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Three-Dimensional Rectification of a Gradient Force in a Strong Nonmonochromatic Field

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Abstract—A new three-dimensional scheme for rectifying a gradient force is proposed and analyzed. The scheme is based on the use of a strong, partially coherent optical field involving a component with a fluctuating phase. It is shown that the rectification of a gradient force acting on atoms with a nondegenerate ground state is a second-order effect with respect to field strength in this scheme, whereas an analogous effect is third-order in coherent bichromatic fields. Conditions for three-dimensional confinement of atoms are obtained by using the velocity dependence of the rectified radiative force. For a large class of atoms, such as even-even isotopes of ytterbium and alkaline-earth elements, these conditions can be implemented at a relatively high effective temperature (of the particle ensemble) of about 10 K. This finding can be used to widen substantially the range of energies of atoms amenable to effective three-dimensional optical manipulation. © *2004 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

Resonant atoms placed in a strong nonmonochromatic nonuniform optical field (as a standing wave) are subjected to a strong rectified gradient force (RGF) [1–5]. Its sign remains invariant over a distance much larger than the optical wavelength λ. The force does not saturate with increasing field intensity (in contrast to spontaneous radiation pressure).

Various theoretical aspects and implementation schemes for rectifying radiative forces were considered in numerous studies (e.g., see [6–11] and references cited therein). In particular, their results suggest that RGF can be used to create extremely deep potential wells [12] and provide dissipative optical traps for confining relatively "hot" atoms with energies well above typical lower limits for laser cooling. Their practical implementation can substantially widen the range of energies of atoms amenable to effective three-dimensional optical manipulation. However, optimization of necessary physical conditions must rely on an analysis of three-dimensional models of rectification that allow for polarization phenomena in mechanical effects of light [4].

In this paper, we propose and analyze a new three-dimensional scheme for rectifying a gradient force in a strong nonmonochromatic field involving a component with a fluctuating phase. The analysis is performed for atoms with $J = 0 \longrightarrow J = 1$ transitions (as even–even Yb and alkaline-earth isotopes), which are deemed promising for new experiments on laser cooling (e.g., see 13–16] and references cited therein). In this scheme, the effects due to the RGF and the delayed gradient force (radiative friction) are only of sixth order (!) in the amplitude of the acting field in the limit case of weakly saturated population of excited levels when a coherent field is used [1–4]. For this reason, analysis is complicated and the radiative force has to be modified. The scheme differs from those with atoms with degenerate ground states [8, 17].

We show that rectification of a gradient force in a strong, partially coherent field is a fourth-order effect with respect to the field amplitude (i.e., a second-order one in intensity). We derive expressions for RGF and delayed gradient force (DGF) in a 3D nonmonochromatic field and use them to determine conditions for stable 3D confinement of resonant particles with an effective temperature T of at least several kelvins (much higher than the known lower limits for laser cooling in similar problems).

We note that the opposite limit case of weak coherent bichromatic field and particles with $T \ll 1$ K was considered in previous studies [18, 19] (also devoted to three-dimensional rectification of radiative forces for atoms with strong singlet–singlet transitions and weak $J = 0 \longrightarrow J = 1$ transitions).

2. MODEL

Consider an atom of mass m moving with velocity $\mathbf{v}$ in an electromagnetic field

$$E(\mathbf{r}, t)e^{-i\omega_0 t} + \text{c.c.}$$

with carrier frequency ω_0 tuned to resonance with the $|J_g = 0, M_g = 0\rangle \longrightarrow |J_e = 1, M_e = 0, \pm 1\rangle$ atomic transi-

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tion, where J_α is the total angular moment and M_α denotes its projections in the ground ($\alpha = g$) and excited ($\alpha = e$) states.

The field is the superposition of coherent quasi-resonant components with three different frequencies polarized in mutually perpendicular directions and a partially coherent (fluctuating) resonant field $\mathbf{E}'$ with a bandwidth Γ:

$$\mathbf{E}(\mathbf{r}, t) = \sum_{j = x, y, z} E_{j1}\mathbf{e}_j \exp[-i\Delta_j t] + \mathbf{E}'(\mathbf{r}, t), \quad (2.1)$$

where $\mathbf{e}_j$ denotes the unit basis vectors of a Cartesian coordinate system and Δ_j is the detuning from the resonant frequency ω_0.

In accordance with the original concept of gradient-force rectification [1], assume the following hierarchy of characteristic frequencies:

$$\begin{aligned} &|\Delta_j|, \quad |\Delta_j - \Delta_l| \gg |V_{j1}|, \\ &\Gamma \gg |U_j|, \quad \frac{|V_{j1}|^2}{\Delta_j}, \quad |\delta|, \end{aligned} \quad (2.2)$$

$$\frac{|V_{j1}|^2}{\Delta_j} \gg \gamma, \quad kv, \quad \frac{|U_j|^2}{\Gamma}, \quad (2.3)$$

where l and $j \neq l$ denote indices x, y, or z; $V_{j1}(\mathbf{r})$ and $U_j(\mathbf{r}, t)$ are the Rabi frequencies defined as

$$V_{j1} = \frac{dE_{j1}}{\hbar}, \quad U_j = \frac{d(\mathbf{e}_j\mathbf{E}'(\mathbf{r}, t))}{\hbar},$$

with $d = \|d\|/\sqrt{3}$ ($\|d\|$ is the reduced dipole transition matrix element); $k = \omega_0/c$ is the wave number; γ is the decay rate for the excited state; and δ is the fluctuating-component detuning from the resonant frequency. Inequality (2.2) implies that the coherent components of $\mathbf{E}_1$ are "quasi-resonant," i.e., give rise to a spatially nonuniform Stark shift, and the fluctuating component is "resonant," i.e., ensures excitation of the atom.[1] Condition (2.3) means that the coherent field $\mathbf{E}_1$ is sufficiently strong to ensure that the light-induced Stark shifts exceed the optical resonance width. The opposite limit of a weak coherent bichromatic field was considered in [18, 19]. Note that superposition (2.1) a fortiori admits a 3D acting-field configuration (cf. [1–3]).

An atom placed in field (2.1) is driven by the force [4, 20]

$$\mathbf{F} = \hbar\sum_j (\rho_j \nabla \hat{V}_j^* + \text{c.c.}), \quad (2.4)$$

where

$$\hat{V}_j(\mathbf{r}, t) = V_{j1}\exp(-i\Delta_j t) + U_j,$$

and ρ_j denotes the projections of the induced dipole moment measured in d, which are determined by solving the optical Bloch equations written for a prescribed unperturbed classical trajectory $\mathbf{r} = \mathbf{v}t$. In the "Cartesian" representation adapted to the present problem [4, 19, 21], these equations and the expression for the force are averaged over oscillations of frequency Δ_j (cf. [22]) to obtain (using the same notation for averaged quantities)

$$\begin{aligned} &i\left(\frac{d}{dt} + \gamma_\perp - i\hat{\Delta}_i(\mathbf{r})\right)\rho_i = \sum_j q_{ij}U_j, \\ &j, i = x, y, z, \\ &i\left(\frac{d}{dt} + \gamma - i\hat{\Delta}_{ij}(\mathbf{r})\right)q_{ij} = -i\gamma\delta_{ij} \\ &+ (\rho_i U_j^* - U_i\rho_j^*) - \delta_{ij}\sum_{l = x, y, z}(\rho_l^* U_l - \text{c.c.}), \end{aligned} \quad (2.5)$$

$$\mathbf{F} = \hbar\sum_j (\rho_j \nabla U_j^* + \text{c.c.}) + \hbar\sum_j q_{jj}\frac{\nabla|V_{j1}|^2}{\Delta_j}, \quad (2.6)$$

where q_{ij} is the population difference between the excited and ground states, q_{ij} (with $i \neq j$) characterize the coherence of excited atomic states,

$$\hat{\Delta}_i(\mathbf{r}) = \frac{2|V_{i1}(\mathbf{r})|^2}{\Delta_i} + \sum_{l \neq i}\frac{|V_{l1}(\mathbf{r})|^2}{\Delta_l}$$

are the effective spatially nonuniform detunings due to light-induced Stark shifts, and

$$\hat{\Delta}_{ij}(\mathbf{r}) = \hat{\Delta}_i(\mathbf{r}) - \hat{\Delta}_j(\mathbf{r}).$$

Next, Eqs. (2.5) and (2.6) are averaged over fluctuations of $\mathbf{E}'$.[2] Bloch equations (2.5) constitute a system of multiplicative linear equations, and the averaging over the ensemble of random processes U_j conditioned on the right-hand inequality in (2.2) can be performed by using the expansions of their solution in terms of $\zeta \ll 1$, which is proportional to the autocorrelation time $\tau_c \sim \Gamma^{-1}$ [25]:

$$|U_j|\tau_c, \quad |\hat{\Delta}_j|\tau_c, \quad kv\tau_c, \quad \gamma\tau_c \leq \zeta \ll 1.$$

By assuming that $\langle\langle U_j \rangle\rangle = 0$ and the $\mathbf{E}'$ components with different polarizations fluctuate independently, i.e.,

$$\langle\langle U_j(\mathbf{r}, t)U_i(\mathbf{r}, t + \tau)\rangle\rangle = \langle\langle U_j(\mathbf{r}, t)U_i^*(\mathbf{r}, t + \tau)\rangle\rangle = 0,$$
$$i \neq j.$$

[1] In the scheme considered in [1], this is achieved by using a "controlling" coherent field component with a small detuning.

[2] In the theory of resonant radiation pressure, radiative forces due to fluctuating fields with finite bandwidths were originally considered in [23, 24].

Eqs. (2.5) and (2.6) are reduced to

$$\bar{\mathbf{F}} = \langle\langle \mathbf{F} \rangle\rangle = \mathbf{F}_g + \mathbf{F}_s,$$

$$\mathbf{F}_g = \sum_i \frac{\nabla |V_{i1}|^2}{\Delta_i} Q_i, \tag{2.7}$$

$$\mathbf{F}_s = -\hbar i$$

$$\times \sum_i Q_i \left(\int_{-\infty}^{0} d\tau \langle\langle \nabla U_i^*(\mathbf{r}, t) U_i(\mathbf{r}, t+\tau) \rangle\rangle - \text{c.c.} \right), \tag{2.8}$$

$$\left[\frac{d}{dt} + \gamma + 2R_i(\mathbf{r})\right] Q_i + \sum_{l \neq i} R_e(\mathbf{r}) Q_e = -\gamma, \tag{2.9}$$

where $Q_i = \langle\langle q_{ii} \rangle\rangle$ and the rate constants for transitions between the ground and excited atomic states induced by the field **E**' are determined by the correlators

$$R_j(\mathbf{r}) = 2\text{Re} \int_{-\infty}^{0} \langle\langle U_j(\mathbf{r}, t) U_j^*(\mathbf{r}, t+\tau) \rangle\rangle d\tau. \tag{2.10}$$

Note also that U_j is treated as a stationary random process and only first-order terms in $\zeta \ll 1$ are retained in the reduced equations.

Equations (2.7)–(2.9) show that, under condition (2.3) of strong quasi-resonant field,

$$|V_{j1}^2/\Delta_{j1}| \gg |U_e|^2/\Gamma,$$

the radiative force $\mathbf{F}_s$ exerted on the atom by the fluctuating field is weak as compared to the gradient force $\mathbf{F}_g$, which is proportional to the sum of the population differences multiplied by the gradients of E_{j1} components:

$$|\mathbf{F}_s| \ll |\mathbf{F}_g|.$$

Accordingly, Eqs. (2.7)–(2.9) expose the roles played by the fields $\mathbf{E}_1$ and **E**' in the present model. The fluctuating field **E**' is responsible for incoherent mixing of atomic states, and the quasi-resonant coherent field $\mathbf{E}_1$ induces the effective potentials that determine the motion of the atom: the excited atom moves in the field with

$$|V_{i1}(\mathbf{r})|^2/\Delta_{i1}, \quad i = x, y, z;$$

the unexcited one, in the field with

$$-\sum_i |V_{i1}(\mathbf{r})|^2/\Delta_{i1}, \quad i = x, y, z.$$

An analogous model (in the basis of adiabatic states) describes a two-level atom moving in a coherent bichromatic field [3, 22]. It is obvious that a rectified force

$$\mathbf{F}_R = \langle \mathbf{F}_g \rangle = \sum_i \left\langle Q_i \frac{\nabla |V_{i1}|^2}{\Delta_i} \right\rangle \neq 0 \tag{2.11}$$

exists when $R_i = R_i(\mathbf{r})$ (transition rates are spatially modulated), which is possible only if the coherent field has mutually interfering components. (Hereinafter, angle brackets denote averaging over oscillations with periods comparable to the optical wavelength.)

Finally, note that Eqs. (2.9), where the effect of $\mathbf{E}_1$ on transition saturation is ignored, are derived under conditions (2.2) and (2.3) supplemented with a refined quasi-resonance condition for $\mathbf{E}_1$:

$$\left|\frac{V_{j1}}{\Delta_j}\right|^2 \sim g_j^2 \ll \frac{R_j}{\gamma}.$$

This makes it possible to restrict analysis to the first approximation (i.e., Eqs. (2.5)) in averaging the original Bloch equations over oscillations with frequencies comparable to Δ_j (higher order approximations for a related problem were discussed in [22]).

3. RECTIFIED GRADIENT FORCE AND THREE-DIMENSIONAL CONFINEMENT

To obtain expressions for the RGF, we specify the fields **E**' and $\mathbf{E}_1$ as superpositions of plane waves ($j = x, y, z$):

$$U_j(\mathbf{r}, t) = \frac{U}{2}$$

$$\times \{ \exp(i\phi_j(t))[\exp(i\mathbf{k}_{j1} \cdot \mathbf{r}) + \exp(i\mathbf{k}_{j2} \cdot \mathbf{r})] \tag{3.1}$$

$$+ \exp(i\psi_j(t))[\exp(-i\mathbf{k}_{j1} \cdot \mathbf{r}) + \exp(-i\mathbf{k}_{j2} \cdot \mathbf{r})] \},$$

$$V_{j1}(\mathbf{r}) = \frac{V_j}{2}[\exp(i(\mathbf{q}_{j1} \cdot \mathbf{r}) + \eta_{j1}) \tag{3.2}$$

$$+ \exp(i(\mathbf{q}_{j2} \cdot \mathbf{r} + \eta_{j2}))],$$

where V_j and $\eta_{j\alpha}$ are the amplitudes and initial phases of the coherent field components, and $\phi_j(t)$ and $\psi_j(t)$ are independent fluctuating phases (with delta-correlated zero-mean derivatives), which determine the correlators of **E**' components by the relations

$$\langle\langle \exp i[\phi_j(t) - \phi_j(t+\tau)] \rangle\rangle$$

$$= \langle\langle \exp i[\psi_j(t) - \psi_j(\tau+\tau)] \rangle\rangle = \exp(-\Gamma|\tau|), \tag{3.3}$$

$$\langle\langle \exp i[\psi_j(t) - \phi_j(t+\tau)] \rangle\rangle = 0$$

in a model of radiation with phase diffusion [23, 26].

Thus, each Cartesian component of **E**' consists of two independent fluctuating components. Their structure implies that $\mathbf{F}_s = 0$ (in approximation (2.8)), and the field **E**' has a Lorentzian spectral profile with bandwidth Γ:

$$I(\omega) \propto \frac{2\Gamma}{(\omega - \omega_0)^2 + \Gamma^2}.$$

Note that representation (3.1) in the region occupied by atoms is valid only if the coherence length

$$l_c = c\tau_c = c/\Gamma$$

is much greater than the diameter of the region:

$$l_c \gg b$$

(see [23]). Moreover, if correlated light beams with wave vectors $\mathbf{k}_{j1}$ and $\mathbf{k}_{j2}$ are obtained from the same source by using an appropriate optical system, then the optical path difference between them must also be much smaller than l_c.

The vectors $\mathbf{k}_{j\alpha}$ and $\mathbf{q}_{j\alpha}$ in (3.1) and (3.2), with the magnitudes

$$|\mathbf{k}_{j\alpha}| = k = \omega_0/c, \quad |\mathbf{q}_{j\alpha}| = q_j = |\omega_0 + \Delta_j|/c$$

lie in the planes perpendicular to the corresponding basis vectors of the Cartesian coordinate system:

$$\mathbf{k}_{j\alpha}\cdot\mathbf{e}_j = \mathbf{q}_{j\alpha}\cdot\mathbf{e}_j = 0.$$

To be specific, suppose that

$$\begin{aligned}\Delta\mathbf{q}_x &= \Delta q\mathbf{e}_y, \quad \Delta\mathbf{q}_y = \Delta q\mathbf{e}_z, \quad \Delta\mathbf{q}_z = \Delta q\mathbf{e}_x,\\ \Delta\mathbf{k}_x &= \Delta k\mathbf{e}_y, \quad \Delta\mathbf{k}_y = \Delta k\mathbf{e}_z, \quad \Delta\mathbf{k}_z = \Delta k\mathbf{e}_x,\end{aligned} \tag{3.4}$$

$$|\delta k| \ll |\Delta k|, \quad |\Delta q|, \tag{3.5}$$

where

$$\delta k = \Delta q - \Delta k, \quad \Delta\mathbf{q}_j = [\mathbf{q}_{j2} - \mathbf{q}_{j1}]/2,$$

$$\Delta\mathbf{k}_j = [\mathbf{k}_{j2} - \mathbf{k}_{j1}]/2,$$

and the values of Δ_q, Δk, and δk are determined by prescribing the angles β and β_j between the wave propagation directions in (3.1) and (3.2), i.e., between the pairs $\{\mathbf{k}_{j2}, \mathbf{k}_{j1}\}$ and $\{\mathbf{q}_{j2}, \mathbf{q}_{j1}\}$:

$$\Delta k = k\sin(\beta/2), \quad \Delta q_j = q_j\sin(\beta_j/2).$$

Consequently, the "microscopic" and "macroscopic" length scales, λ_M and Λ ($\lambda_M \ll \Lambda$, see [3, 22]), are estimated as $\lambda_M = \pi/\Delta q \sim \lambda$ and $\Lambda = \pi/\delta k$ in this problem and are parameters that can be adjusted by choosing values β and β_j. The optical field configuration is schematized in the figure.

Expressions for the transition rates $R_j(\mathbf{r})$ and the effective potentials $|V_{j1}(\mathbf{r})|^2/\Delta_j$ are obtained by combining (2.10), (3.1), and (3.2):

$$\begin{aligned}R_j(\mathbf{r}) &= R\cos^2(\Delta\mathbf{k}_j\cdot\mathbf{r}), \quad R = 4|U|^2/\Gamma,\\ \frac{|V_{j1}(\mathbf{r})|^2}{\Delta_j} &= \frac{|V_j|^2}{\Delta_j}\cos^2[\Delta\mathbf{q}_j\cdot\mathbf{r} + \xi_j], \quad \xi_j = [\eta_{j2} - \eta_{j1}]/2.\end{aligned} \tag{3.6}$$

When the transition is not saturated, i.e.,

$$4R/\gamma < 1,$$

the steady-state solution to Eqs. (2.9) (at $t > \gamma^{-1}$) can be represented as a convergent series in powers of the $\mathbf{E}'$ wave intensity:

$$Q_j = \sum_{j=0}^{\infty} Q_j^{(n)}, \quad Q_j^{(0)} = -1, \tag{3.7}$$

where $Q_j^{(n)}(\mathbf{r})$ are defined by the recursive relations

$$\begin{aligned}Q_j^{(n)}(\mathbf{r}) = -\int_{-\infty}^{0}\Bigg[&2R_j(\mathbf{r}+\mathbf{v}\tau)Q_j^{(n-1)}(\mathbf{r}+\mathbf{v}\tau)\\ &+ \sum_{l\neq j} R_l(\mathbf{r}+\mathbf{v}\tau)Q_l^{(n-1)}(\mathbf{r}+\mathbf{v}\tau)\Bigg]e^{\gamma\tau}d\tau.\end{aligned} \tag{3.8}$$

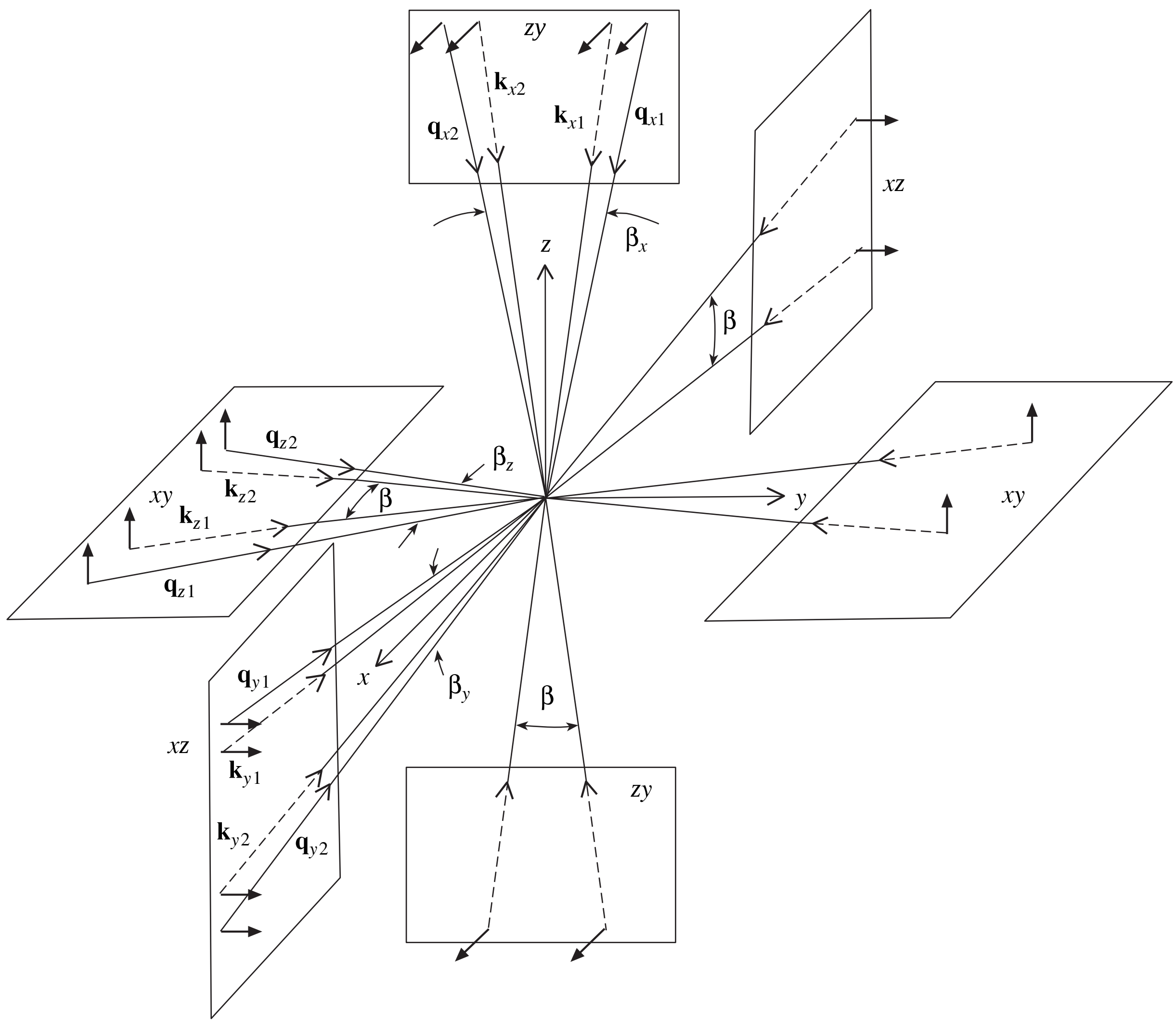
In the linear approximation with respect to the $\mathbf{E}'$ wave intensity, (3.7) and (3.8) yield an expression for the population difference,

$$\begin{aligned}Q_j(\mathbf{r}) \approx &\frac{\gamma^2 R}{(\gamma^2 + 4(\Delta\mathbf{k}_j\cdot\mathbf{v})^2)}\\ &\times\left[\frac{1}{\gamma}\cos(2\Delta\mathbf{k}_j\cdot\mathbf{r}) + \frac{2\Delta\mathbf{k}_j\cdot\mathbf{v}}{\gamma^2}\sin(2\Delta\mathbf{k}_j\cdot\mathbf{r})\right]\\ &+ \sum_{l\neq j}\frac{\gamma^2 R}{2(\gamma^2 + 4(\Delta\mathbf{k}_l\cdot\mathbf{v})^2)}\\ &\times\left[\frac{1}{\gamma}\cos(2\Delta\mathbf{k}_l\cdot\mathbf{r}) + \frac{2\Delta\mathbf{k}_l\cdot\mathbf{v}}{\gamma^2}\sin(2\Delta\mathbf{k}_l\cdot\mathbf{r})\right],\end{aligned}$$

which can be combined with (2.7), (3.4), and (3.5) to find the rectified radiative force (after averaging $\bar{\mathbf{F}}$ over spatial oscillations of period λ_M):

$$\begin{aligned}\mathbf{F}_R &= \langle\bar{\mathbf{F}}\rangle = \sum_i (F_{0i} + F_{1i})\mathbf{e}_i,\\ F_{0i} &= -\frac{\hbar\Delta k\Delta_j}{1 + (v_i/v_c)^2}\frac{R}{2\gamma}g_j^2\sin\Phi_i,\\ \Phi_i &= 2\delta k r_i + 2\xi_j,\\ F_{1i} &= -\frac{m\chi_i v_i}{1 + (v_i/v_c)^2}, \quad \chi_i = \kappa_i\cos\Phi_i,\\ \kappa_i &= \frac{\hbar\Delta k^2 g_j^2 R\Delta_j}{m\gamma^2}.\end{aligned} \tag{3.9}$$

In accordance with (3.4), the pairs of indices (i, j) are



Three-dimensional optical field configuration corresponding to superposition (3.1), (3.2) and satisfying conditions (3.4): long dashed and solid arrows indicate the directions of propagation of partially coherent and coherent waves (with wave vectors $\pm\mathbf{k}_{j1}$, $\pm\mathbf{k}_{j2}$ and $\mathbf{q}_{j1}$, $\mathbf{q}_{j2}$), respectively; short arrows, polarization directions; β and β_j are angular widths.

(x, z), (y, x), or (z, y),

$$g_j^2 = |V_j/\Delta_j|^2,$$

$$r_i = \mathbf{e}_i \cdot \mathbf{r}, \quad r_x = x, \quad r_y = y, \quad r_z = z,$$

$$v_i = \mathbf{e}_i \cdot \mathbf{v}, \quad v_c = \gamma/2\Delta k,$$

$\mathbf{F}_0$ is the rectified gradient force, and $\mathbf{F}_1$ is the delayed gradient force (radiative friction) (by the terminology of [4]).

It is clear from (3.9) that both RGF and DGF are second-order quantities with respect to field strength here, whereas third-order analogous quantities are obtained in coherent bichromatic fields [3, 4]. The velocity dependence of RGF has a Lorentzian profile with a width determined by the “microscopic” length scale:

$$v_c = \gamma\lambda_M/2\pi.$$

When $v_i \gg v_c$, the RGF scales with the inverse square of particle velocity; when $v_i \ll v_c$, it is virtually independent of the velocity. In the latter case, macroscopic potential wells are created, with depths greater than the characteristic depth $\hbar|V_{j1}|^2/\Delta_j$ of microscopic potential wells. Note also that the DGF is a nonlinear function of both velocity and coordinate of the atom.

It is remarkable that the “macroscopic” motions of particles along the axes of the Cartesian coordinate system induced by RGF and DGF are mutually independent. When

$$mv_i^2/2 = T_i \gg \hbar|V_{j1}|^2/\Delta_j,$$

they are governed by the equations

$$m\left[1+\frac{v_i^2}{v_c^2}\right]\frac{dv_i}{dt} = -\frac{\partial\Pi(r_i)}{\partial r_i} - m\kappa v_i\cos\Phi_i, \qquad (3.10)$$
$$\frac{dr_i}{dt} = v_i, \quad i = x, y, z,$$

i.e., by the Newton equations with a "renormalized" (velocity-dependent) mass, where

$$\Pi(r_i) = \Pi_0(1-\cos\Phi_i), \quad \Pi_0 = \hbar\omega_0\frac{R\Lambda\Delta k}{4\pi ck}\sqrt{\frac{Ig^2}{I_s}},$$

$I_s = \hbar\omega_0\gamma k^2/6\pi$ is the wave intensity that saturates the atomic transition, $I = I_x$, $g^2 = g_x^2$, I_i denotes the intensity of a plane-wave component in superposition (3.2) polarized along $\mathbf{e}_j$, and $\Delta k > 0$. Furthermore, all detunings $\Delta_j > 0$ induced by the RGF also supposed to be similarly distributed along each Cartesian coordinate axis:

$$|V_l|^2/\Delta_l = |V_i|^2/\Delta_i.$$

Therefore,

$$I_l g_l^2 = I_i g_i^2 = Ig^2$$

for every pair of indices l and i, and

$$\kappa_i \equiv \kappa.$$

Thus, the model of three-dimensional confinement is reformulated as a nonlinear model of one-dimensional motion. Under the conditions $\Delta_j > 0$, the minima of the potential $\Pi(r)$ are found by solving the equation

$$\cos\Phi_i(r_m) = 1.$$

It is clear that each point A_m with phase-space coordinates ($r_i = r_m$, $v_i = 0$) is a stable stationary point (attractor) of system (3.10). However, a particle moving in the vicinity of the RGF node located at r_m is confined in its region of attraction G_m only if its kinetic energy $mv_i^2/2 = T_i$ does not exceed a certain critical T_k determined not only by the potential-barrier height $2\Pi_0$, but also by the profile width of the RGF as a function of velocity. If $mv_c^2/2 = T_c \ll \Pi_0$, then $T_k \ll \Pi_0$ (since the RGF rapidly decreases at $v_i \gg v_c$). If $T_c \gg \Pi_0$, then T_k is comparable to Π_0, but is substantially lower than T_c. Both G_m and T_k are difficult to determine because the sign of the friction coefficient depends on the particle's location.

Let us find sufficient conditions for three-dimensional confinement of atoms and estimate T_k, using the fact that DGF plays the role of friction only in the regions Ω_m where

$$\kappa\cos\Phi_i > 0, \qquad (3.11)$$

i.e., when

$$r_i \in (r_m - \pi/4\delta k, r_m + \pi/4\delta k).$$

Define the generalized energy

$$\mathcal{E}(r_i, v_i) = T_i(1+T_i/2T_c) + \Pi(r_i) \qquad (3.12)$$

(when $\kappa = 0$, $\mathcal{E}$ is an integral of motion). Alternatively, $\mathcal{E}$ is interpreted as the Lyapunov function of system (3.10) in the phase-space domain N_m bounded by the closed contour $\mathcal{E}(r_i, v_i) = \Pi_0$ encompassing the attractor A_m. Indeed, (3.10) implies that its derivative along the trajectory $r_i = r_i(t)$, $v_i = v_i(t)$ almost everywhere in N_m (except for A_m) satisfies the differential inequality

$$\frac{d\mathcal{E}(r_i, v_i)}{dt} = -2\kappa T_i\cos\Phi_i < 0, \qquad (3.13)$$

because the condition for particle confinement inside N_m,

$$\mathcal{E}(r_i, v_i) < \Pi_0, \qquad (3.14)$$

entails (3.11) and, therefore, $r_i \in \Omega_m$. Note that the function $\mathcal{E}$ is positive definite everywhere in N_m except for A_m (where $\mathcal{E} = 0$).

Thus, every trajectory passing through N_m asymptotically approaches the point A_m as $t \longrightarrow \infty$, crossing the closed contours of constant $\mathcal{E}$ inwards, and inequality (3.14) is a sufficient condition for confinement of atoms at the nodes of RGF. Note that, even though $N_m \in G_m$ (i.e., N_m is just a subregion of the region of attraction of A_m, as shown numerically), the availability of analytical representation (3.14) facilitates analysis of the confinement conditions.

Condition (3.14) entails a constraint on the kinetic energy of particles and an estimate for T_k:

$$T_i < T_M = \frac{2\Pi_0}{1+\sqrt{1+2\Pi_0/T_c}} < T_k.$$

This means that an atom that passes through the RGF node located at r_m and has an energy not higher than T_M will be trapped in its vicinity. On the other hand, an atom with energy $T_i \ll T_M$ confined in a small neighborhood of an RGF node cannot be released from the region of attraction by a sudden perturbation (e.g., by a single collision with a "hot" particle) if the resulting increase in its energy is not greater than T_M.

Since

$$\Pi_0 \propto \Delta k/k, \quad \Pi_0/T_c \propto (\Delta k/k)^3,$$

the quantity $T_M = T_M(\Delta k)$ as a function of the parameter $\Delta k = \pi/\lambda_M$ reaches a maximum value T'_M, which can be expressed in a form suitable for estimation:

$$\max_{\Delta k} T_M = T'_M \approx \frac{T_0}{3.2}\left[\frac{\hbar\omega_0 Rg}{2\pi T_0 c}\right]^{2/3}\left[\frac{I\Lambda^2}{I_s}\right]^{1/3}, \quad (3.15)$$

where $T_0 = m\gamma^2/2k^2$.

The maximum is reached when

$$\Delta k/k = \sqrt{T_0/2T'_M} < 1$$

and is associated with a specific relation between the profile width of the RGF as a function of velocity and its magnitude:

$$\Pi_0 = 4T_c.$$

By solving Eq. (3.10) numerically, a simple relation is found:

$$T_k/T'_M = \eta \approx 2.07.$$

Note that the value of T_k is lower than $\max\Pi(r_i) = 2\Pi_0$ approximately by half in the optimal regime considered here.

Thus, under an optimal choice of the field configuration, the RGF can be used to confine particles in three-dimensional traps of size smaller than Λ if their effective temperature satisfies the condition

$$T_{\text{eff}} < T = 2\eta T'_M/k_B,$$

where k_B is the Boltzmann constant, T'_M is defined by (3.15), and $\eta \sim 2$.

It is important that T increases with the coherent-field intensity even when both g and R/γ are held constant. As an example consider an ytterbium atom with the $^1S_0$–$^1P_1$ singlet–singlet transition ($\lambda = 398.8$ nm, $\gamma = 1.8 \times 10^8$ s^{-1}). If $R/\gamma \approx 0.2$ and $g^2 \approx 0.05$ are taken as estimated values, then (3.15) yields a simple expression for the limit temperature (in kelvins) for atoms confined by means of the RGF:

$$T \approx 2\left[\frac{I\Lambda^2}{I_s}\right]^{1/3},$$

where I and I_s are measured in W/cm^2 and Λ in centimeters. In particular, if $\Lambda \approx 0.5$ cm, $I/I_s = 10^3$, and $\Lambda^2 I \approx 25$ W, then $T \approx 12$ K. In this case, $\Delta k/k \sim 0.38$, $\Delta_j \gtrsim 2 \times 10^{10}$ s^{-1}, and all starting conditions of the problem are satisfied if the fluctuating-field intensity is $I' \approx 5I_s$ and its bandwidth is $\Gamma \approx 5 \times 10^9$ s^{-1}. For comparison, note that $T_0 \approx 1.5$ K in the example considered here, whereas the lower temperature limit for confined atoms corresponding to quantum fluctuations of radiative forces does not exceed

$$T_1 \approx \hbar|V_{i1}|^2/\Delta_i \approx 0.01 \text{ K}.$$

4. CONCLUSIONS

The scheme of the ponderomotive effect of a strong, partially coherent field on atoms with a $J = 0 \longrightarrow J = 1$ transition analyzed here is remarkable in two respects. First, both rectified gradient force and friction force are second-order quantities with respect to the field intensity. Second, the light-induced motion of a particle is (on a macroscopic scale) a superposition of independent one-dimensional motions along three mutually orthogonal axes. Each of these motions is controlled only by field components having a certain polarization in the plane perpendicular to the direction of motion.[3] This finding can be used to simplify optical control of three-dimensional particle motion by independently varying the parameters and geometry of field components with mutually orthogonal polarizations.

In principle, the proposed scheme for rectifying the gradient force makes it possible to implement three-dimensional confinement of relatively “hot” particles with temperatures as high as several kelvins under an optimal choice of the optical field geometry and parameters. In particular, deep traps of this kind may help to solve the challenging problem of optical trapping of an ultracold electron–ion plasma with ions in resonance with laser light, because its electron subsystem may have a relatively high temperature of 1 to 10 K (even when its density is low) [27].

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