Effects of the dimensionality of inhomogeneities on the wave spectrum of superlattices

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Dependences of the dispersion laws and damping of waves in an initially sinusoidal superlattice on the dimensionality of inhomogeneities modulating the period of the superlattice are studied. The cases of one- and three-dimensional modulations, as well as modulation by a mixture of inhomogeneities of both of these dimensionalities, are considered. The correlation function of the superlattice $K(r)$ has the form of a product of the same periodic function and a decreasing function that is significantly different for these different cases. For $r\rightarrow\infty$, the decreasing function goes to zero for the one-dimensional inhomogeneities and to a nonzero asymptote for the three-dimensional ones. Consequently, the transition from the disordered to ordered states is accompanied in the three-dimensional case not only by an increase of the correlation radius as in the one-dimensional case, but also by a change in the relationship between the volumes of the superlattices with finite and infinite correlation radii. The decreasing part of the correlation function for the mixture of inhomogeneities of different dimensionalities has the form of a product of the decreasing parts of the correlation functions of the components of the mixture. This leads to the nonadditivity of the contributions of the components of different dimensionalities to the resulting modification of the parameters of the wave spectrum that are due to the inhomogeneities (the damping of waves for the mixture of these components is smaller than the sum of the dampings of the components, the maximum gap in the spectrum corresponds to the simultaneous presence of both components, and so on).

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I. INTRODUCTION

Investigations of the spectrum of waves in partially randomized superlattices (SL's) have been carried out very intensively in recent years. This is due to the wide use of these materials in various high technology devices as well as due to the fact that they are convenient models for developing new methods of theoretical physics for studying media without translation symmetry. Several methods exist now for developing a theory of such SL's: the modeling of the randomization by altering the order of successive layers of two different materials;[1–7] the numerical modeling of the random deviations of the interfaces between layers from their initial periodic arrangement;[8–10] the postulation of the form of the correlation function of a SL with inhomogeneities;[11,12] the application of the geometrical optics approximation;[13] and the development of the dynamic composite elastic medium theory.[14]

One more method for the investigation of the influence of inhomogeneities on the wave spectrum of a SL was suggested in Ref. 15, the method of the random spatial modulation (RSM) of the period of the SL. This method is an extension of the well-known theory of the random frequency (phase) modulation of a radio signal[16,17] to the case of spatial inhomogeneities in the SL. The advantage of this method is that the form of the correlation function (CF) of the SL is not postulated but is developed from the most general assumptions about the nature of a random spatial modulation of the SL period. It appeared that in the general case, this function had a quite complicated form that depended on the dimensionality of the inhomogeneities and the values of their rms fluctuations and correlation radii. Knowledge of the CF corresponding to a particular type and dimensionality of the inhomogeneities permitted us to apply methods of investigation of averaged Green functions to find the energy spectrum and other characteristics of the waves.[15,18,21] In carrying out these investigations, we also used another advantage of the RSM method. This method permitted us to study the wave spectrum for two models of SL's in the framework of the same approach: SL's with an initially sinusoidal dependence of material parameters on a coordinate,[15,18,21] and SL's with a dependence in the form of rectangular spatial pulses.[19,20] These models, which are widely used in the literature, correspond to the two limiting cases of the relation between the thickness of the interfaces d and the period l of the multilayer structure. To describe the general situation, we introduced[22] a model of a SL with an arbitrary relation between d and l; applying the RSM method to this model, we studied for the first time the effects of inhomogeneities on the wave spectrum of multilayers with finite interface thicknesses.[23] The RSM method also permitted us to consider inhomogeneities of different dimensionalities in the framework of the same model. Effects of one-dimensional (1D) and three-dimensional (3D) inhomogeneities on the wave spectrum were studied for sinusoidal SL's, SL's with sharp interfaces, and SL's with arbitrary thicknesses of interfaces.

However, one of the most significant aspects of this problem was not considered clearly enough, namely, the interrelation between the form of the CF of the SL corresponding to inhomogeneities of a given dimensionality and the nature of the modifications in the wave spectrum that are due to these inhomogeneities. In addition, the influence of inhomogeneities of each dimensionality was studied separately. So, the other significant aspect of the problem that was not considered up to now is the situation when inhomogeneities of different dimensionalities are present simultaneously in a superlattice. The study of both of these aspects is the objective of the present work.

II. MODEL AND CORRELATION FUNCTION

A SL is characterized by the dependence of some material parameter A on the coordinates $\mathbf{x}=\{x,y,z\}$. The physical nature of the parameter $A(\mathbf{x})$ can be different. This parameter can be a density of matter or a force constant for the elastic system of a medium, the magnetization, anisotropy, or exchange for a magnetic system, and so on. We represent $A(\mathbf{x})$ in the form

$$A(\mathbf{x})=A[1+\gamma\rho(\mathbf{x})], \tag{1}$$

where A is the average value of the parameter, γ is its relative rms variation, $\rho(\mathbf{x})$ is a centered ($\langle\rho(\mathbf{x})\rangle=0$) and normalized ($\langle\rho^2(\mathbf{x})\rangle=1$) function. The function $\rho(\mathbf{x})$ describes the periodic dependence of the parameter along the SL axis z, as well as the random spatial modulation of this parameter which, in the general case, can be a function of all three coordinates $\mathbf{x}=\{x,y,z\}$.

We will consider in this paper a SL that has a sinusoidal dependence of the material parameter on the coordinate z in the initial state where inhomogeneities are absent. According to the RSM method, we represent the function $\rho(\mathbf{x})$ in the form

$$\rho(\mathbf{x})=\sqrt{2}\cos\{q[z-u_1(z)-u_3(\mathbf{x})]+\psi\}, \tag{2}$$

where $q=2\pi/l$ is the SL wave number. We assume in the RSM method that the function $u_1(z)$ models 1D displacements of the interfaces from their initial periodic arrangement. The function $u_3(\mathbf{x})$ is introduced in Eq. (2) to model a random deformation of the surfaces of the interfaces. At first glance, it would seem that this function must depend only on the two coordinates x and y. But the function $u(x,y)$ describes in the RSM method a 2D deformation that is uniform for all interfaces of the SL, that is, which has an infinite value of the correlation radius along the z coordinate. The directly opposed cases are of interest in reality, namely, the cases where the deformations of two nearest interfaces are uncorrelated (the correlation radius along z is much smaller than $l/2$) or only several interfaces are correlated. That is why, $u_3(\mathbf{x})$ must be a random function of all three coordinates x, y, and z. In the general case, this function has an anisotropy of correlation properties because the values of the correlation radii in the xy plane and along the z axis are determined by different physical reasons. But we restrict ourselves here to the simplest case and assume that $u_3(\mathbf{x})$ is a 3D random function with isotropic correlation properties. A coordinate-independent random phase ψ is introduced into Eq. (2) to ensure the fulfillment of the condition of ergodicity for the function $\rho(\mathbf{x})$ (see Ref. 15); it is characterized by a uniform distribution in the interval $(-\pi,\pi)$. After averaging the product of the functions $\rho(\mathbf{x})$ and $\rho(\mathbf{x}+\mathbf{r})$ over the phase ψ, we obtain

$$\langle\rho(\mathbf{x})\rho(\mathbf{x}+\mathbf{r})\rangle_\psi=\cos(qr_z-\chi_1-\chi_3), \tag{3}$$

where

$$\chi_1=q[u_1(z+r_z)-u_1(z)],\quad \chi_3=q[u_3(\mathbf{x}+\mathbf{r})-u_3(\mathbf{x})]. \tag{4}$$

We assume that the random functions χ_1 and χ_3 are mutually uncorrelated and that each of them is a Gaussian random process.

After averaging Eq. (3) over χ_1 and χ_3, we obtain a general expression for the CF of the SL in the form

$$K(\mathbf{r})=\cos(qr_z)K_1(r_z)K_3(r), \tag{5}$$

where

$$K_1(r_z)=\exp\left[-\frac{1}{2}Q_1(r_z)\right], \tag{6}$$

$$K_3(r)=\exp\left[-\frac{1}{2}Q_3(r)\right], \tag{7}$$

and the structure functions $Q_i(\mathbf{r})$ are defined by the equations

$$Q_1(r_z)=\langle\chi_1^2\rangle,\quad Q_3(r)=\langle\chi_3^2\rangle. \tag{8}$$

One can see from Eqs. (6)–(8) that $K_1(r_z)$ and $K_3(r)$ are the decreasing parts of the CF's of the SL's with 1D or 3D inhomogeneities [recall that the complete CFs for these cases have the form of the product of $\cos(qr_z)$ and $K_1(r_z)$ or $K_3(r)$, respectively[15]]. So, the decreasing part of the CF of a SL with a mixture of mutually uncorrelated phase inhomogeneities of different dimensionalities has the form of the product of the decreasing parts of the CF's of the components of this mixture.

For finding the structure functions $Q_1(r_z)$ and $Q_3(r)$, we must model the correlation properties of the modulating functions $u_1(z)$ and $u_3(\mathbf{x})$ or, more precisely, the correlation properties of their gradients. Both $Q_1(r_z)$ and $Q_3(r)$ were found in Ref. 15 (see also some refinements of the coefficients in these expressions in Ref. 21) by the use of different forms of the model CF's for the random modulation. It was shown that the forms of the functions Q_i do not depend asymptotically (for both small and large values of r) on the form of the model CF, but strictly depend on the dimensionalities of the inhomogeneities. For the exponential model CF's for $u_1(z)$ and $u_3(\mathbf{x})$, the structure functions were obtained in the forms

$$Q_1(r_z)=2\gamma_1^2[\exp(-k_\parallel r_z)+k_\parallel r_z-1], \tag{9}$$

$$Q_3(r)=6\gamma_3^2\left[1-\frac{2}{k_0r}+\left(1+\frac{2}{k_0r}\right)\exp(-k_0r)\right], \tag{10}$$

where $k_\parallel$ and k_0 are the correlation wave numbers of the random modulations $u_1(z)$ and $u_3(\mathbf{x})$, respectively. The functions $u_1(z)$ and $u_3(\mathbf{x})$ are inhomogeneous random functions and their rms fluctuations depend on the coordinates. The coordinate-independent quantities γ_1 and γ_3 in Eqs. (9) and (10) are the coefficients of the relative rms fluctuations of the functions $u_1(z)$ and $u_3(\mathbf{x})$, respectively. More precisely,[21] γ_1 and γ_3 are determined from the relationships $\gamma_1=\sigma_1 q/k_\parallel$ and $\gamma_3=\sigma_3 q/\sqrt{3}k_0$, where σ_1 and σ_3 are the rms fluctuations of the gradients of $u_1(z)$ and $u_3(\mathbf{x})$.

After the substitution of Eqs. (9) and (10) into Eqs. (6) and (7), the latter become quite complicated. That is why, approximate expressions for $K_1(r_z)$ and $K_3(r)$ were suggested for the 1D and 3D inhomogeneities (see Refs. 15 and 23, respectively):

$$K_1(r_z)=\exp(-\gamma_1^2 k_\parallel r_z), \tag{11}$$

$$K_3(r)=(1-L)\exp(-\gamma_3^2 k_0 r)+L, \tag{12}$$

where $L=\exp(-3\gamma_3^2)$ is the asymptotic form of $K_3(r)$ when $r\to\infty$.

According to these equations, effective correlation radii of the SL can be introduced for the 1D and 3D cases, respectively,

$$r_1=(\gamma_1^2 k_\parallel)^{-1}, \quad r_3=(\gamma_3^2 k_0)^{-1}. \tag{13}$$

One can see that the effective correlation radii of the SL depend not only on the correlation radii $k_\parallel^{-1}$ or k_0^{-1} of the corresponding modulating functions u_1 or u_3 but also on the rms fluctuations of these functions, γ_1 or γ_3.

The approximate forms of $K_1(r_z)$ and $K_3(r)$ given by Eqs. (11) and (12) are shown in Figs. 1(a) and 1(b), respectively, by dashed curves. A comparison of these curves with the exact dependences described by Eqs. (6), (7), (9), and (10) (depicted by solid curves) shows that Eqs. (11) and (12) are good approximations in the entire region $\gamma_i^2<1$.

In Figs. 2(a,b), the functions $K_1(\mathbf{r})$ and $K_3(\mathbf{r})$ are shown as functions of the two coordinates, r_z and $r_\perp=(r_x^2+r_y^2)^{1/2}$. It is seen from these figures that there is a cardinal difference between the asymptotic behaviors of $K_1(\mathbf{r})$ and $K_3(\mathbf{r})$. The function $K_1(\mathbf{r})$ goes to zero when $r\equiv(r_z^2+r_\perp^2)^{1/2}$ goes to infinity in all directions, characterized by the angle $\theta=\arccos(r_z/r)$, excluding the point $\theta=\pi/2$, while the function $K_3(\mathbf{r})$ goes to the nonzero asymptotic value $L(\gamma_3)$ when $r\to\infty$. Consequently, in the 1D case the inhomogeneities have a finite correlation radius r_1 in the entire volume of the SL. In contrast to this, in the 3D case volumes with an infinite correlation radius exist side by side with volumes with the finite correlation radius r_3.

In Fig. 3, the decreasing part of the CF of the SL containing a mixture of 1D and 3D inhomogeneities $K_{13}(\mathbf{r})=K_1(r_z)K_3(r)$ is depicted. One can see that this function goes to zero when $r\to\infty$ for all values of θ, except $\theta=\pi/2$. This property makes $K_{13}(\mathbf{r})$ approach the function $K_1(\mathbf{r})$. But in the direction $\theta=\pi/2$, the function $K_{13}(\mathbf{r})$ becomes similar to the function $K_3(\mathbf{r})$.

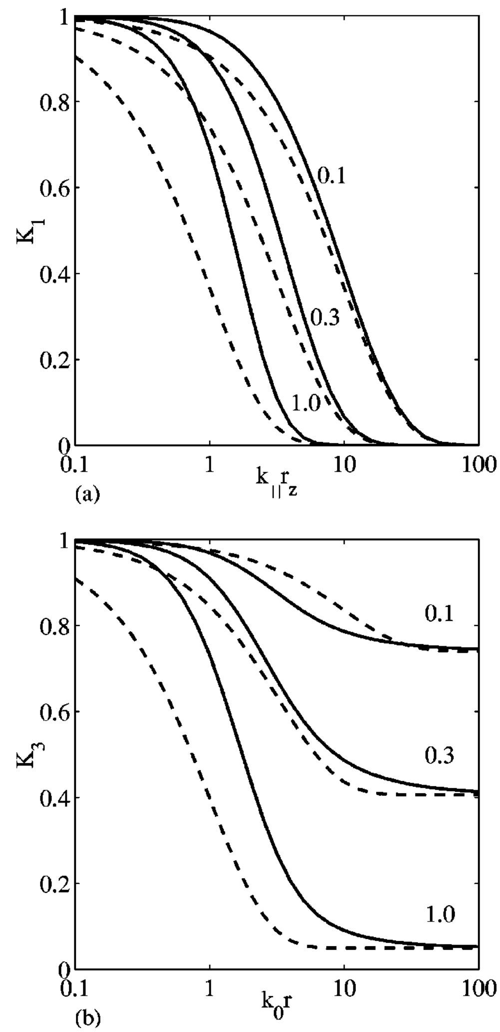

FIG. 1. Decreasing parts of the correlation functions corresponding to the 1D (a) and 3D (b) inhomogeneities for the different values of γ_1^2 and γ_3^2 that are shown at the curves. The solid curves correspond to exact expressions for $K_1(r_z)$ and $K_3(\mathbf{r})$, the dashed curves correspond to the approximate expressions for these functions given by Eqs. (11) and (12), respectively. Note that the scales along the abscissa axes are logarithmic.

III. DISPERSION LAW AND DAMPING OF WAVES

We consider the equation for waves in the superlattice in the form

$$\nabla^2\mu+[\nu-\varepsilon\rho(\mathbf{x})]\mu=0, \tag{14}$$

where the expressions for the parameters ε and ν and the variable μ are different for waves of different natures. For spin waves, when the parameter of the superlattice $A(\mathbf{x})$ in Eq. (1) is the value of the magnetic anisotropy $\beta(\mathbf{x})$, we have[15]

$$\nu=\frac{\omega-\omega_0}{\alpha g M}, \quad \varepsilon=\frac{\gamma\beta}{\alpha}, \tag{15}$$

where ω is the frequency, $\omega_0=g(H+\beta M)$, g is the gyromagnetic ratio, α is the exchange parameter, H is the

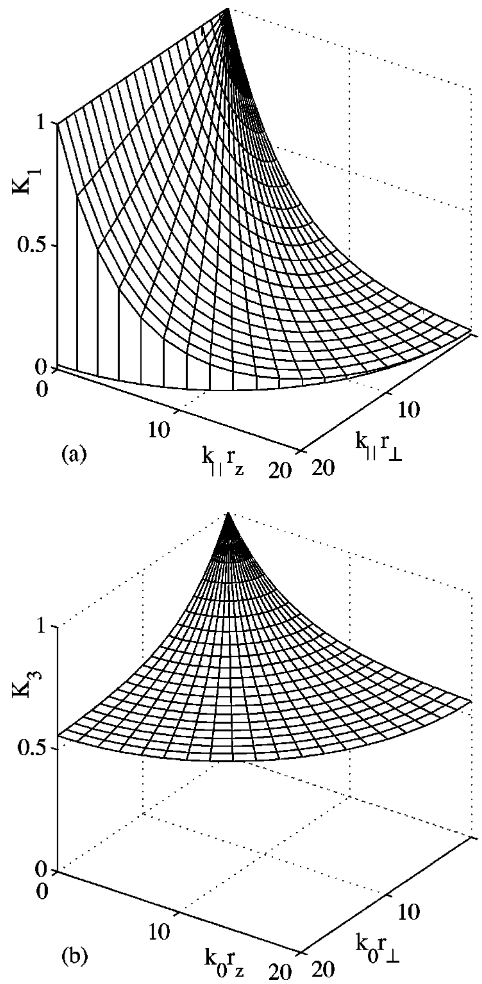

FIG. 2. Decreasing parts of the correlation functions for the 1D (a) and 3D (b) inhomogeneities in the r_z and $r_\perp=(r_x^2+r_y^2)^{1/2}$ coordinates for $\gamma_1^2=\gamma_3^2=0.3$.

magnetic-field strength, M is the value of the magnetization, β is the average value of the anisotropy, and γ is its relative rms variation. For elastic waves in the scalar approximation, we have

$$\nu=(\omega/v)^2, \quad \varepsilon=\gamma\nu, \tag{16}$$

where γ is the rms fluctuation of the density of the material and v is the wave velocity. For electromagnetic waves in the same approximation, we have

$$\nu=\varepsilon_e(\omega/c)^2, \quad \varepsilon=\gamma\nu, \tag{17}$$

where ε_e is the average value of the dielectric permeability, γ is its rms deviation, and c is the speed of light in vacuum.

Laws of the dispersion and damping of the averaged waves are determined by the equation for the complex frequency $\nu=\nu'+i\xi$, which follows from the vanishing of the denominator of the Green function of Eq. (14). In the Bourret approximation,[24] this equation has the form[21]

$$\nu-k^2=-\frac{\varepsilon^2}{4\pi}\int K(\mathbf{r})\exp[-i(\mathbf{kr}+\sqrt{\nu}r)]\frac{d\mathbf{r}}{r}. \tag{18}$$

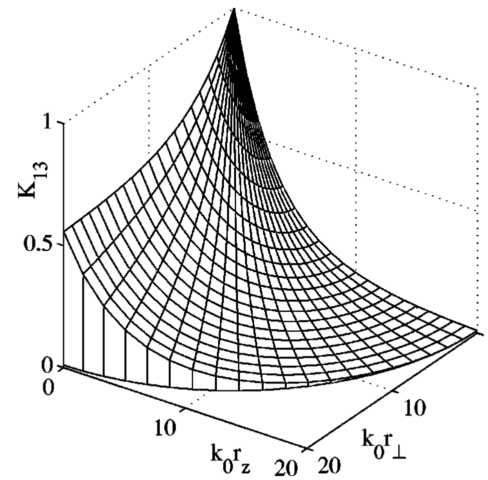

FIG. 3. Decreasing part of the correlation function of the mixture of the 1D and 3D inhomogeneities in the r_z and $r_\perp$ coordinates. It is assumed that $k_\parallel=k_0$, $\gamma_1^2=\gamma_3^2=0.3$.

Substituting Eq. (5) into Eq. (18) and approximating $K_1(r_z)$ and $K_3(r)$ by Eqs. (11) and (12), we obtain an exactly integrable expression. Upon integrating this expression with respect to $\mathbf{r}$, we obtain an explicit form of the equation for ν:

$$\nu-k^2=\frac{\varepsilon^2}{2}\left\{(1-L)\frac{P_{13}}{P_3}\left[\frac{1}{P_{13}^2-(k-q)^2}+\frac{1}{P_{13}^2-(k+q)^2}\right]+L\frac{P_1}{\sqrt{\nu}}\left[\frac{1}{P_1^2-(k-q)^2}+\frac{1}{P_1^2-(k+q)^2}\right]\right\}, \tag{19}$$

where

$$P_1=\sqrt{\nu}-ik_\parallel\gamma_1^2, \quad P_3=\sqrt{\nu}-ik_0\gamma_3^2,$$

$$P_{13}=\sqrt{\nu}-i(k_\parallel\gamma_1^2+k_0\gamma_3^2). \tag{20}$$

We consider this equation at the Brillouin-zone boundary $k=k_r\equiv q/2$. Under the conditions that ε, $(k_\parallel\gamma_1^2)^2$, and $(k_0\gamma_3^2)^2$ are much smaller than $\nu_r=k_r^2$, we obtain Eq. (19) in the form of a cubic equation in ν:

$$\nu-k_r^2=\frac{\varepsilon^2}{2}\left[\frac{1-L}{\nu-2ik_r(k_\parallel\gamma_1^2+k_0\gamma_3^2)-k_r^2}+\frac{L}{\nu-2ik_rk_\parallel\gamma_1^2-k_r^2}\right]. \tag{21}$$

Both limiting cases of this equation corresponding to 1D $(\gamma_1\neq 0,\gamma_3=0)$ and 3D $(\gamma_1=0,\gamma_3\neq 0)$ inhomogeneities have been considered in our previous works. For the 1D case, Eq. (21) reduces to a quadratic equation[15] and has the following solutions for the eigenfrequencies $\nu'_\pm$ and dampings $\xi_\pm$:

$$\nu'_\pm=k_r^2\pm\frac{\Lambda}{2}\mathrm{Re}[1-(\eta_1\gamma_1^2)^2]^{1/2}, \tag{22}$$

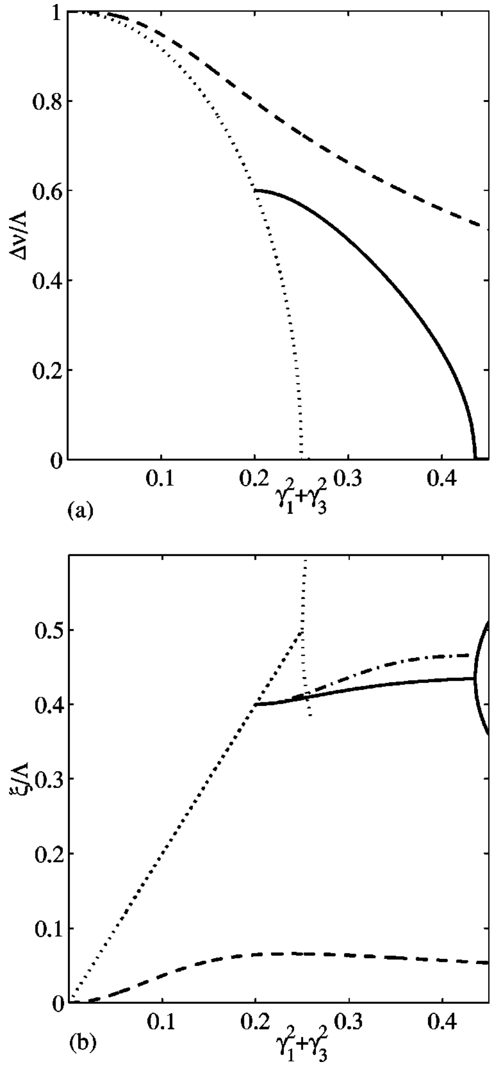

FIG. 4. The width of the gap (a) and the damping (b) as functions of the sum $\gamma_1^2+\gamma_3^2$ for different situations: $\gamma_1^2\neq 0,\ \gamma_3^2=0$ (dotted curves); $\gamma_1^2=0,\ \gamma_3^2\neq 0$ (dashed curves); $\gamma_1^2=0.2,\ \gamma_3^2\neq 0$ (solid curves). The explanation of the dotted-dashed curve in (b) is given in the text.

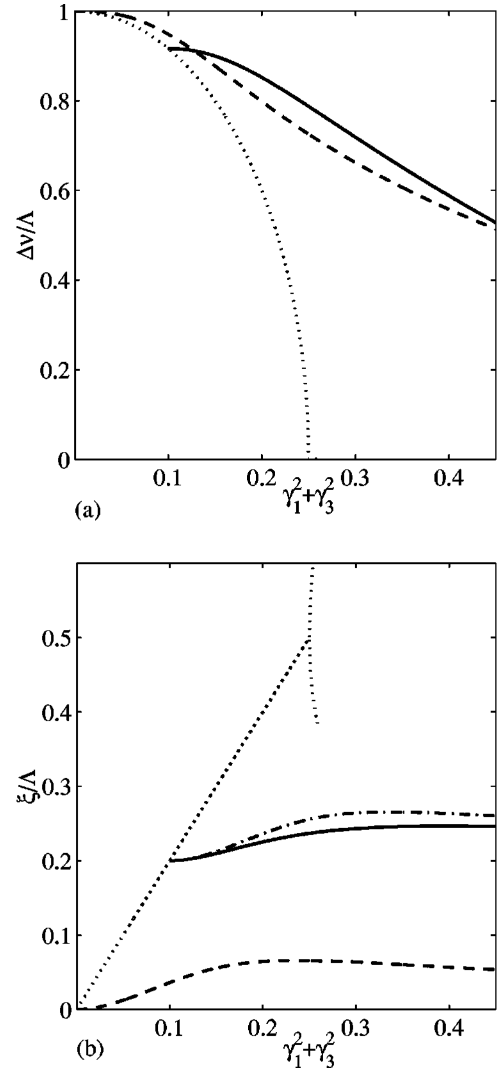

FIG. 5. The width of the gap (a) and the damping (b) as functions of the sum $\gamma_1^2+\gamma_3^2$ for different situations: $\gamma_1^2\neq 0,\ \gamma_3^2=0$ (dotted curves); $\gamma_1^2=0,\ \gamma_3^2\neq 0$ (dashed curves); $\gamma_1^2=0.1,\ \gamma_3^2\neq 0$ (solid curves). The explanation of the dotted-dashed curve in (b) is given in the text.

$$\xi_{\pm}=\frac{\Lambda}{2}\{\eta_1\gamma_1^2\pm \mathrm{Im}[1-(\eta_1\gamma_1^2)^2]^{1/2}\}, \tag{23}$$

where $\Lambda=\sqrt{2}\varepsilon$ is the width of the gap at the first Brillouin-zone boundary (forbidden bandwidth) for the ideal SL in the absence of any inhomogeneities, and $\eta_1=k_{\parallel}q/\Lambda$ is the normalized correlation wave number of the 1D inhomogeneities. For the 3D case, Eq. (21) remains a cubic equation and was investigated by numerical methods.[23]

General equation (21) for the mixture of 1D and 3D inhomogeneities has been investigated also by numerical methods. The results of this investigation are shown in Figs. 4–6 by solid curves. Dotted and dashed curves in these figures correspond to the limiting cases of the presence of only 1D or 3D inhomogeneities, respectively. All figures correspond to the same correlation wave numbers for 1D ($\eta_1\equiv k_{\parallel}q/\Lambda=4$) and 3D ($\eta_3\equiv k_0q/\Lambda=4$) inhomogeneities. Different situations are shown in these figures.

Figure 4(a) shows the decrease of the gap $\Delta\nu=\nu'_+-\nu'_-$ with the increase of γ_1^2 or γ_3^2. If $\gamma_3=0$, the increase of γ_1^2 leads to the closing of the gap at $\gamma_1^2=0.25$ (dotted curve). Simultaneously, the damping of both eigenfrequencies increases linearly till the point $\gamma_1^2=0.25$ [dotted curve in Fig. 4(b)]. For $\gamma_1^2>0.25$, two degenerate eigenfrequencies $\nu'_+=\nu'_-$ exist with different dampings, $\xi_+\neq\xi_-$. If $\gamma_1^2=0$, the increase of γ_3^2 also leads to the decrease of the gap [dashed curve in Fig. 4(a)] but significantly more slowly than under the action of the 1D inhomogeneities. For example, a large gap exists for $\gamma_3^2=0.25$, while the gap closes when γ_1^2 has the same value. In line with this, the damping increases very slightly with the increase of γ_3^2 [dashed curve in Fig. 4(b)].

To show the effects of the mixture of inhomogeneities of different dimensionalities, the following situation is depicted in Figs. 4(a,b). Let us have only 1D inhomogeneities with $\gamma_1^2=0.2$ and, correspondingly, the spectrum gap is $\Delta\nu/\Lambda$

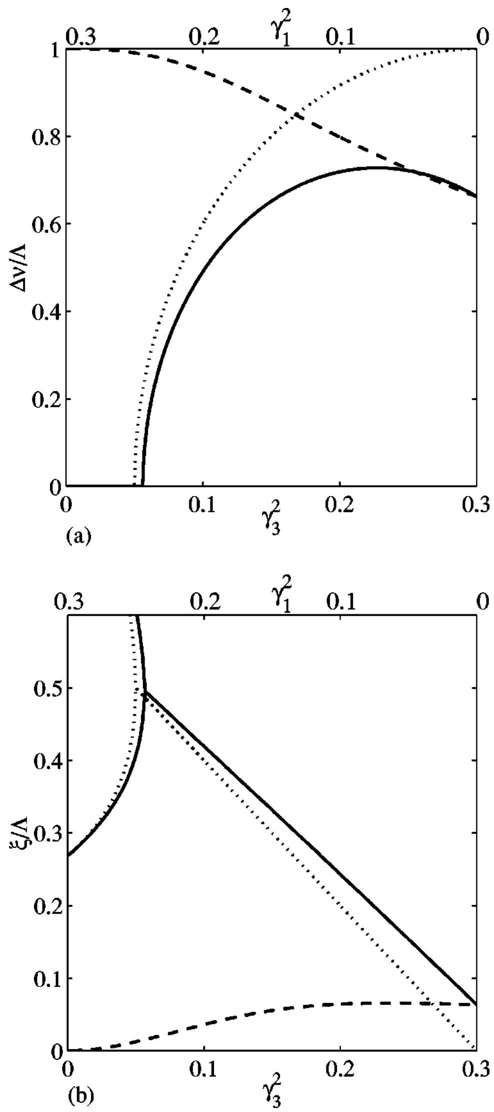

FIG. 6. The width of the gap (a) and the damping (b) under the condition $\gamma_1^2+\gamma_3^2=0.3$ (solid curves) and for the situations when γ_3^2 increases for $\gamma_1^2=0$ (dashed curves, the scale is under the figure), and when γ_1^2 decreases for $\gamma_3^2=0$ (dotted curves, the scale is above the figure).

$=0.6$. Then we add 3D inhomogeneities increasing γ_3^2 and keeping $\gamma_1^2=0.2$. One can see that the gap decreases slowly and closes at $\gamma_1^2+\gamma_3^2=0.45$ [solid curve in Fig. 4(a)]. Simultaneously, the increase of the damping slows down [solid curve in Fig. 4(b)]. The dashed-dotted curve in Fig. 4(b) corresponds to the unreal situation that would have been realized if the damping of the mixture of 1D inhomogeneities with $\gamma_1^2=0.2$ and 3D inhomogeneities with γ_3^2 were equal to the simple sum of the damping of the components of the mixture. One can see that in reality, the additional contribution to the damping due to 3D inhomogeneities in the presence of the 1D inhomogeneities is approximately two times smaller than in the absence of the latter.

In Figs. 5(a,b) the same situations as in Figs. 4(a,b) are depicted, but the value $\gamma_1^2=0.1$ instead of $\gamma_1^2=0.2$ is chosen for the 1D component of the mixture. One can see that the width of the gap for the mixture of inhomogeneities can be larger than the widths corresponding to the components of the mixture.

Quite another situation is shown in Figs. 6(a,b) by solid curves. We assume here that the sum $\gamma_1^2+\gamma_3^2$ remains constant (and equal to 0.3 in these graphs) when γ_1^2 and γ_3^2 are varied. In other words, we consider a gradual replacement of the 1D inhomogeneities by 3D inhomogeneities with the same values of rms fluctuations. For comparison, the functions $\Delta\nu$ and ξ are shown in Figs. 6(a,b) separately for the 1D and 3D inhomogeneities. The origin of the coordinates corresponds to $\gamma_3^2=0$ (the scale is under the figure) and $\gamma_1^2=0.3$ (the scale is above the figure). The width $\Delta\nu$ of the gap is equal to zero for the 1D inhomogeneities and to Λ for the 3D inhomogeneities. The dashed curve in Fig. 6(a) shows the decrease of $\Delta\nu$ when γ_3^2 increases for $\gamma_1^2=0$. The dotted curve in this figure shows the opening and increase of $\Delta\nu$ when γ_1^2 decreases for $\gamma_3^2=0$. The solid curve shows the dependence of $\Delta\nu$ on γ_3^2 under the condition $\gamma_1^2+\gamma_3^2=0.3$. One can see that the maximum of $\Delta\nu$ corresponds to some point corresponding to the presence of both components of the mixture ($\gamma_1^2\neq 0, \gamma_3^2\neq 0$) but not to the absence of the 1D inhomogeneities ($\gamma_1^2=0, \gamma_3^2=0.3$), as might be expected from the general point of view.

IV. CONCLUSION

The method of the random spatial modulation of the superlattice period[15] permits developing the CF of a SL with a 1D random modulation (which models random displacements of the interfaces from their initial periodic arrangement), 3D modulation (which models random deformations of the interfaces), and the simultaneous presence of both kinds of these modulations (which models the mixture of the 1D and 3D inhomogeneities of the SL structure).

For the initially sinusoidal SL, the CF in all these cases is a product of the same harmonic function $\cos(qr_z)$ and a decreasing function that has different forms for the different cases. The main difference between the CF's for 1D and 3D inhomogeneities is that the decreasing function goes to zero when $r_z\to\infty$ in the 1D case, while the decreasing function in the 3D case goes to the nonzero asymptotic value $L=\exp(-3\gamma_3^2)$ when $r\to\infty$. Because of this, the 1D inhomogeneities have a finite correlation radius $r_1=(\gamma_1^2k_\parallel)^{-1}$ in the entire volume of the superlattice, while for the 3D case volumes with a finite correlation radius $r_3=(\gamma_3^2k_0)^{-1}$ exist side by side with volumes with an infinite correlation radius. If we set the total volume equal to unity, the contribution of the volumes with the infinite correlation radius is proportional to L, while the contribution of the volumes with the finite correlation radius is $1-L$. In the 1D case, the transition to the ideal periodic structure with the decrease of the rms fluctuation γ_1 is achieved by increasing the correlation radius r_1; it is the usual transition from a random to a periodic system. In the 3D case, we have another kind of transition. The correlation radius r_3 increases with the decrease of γ_3 as in the 1D case. But in parallel with this, the changing of the relationship between the volumes with finite and infinite correlation radii goes on with the decrease or increase of the disorder. This leads to the essential differences between the effects on the wave spectrum that are due to the 1D or 3D inhomogeneities (the much smaller damping of waves in the 3D case, and so on).

The decreasing part of the CF of the SL in the presence of the mixture of the 1D and 3D inhomogeneities has the form of the product of the decreasing parts of the CF's of the components of the mixture $K_1(r_z)$ and $K_3(\mathbf{r})$. This leads to the conclusion that the general form of the CF of the mixture is determined mainly by the 1D inhomogeneities. The changing of the relationship between the volumes of the medium with and without the 3D correlations is going on with the changing of γ_3 in this case as well. However, the 1D correlations are present in both of these volumes. As a consequence of this, the CF of the mixture goes to zero when $r \to \infty$ for all directions except $\theta = \pi/2$ as in the 1D case; the presence of the 3D inhomogeneities leads only to a decrease of the effective correlation radii of the mixture along all coordinate axes.

The dependence of the widths of the gap in the spectrum and the damping of waves on the relationship between rms fluctuations γ_1 and γ_3 of the 1D and 3D inhomogeneities have been studied at the boundary of the first Brillouin zone. On addition of the 3D inhomogeneities to the SL containing only 1D inhomogeneities, the damping of waves increases. But this additional damping is approximately half as large as the damping that is due to the inhomogeneities with the same value of γ_3^2 in the absence of the 1D inhomogeneities. The situation has also been considered when a gradual changing of inhomogeneities of one dimensionality by inhomogeneities of the other dimensionality subject to the condition $\gamma_1^2 + \gamma_3^2 = \text{const}$ is occurring. It has been shown that the maximum value of the gap corresponds to some relationship between γ_1^2 and γ_3^2 but not to $\gamma_1^2 = 0$ as one could expect from general considerations. This phenomenon as well as the phenomenon of the reduction of the damping induced by the 3D inhomogeneities in the presence of 1D inhomogeneities are due to the fact that the decreasing parts of the CF's of the components of the mixture $K_1(r_z)$ and $K_3(\mathbf{r})$, as for the mixture of any phase inhomogeneities, enter into the CF of the SL in the form of a product, not a sum.

For the experimental observations of the effects predicted in this paper, it is desirable to create model SL's in which the dimensionalities and all parameters of the inhomogeneities of the structure are known. Not only the sinusoidal SL's that have been considered in this paper but also SL's with a rectangular initial profile could be used for this purpose, because the effects in the spectrum at the boundary of the first Brillouin zone for both of these models of SL's differ insignificantly.[23] All the effects considered in this paper could be observed on the three types of the model samples: samples with controlled random deviations of the plane interfaces from their initial periodic arrangement; samples with controlled deformations of the surfaces of the interfaces preserving their periodic arrangement; and samples with the simultaneous presence of both of these types of inhomogeneities. The results of such model experiments could be used for the determination by spectral methods of the dimensionalities, rms fluctuations γ_1 and γ_3, and correlation radii $k_\parallel$ and k_0 in real SL's for which these characteristics are unknown.

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