{v}^2}{2} + U(\mathbf{r}),$$
$$T = \frac{mD}{\kappa} = -\frac{\hbar}{2}(g_0 + g_1)/\left(\frac{g_0\Delta_0}{|\nu_0|^2} + \frac{g_1\Delta_1}{|\nu_1|^2}\right), \tag{17}$$
$$g_\alpha = V_\alpha^2/|\nu_\alpha|^2.$$

The condition for deep localization of atoms in superlattice cells follows from Eqs. (17):

$$\eta = U_0/T \gg 1.$$

If this condition is satisfied, the sizes of the localized bunches of atoms are estimated as $r_0 \sim 1/\delta k\eta^{1/2} \ll 1/\delta k \sim L$. Thus, the localization parameter η is a complicated function of the field amplitudes and frequency detunings: $\eta = \eta(\Delta_1, \Delta_2, V_1, V_2)$. The stability of deep ($\eta \gg 1$) atom localization is characterized by the mean lifetime of an atom in an individual superlattice cell (determined by the time of particle diffusion from one well into another). The latter is estimated as

$$\tau > \tau_0\frac{\pi^{5/2}}{8}\left(\frac{1}{\eta}\right)^{3/2} e^{\eta}, \quad \tau_0 = (4D_s\delta k^2)^{-1}, \tag{18}$$

where $D_s \simeq D/\kappa^2$ is the coefficient of spatial diffusion of the atoms in the field of radiative forces and τ_0 has the meaning of the particle lifetime in the so-called optical molasses with sizes $L/2$ (see, e.g., [2]). The estimate (18) was obtained from (10) in the limit ($\varepsilon \ll 1$) of overdamped particle motion (which is reached in most real situations [8]) based on the approximation of the boundary of the region of attraction of a stable RRF node by an atom-absorbing sphere of radius $L/2$. Clearly, it makes sense to speak about stable localization of atoms in the superlattice if τ is much longer than the lifetime of viscous confinement, i.e., when $(1/\eta)^{3/2}e^{\eta} \gg 1$.

To obtain specific estimates showing the real possibility of deep atom localization, we choose field frequencies and amplitudes to satisfy the conditions

$$\Delta_0 = -\frac{\gamma}{2}, \quad |\Delta_1| \gg \gamma,$$
$$|V_{j1}|^2/|\nu_1\nu_0| \sim |V_{j0}/\nu_0|^2 \sim g \simeq 2 \times 10^{-2} \ll 1. \tag{19}$$

In that case, only the field $\mathbf{E}_0$ is responsible for cooling (because $g_1 \ll g_0$), $T \simeq \hbar\gamma/2$, the frequency detuning Δ_1 of the field $\mathbf{E}_1$ determines the superlattice spacing (because $\delta k \simeq \Delta_1/c$), and all conditions (6) for the application of perturbation theory ($|\rho_j^{(3)}/\rho^{(1)}| \sim 0.1$) are satisfied. The atom localization parameter η is determined only by the ratio of the transition frequency to the detuning Δ_1,

$$\eta \simeq \frac{\omega_0}{|\Delta_1|} \times 6 \times 10^{-4},$$

and the field intensities J_α with the frequency detuning Δ_α required for particle localization are related to the radiation intensity saturating the quantum transition, J_s, by simple formulas, $J_0 = J_s \times 10^{-2}$ and $J_1 = \sqrt{2} \times 10^{-2}$ $(|\Delta_1|/\gamma)J_s$. For example, for the $^1S_0$–$^1P_1$ singlet transition of the ytterbium atom with $\lambda = 398.8$ nm, $\gamma = 1.8 \times 10^8$ s^{-1}, and detuning $|\Delta_1| \sim 2 \times 10^{11}$ s^{-1}, we have the following estimates: $L \simeq 0.5$ cm, $\eta = 14$, $r_0 \sim 0.1$ cm, $\tau_0 \simeq 0.01$ s, $\tau \sim 250$ s, $T = 7.2 \times 10^{-4}$ K, $J_0 \sim 0.6$ mW cm^{-2}, and $J_1 \simeq 0.8$ W cm^{-2}. For the same detuning Δ_1 of the quasi-resonant field and for the $^1S_0$–$^3P_1$ intercombination transition [11] with $\lambda = 555.6$ nm and $\gamma \sim 1.2 \times 10^6$ s^{-1}, we have $L \simeq 0.5$ cm, $\eta \sim 10$, $r_0 \sim 0.1$ cm, $\tau_0 \simeq 1$ s, $\tau \sim 250$ s, $T = 5$ μK (!), $J_0 \sim 1.4 \times 10^{-6}$ W/cm^2, and $J_1 \simeq 280$ mW/cm^2.

Thus, the atom localization conditions are satisfied for extremely low intensities. By decreasing the detuning $|\Delta_1|$, we can increase the localization parameter η and decrease the quasi-resonant field intensity J_1. In this case, the confinement of atoms may require increasing the cross-sectional laser-beam sizes R because of the condition $R > 1/\delta k$.

In conclusion, note that the vortex RRF component of atoms in a bichromatic field of the form (2) for arbitrary relative phase shifts of the standing waves can be suppressed by a purposeful choice of field detunings [see expressions (12) and (14)]: $\Delta_1\Delta_0 = -\gamma_\perp^2$. In such a situation, however, the stability condition ($\eta \gg 1$) is very difficult to satisfy for realistic superlattice parameters, because the field frequency detunings are "rigidly" related to each other. In particular, the regime of atom confinement that corresponds to the conditions (19) cannot be achieved; in this regime, cooling to limiting temperatures $\sim\hbar\gamma$ is combined with stable deep localization and a relatively small ($L < 1$ cm) adjustable superlattice spacing.

The vortex RRF component is also suppressed for uncorrelated fluctuating phases of orthogonally polarized standing waves [in superposition (2)], which can be produced, for example, by using independent sources of laser radiation. In this case, the localization conditions deteriorate, because the depths of the light-induced potential wells decrease and the relative phase shifts of the waves completely lose their role of controlling parameters.

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