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Reflection Spectrum of a Cholesteric Liquid Crystal with Structural Defects

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An expression is obtained for the reflection coefficient of circularly polarized light normally incident on the film of a cholesteric liquid crystal with a variable helix pitch. It is shown that, in the case of a single defect (local change in the helix pitch), the spectrum of light reflection from a cholesteric acquires a dip corresponding to the defect mode. New qualitative features appear in the reflection spectrum of a cholesteric with two defects as the distance between them varies. © *2004 MAIK "Nauka/Interperiodica".*

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In dielectric media known as photonic crystals, whose dielectric properties periodically vary within a spatial scale on the order of the optical wavelength, a fundamentally new electromagnetic effect can be realized. This is the phenomenon of light localization in defect modes with discrete frequencies lying in the band gaps of an unperturbed photonic crystal [1, 2]. The defect modes can be used to create narrowband filters [3, 4] in information and communication technologies [5, 6], and for low-threshold lasers [7]. In recent years, interest has been expressed in cholesteric liquid crystals (CLCs), which are one-dimensional photonic crystals with a photon band gap for light propagating along the helix axis of a CLC with a circular polarization coinciding with the sign of the cholesteric helix [8]. Light waves with the opposite circular polarization are transmitted through the cholesteric medium without distortion. For CLCs, two ways of defect introduction were proposed: a thin layer of an isotropic substance introduced between two cholesteric layers [9] and a defect caused by the phase jump in the cholesteric helix at the interface between two layers of a cholesteric polymer film [10]. The fact that a defect mode can be induced in a CLC by introducing a phase jump in the helix was experimentally verified in [11].

In this paper, we investigate the reflection spectrum of a CLC with a defect of a new type: a local change in the helix pitch. We also study the features of the reflection spectrum of a cholesteric with two structural defects depending on the distance between them. The structure under consideration is a CLC film whose width is equal to N helix pitches, and the external medium is isotropic and characterized by the mean refractive index of the cholesteric under study: $n = \frac{1}{2}(n_{\parallel} + n_{\perp})$. We assume that the helix pitch may locally deviate from the pitch P of a perfect CLC, which is commensurate with the light wavelength. We assume that the structure is right-handed and that clockwise-polarized light is normally incident on the film and propagates along the axis of the helix. This geometry of the problem makes it possible to adjust to our needs a simple method of investigation that was developed for X-ray diffraction by perfect crystals and successfully used for describing the reflection spectrum of a perfectly organized CLC [12]. In the general case, the problem stated has a solution in the framework of the rigorous approach based on the theory of electromagnetism with the use of the method developed for describing the optical properties of inhomogeneous anisotropic layered media [13]. However, in the case under consideration, where light is normally incident on the film surface and propagates along the helix axis, the solution is simplified. At the same time, the results of calculating the reflection coefficient for a perfect cholesteric in the absence of dielectric boundaries in terms of the dynamic diffraction theory (Fig. 1) and the rigorous approach based on the electromagnetic theory virtually coincide [8, 12]. In addition, when the external medium outside the cholesteric is isotropic with a refractive index of the cholesteric under study $n = \frac{1}{2}(n_{\parallel} + n_{\perp})$, the Fresnel reflection from the film surface and the interference fringes from the boundary surfaces are weak. Such conditions can easily be realized in an experiment. In terms of the dynamic model, the reflection of light from the cholesteric is only determined by the diffraction by its structure, which allows us to correctly investigate the effect of defects on the reflection

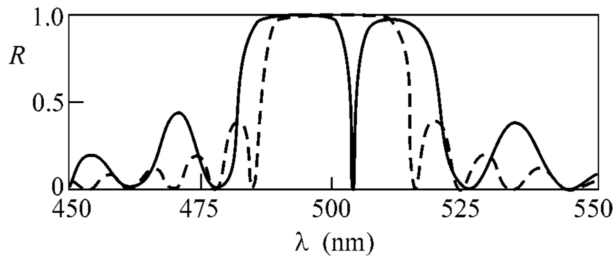

Fig. 1. Dependence of the reflection coefficient on the wavelength at normal incidence of light on the CLC. The dashed line corresponds to a perfect structure, and the solid line, to the structure with a defect. The parameters of the perfect CLC are $n = 1.5$, $\delta n = 0.07$, and $\lambda_0 = nP = 500$ nm, and the film thickness is $25P$. The helix pitch of the defect positioned in the middle of the medium is $P_D = 0.8P$.

spectra. The dynamic theory is inapplicable in the case of a very small film thickness or where the distance within which the amplitude of the incident wave decreases by a factor of e, $l = Pn/\pi\delta n$ (where $\delta n = n_{\parallel} - n_{\perp}$ is the birefringence of the quasi-nematic layer, is comparable with the helix pitch).

We assume that the CLC consists of nonequidistant parallel planes separated from one another by a distance equal to the helix pitch varying along the helix axis. In other words, each plane replaces m layers, and the amplitude reflection coefficient of the plane is expressed as [12]

$$r = -iQ = -i\pi\delta n/n. \tag{1}$$

To take into account the effect of multiple reflections from the planes, it is necessary to write the difference equations. Let A_l and B_l be the complex amplitudes of the incident and reflected waves at a point immediately above the nth plane. Then, the difference equations for a cholesteric with a varying helix pitch have the form

$$\begin{gathered} B_l = -iQA_l + \exp(-i\varphi_l)B_{l+1}, \\ A_{l+1} = \exp(-i\varphi_l)A_l - iQ\exp(-2i\varphi_l)B_{l+1}, \\ l = 0, 1, \ldots, N-1, \end{gathered} \tag{2}$$

where the phase difference φ_l, which arises when light travels the distance between the planes, has the form $\varphi_l = 2\pi nP_l/\lambda$ [12]; here, λ is the light wavelength in vacuum and P_l is the helix pitch corresponding to the lth plane. Note that Eqs. (2) are reduced to the difference equations for a perfect CLC [12] if we ignore the subscript l in the expression for the phase difference φ_l. In deriving Eqs. (2), we assumed that the reflection coefficient is the same on both sides of the plane.

The set of equations (2) can be represented as a matrix equation

$$\begin{pmatrix} B_l \\ A_l \end{pmatrix} = \hat{T}_l \begin{pmatrix} B_{l+1} \\ A_{l+1} \end{pmatrix}, \tag{3}$$

where the transfer matrix has the form

$$\hat{T}_l = \begin{pmatrix} e^{-i\varphi_l} & -iQe^{i\varphi_l} \\ iQe^{-i\varphi_l} & e^{i\varphi_l} \end{pmatrix}. \tag{4}$$

From Eq. (3) it follows that the amplitudes A_0 and B_0 are related to A_N and B_N as follows:

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \hat{M} \begin{pmatrix} B_N \\ A_N \end{pmatrix}, \tag{5}$$

where

$$\hat{M} = \hat{T}_0, \hat{T}_1, \ldots, \hat{T}_{N-1}. \tag{6}$$

The energy reflection coefficient $R(\lambda)$ is determined under the condition that reflection is absent on the right-hand side of the CLC sample:

$$R(\lambda) = \left|\left(\frac{A_0}{B_0}\right)_{B_{N=0}}\right|^2. \tag{7}$$

Using Eq. (5), we obtain

$$R(\lambda) = \left|\frac{M_{12}}{M_{22}}\right|^2, \tag{8}$$

where M_{12} and M_{22} are elements of the matrix $\hat{M}$.

Let us now consider the features of the reflection spectrum of CLC with one and two structural defects by using a numerical solution to the equation for reflection coefficient (8) while varying the parameters of photonic crystal. In what follows, the parameters of the bare structure, which satisfy the conditions of applicability of the dynamic mode, are assumed to be identical to those given in [12] in the study of a perfect CLC film with a thickness of $25P$: $n = 1.5$, $\delta n = 0.07$, and $\lambda_0 = nP = 0.5$ µm.

Figure 1 shows the dependence of the reflection coefficient of the CLC film on the wavelength varying within the region of existence of the band gap for a perfect CLC film and for the case where the helix pitch at the center of the film is smaller than the pitch of the perfect structure and is equal to $0.8P$. The boundaries of the spectral region of the band gap in a perfect cholesteric determine the wavelengths $\lambda_1 = 485$ nm and $\lambda_2 = 515$ nm. The value of the reflection coefficient for the CLC without defects reaches unity in the selective reflection band and decreases with a decrease in both the film thickness and the anisotropy of the cholesteric δn. As is seen from Fig. 1, a dip occurs in the middle of the band gap because of the structural defect, and the width of the band gap in the CLC with the defect proves to be greater than the band gap in the perfect cholesteric. The oscillations of the reflection coefficient

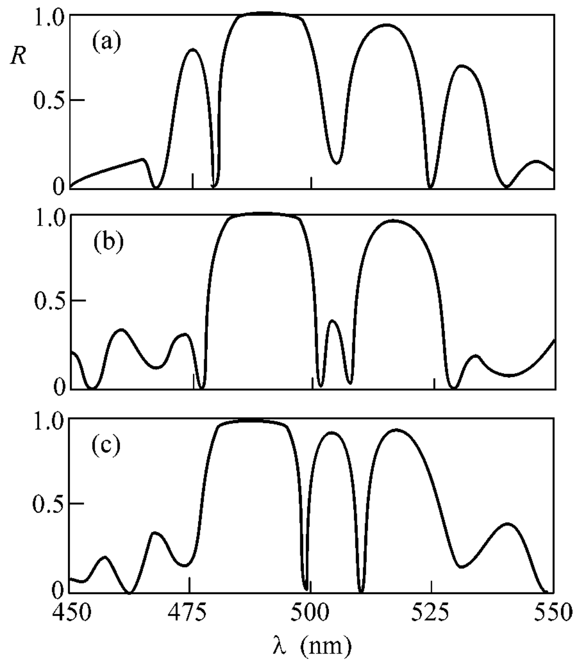

Fig. 2. The dependence of the reflection coefficient on the wavelength at normal incidence of light on the CLC with two identical structural defects. The parameters of the perfect structure are the same as in Fig. 1. The distance between the defects symmetrically positioned with respect to the center of the film is equal to (a) 13*P*, (b) 9*P*, and (c) 5*P*.

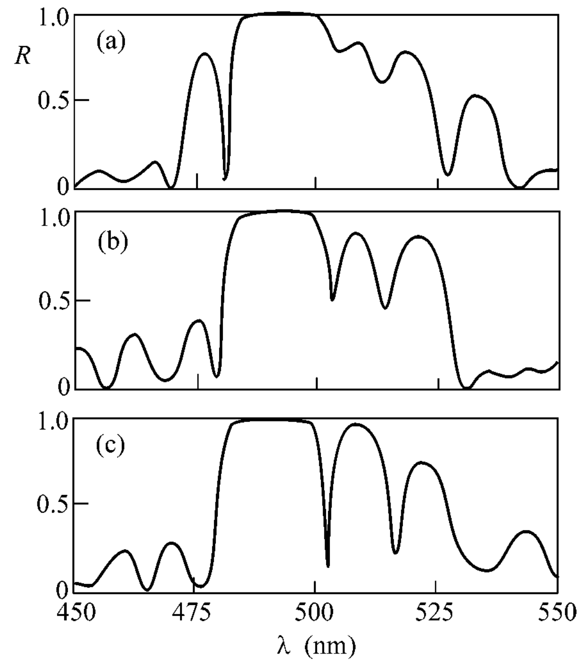

Fig. 3. Dependence of the reflection coefficient on the wavelength at normal incidence of light on the CLC with two different structural defects: 0.8*P* and 0.9*P*. The other parameters are the same as in Fig. 2.

observed as the wavelength moves away from the selective reflection region are caused by the light diffraction by a bounded volume, and, for a semibounded space occupied by the cholesteric, such oscillations of the reflection coefficient are absent [12].

The reflection spectrum of a CLC with two identical structural defects is shown in Fig. 2 for different distances between the defects. One can see that, as the distance between the defects increases, or, in other words, as the mutual effect of electromagnetic modes localized at the defects becomes weaker, the reflection spectrum exhibits qualitative changes. In the selective reflection region, two dips induced by the defects merge into one; i.e., a degeneration of the frequencies of electromagnetic modes localized at the defects takes place, and the positions of the boundaries of the spectral region of the band gap change. The oscillations of the reflection coefficient for the wavelengths near the selective reflection region undergo considerable changes (e.g., for wavelengths smaller than 480 nm and greater than 525 nm in Fig. 2a).

Figure 3 shows the reflection spectrum of a CLC with two different defects with a helix pitch of 0.8*P* and a pitch of 0.9*P*. The defects are at approximately equal distances from the middle of the film, these distances being the same as in the case of two identical defects (Fig. 2). One can see that the dips observed in the selective reflection band do not merge for the defect separations reaching 13*P* (Fig. 3a); i.e., no degeneration of the defect-mode frequencies takes place. In addition, as seen from a comparison of Figs. 3b, 3c with Figs. 2b, 2c, the dip depths are smaller for the CLC with different defects. These are the main features that differentiate these spectra from the reflection spectra shown in Fig. 2, which indicate the resonance nature of the mutual influence of the electromagnetic defect modes in a cholesteric with identical defects.

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