Zeros in single-channel transmission through double quantum dots

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By using a simple model we consider single-channel transmission through a double quantum dot that consists of two single dots coupled by a wire of finite length L. Each of the two single dots is characterized by a few energy levels only, and the wire is assumed to have only one level whose energy depends on the length L. The transmission is described by using S matrix theory and the effective non-Hermitian Hamilton operator H_{eff} of the system. The decay widths of the eigenstates of H_{eff} depend strongly on energy. The model explains the origin of the transmission zeros of the double dot that is considered by us. Mostly, they are caused by (destructive) interferences between neighboring levels and are of first order. When, however, both single dots are identical and their transmission zeros are of first order, those of the double dot are of second order. First-order transmission zeros cause phase jumps of the transmission amplitude by π, while there are no phase jumps related to second-order transmission zeros. In this latter case, a phase jump occurs due to the fact that the width of one of the states vanishes when crossing the energy of the transmission zero. The parameter dependence of the widths of the resonance states is determined by the spectral properties of the two single dots.

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I. INTRODUCTION

Some years ago, the phase of the transmission amplitude has been measured in a double-slit interference experiment [1]. The results showed phase jumps by π between resonances which raised intensive theoretical work for an explanation [2–7]. Most of these calculations associate the sharp phase drops with the occurrence of transmission zeros and relate them to the interference zeros of Fano resonances. In Ref. [6], it was shown, however, that the existence of a transmission zero is, indeed, a necessary condition for the phase jump but not a sufficient one. The sharp phase change bases, according to Ref. [6], on the destructive interference between neighboring resonance states. Destructive interferences between neighbored resonances are considered also in Refs. [4,5].

The Fano resonance phenomenon characterizes the interference between a single resonance with a relatively smooth background [8]. The interference processes in the regime of overlapping resonances are, however, much more complicated than those in the regime of isolated resonances. This has been demonstrated, e.g., in an experimental study of the conductance through a quantum dot (QD) in an Aharonov-Bohm interferometer [9] and in a theoretical study [10]. The Fano parameter becomes complex when there are interferences between short-lived resonances, long-lived resonances, and a smooth background. Numerical calculations with a complex Fano parameter are performed also in Ref. [11]. These results are a hint to the conclusion that the sharp phase changes observed in Ref. [1] are the result of interference processes not only between a narrow Fano resonance and the smooth background but also between neighboring resonances.

In a preceding paper [12], we studied the transmission properties of a double QD system when one lead is attached to the first single QD, another one to the second single QD, and both single QDs are connected by a wire of finite length L. Also in this system, transmission zeros appear when there are more than one state in each single dot. The special situation of a double QD is such that, on the one hand, the transmission zeros of the double dot are determined by the zeros in the transmission through the single QDs to which the leads are attached. The reason for this fact is that the total reflection (zero transmission) is determined by the overlap integrals between leads and single QDs. On the other hand, the resonance picture as a whole is determined by the resonance states of the double QD. As a consequence, the interference processes between the resonance states of the double QD are expected to be more complicated than those for a single QD. Since phase jumps in the transmission amplitudes are related to the transmission zeros, they are related therefore, first of all, to the resonances of the single QDs.

We will show in the present paper that transmission zeros of a single two-dimensional (2D) QD cause phase jumps by π in the amplitude of the transmission through this single QD. This situation corresponds to the usual Fano interference picture between narrow resonances and background or between neighboring resonance states. The situation in double QDs is, however, more complicated. It depends on the spectral properties of both single QDs and on the manner they are connected to the wire and to the leads, whether or not the transmission zeros of the single QDs cause transmission zeros of the same type also in the double QD system. When the single QDs remain true 2D systems in the double

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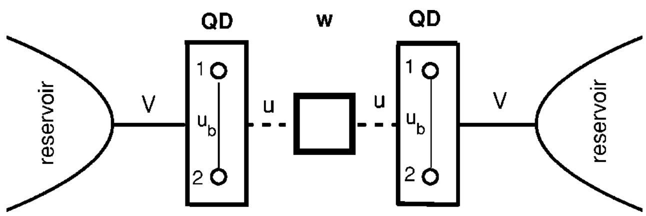

FIG. 1. The double dot system is connected to the reservoirs by the coupling constants v. The single dots are coupled to the wire by the coupling constants u.

QD and have different energy spectra, each transmission zero of each single QD causes a transmission zero of the same type in the double QD system. When the spectra of both single QDs are however equal, the corresponding transmission zeros of the double QD system are of another type. They give rise to two phase jumps, each by π, that compensate each other. When a resonance state crosses this transmission zero at a certain length of the wire, its decay width vanishes and a phase jump appears now due to the extremely narrow resonance. This mechanism differs completely from the simple Fano interference picture.

In Sec. II, we sketch the formalism for describing the double QD in order to provide a minimum of necessary equations. The method used by us is the S matrix represented by means of the eigenvalues and eigenfunctions of the non-Hermitian effective Hamilton operator of the open quantum system. We will neglect the Coulomb interaction in and between the single dots although it can be taken into account [13]. The point is that we are interested in this paper in a study of the origin of the transmission zeros and of the phase jumps of the transmission amplitudes related to them, and not in the quantitative description of the results obtained in a special experimental setup. In Sec. III, we study the transmission zeros appearing in a double dot consisting of two identical single dots coupled by a wire. We compare the results in Sec. IV with those obtained for a double dot with the same structure, but with different spectra of the two single dots. The results show which role the parameter dependence of the widths of the resonance states may play in order to satisfy the unitarity condition. The width of one of the states may even vanish at a certain value of the parameter.

II. BASIC EQUATIONS

The double dot system we will study in the following consists of two single dots connected by a wire; see Fig. 1 for illustration. The single dots have each two levels the energies of which are denoted by ε_{i_L}, ε_{i_R}, $i=1,2$ where L and R stand for the left and right single dot. The energy ϵ of the wire depends on its length L. The coupling strength u between single dot and wire is assumed to be the same for both single dots. It characterizes the "internal" interaction of the double dot system. The coupling strength v of the double dot as a whole to the attached leads characterizes the openess of the system. It may be called "external" interaction. The interplay between internal and external interaction and its role for the transmission through the double dot is studied in Ref. [12].

The S matrix theory of single-channel transport through two QDs coupled by a variable wire is given in Ref. [13]. Different from the standard S matrix theory, the energies and widths as well as the wave functions of the resonance states are obtained, in this approach, by diagonalizing the effective Hamiltonian of the open double QD. We will present here only a few formulas of that theory which are important for the study of the transmission zeros. We will neglect the Coulomb interaction although its influence can be taken into account [13]. The reason for doing this is that we are interested, in the present paper, in the discussion of the origin of the transmission zeros in double QDs. We will not try to describe quantitatively the results that can be obtained in a special experimental device.

When the double dot is a 1D chain or when the single dots have each only one level there do not appear any transmission zeros. This result is obtained in different studies, e.g., Refs. [3,13,14]. We will not consider it here again.

Following Ref. [13], the Hamiltonian of our closed double QD consisting of the two single QDs and the wire is

$$H_B = \begin{pmatrix} \varepsilon_1 & 0 & u & 0 & 0 \\ 0 & \varepsilon_2 & u & 0 & 0 \\ u & u & \epsilon(L) & u & u \\ 0 & 0 & u & \varepsilon_2 & 0 \\ 0 & 0 & u & 0 & \varepsilon_1 \end{pmatrix}. \tag{1}$$

For simplicity we have assumed that all the coupling constants between the wire and the single QD are the same and are given by the constant value u. The Coulomb interaction is ignored; see above.

Two eigenvalues of the Hamiltonian H_B coincide with the energies ε_1 and ε_2 of the single QDs [13]. The other three eigenvalues of Eq. (1) can be found by solving a cubic equation. Also the finding of the eigenstates of Eq. (1) is a formidable task. We consider therefore the transmission through a system with two states in each single QD numerically.

The Hamiltonian (1) is written in the energy representation. In order to specify the connection between the reservoirs and the single QDs, we have, however, to know the eigenstates of Eq. (1) also in the site representation. The Hamiltonian of the single QD in the site representation is

$$H_b = \begin{pmatrix} \varepsilon_0 & u_b \\ u_b & \varepsilon_0 \end{pmatrix}. \tag{2}$$

The hopping matrix elements u_b are shown in Fig. 1 by thin solid lines. The eigenfunctions and eigenvalues are the following:

$$\langle j|\varepsilon_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \langle j|\varepsilon_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

$$\varepsilon_{1,2} = \varepsilon_0 \mp u_b. \tag{3}$$

We introduce the projection operators

$$P_L = \sum_{b_L} |\varepsilon_{b_L}\rangle\langle\varepsilon_{b_L}|, \quad P_w = |1_w\rangle\langle 1_w|, \quad P_R = \sum_{b_R} |\varepsilon_{b_R}\rangle\langle\varepsilon_{b_R}|, \tag{4}$$

where $b_L=1,2$, $b_R=1,2$, and $|1_w\rangle$ is the one-dimensional eigenstate of the wire. Let E_m and $|m\rangle$ with $m=1,\ldots,5$ denote the five eigenenergies and eigenstates of Eq. (1), $H_B|m\rangle=E_m|m\rangle$. The elements of the left coupling matrix are

$$\langle L,E|V|m\rangle = \langle L,E|VP_L|m\rangle = \sum_{b_L} \langle L,E|V|\varepsilon_{b_L}\rangle\langle\varepsilon_{b_L}|m\rangle$$
$$= \sum_{j_L=1,2} \sum_{b_L} \langle L,E|V|j_L\rangle\langle j_L|\varepsilon_{b_L}\rangle\langle\varepsilon_{b_L}|m\rangle. \tag{5}$$

Similar expressions can be derived for the right coupling matrix. Here we used the assumption that the left reservoir is connected only to the left single QD and the right reservoir only to the right single QD. As previously, the reservoirs are assumed to be semi-infinite one-dimensional wires. Next we have to specify which sites of the left (right) single QD are connected to the left (right) reservoir. There are two possibilities.

(i) Assume the left reservoir is connected only to the first site $j_L=1$ of the left single QD. Then, with account of Eq. (3), Eq. (5) becomes

$$\langle L,E|V|m\rangle = v\sqrt{\frac{\sin k}{2\pi}} \sum_{b_L} \langle\varepsilon_{b_L}|m\rangle. \tag{6}$$

A corresponding expression can be written down for the right coupling matrix if the right reservoir is connected to the first site of the right single QD.

(ii) We can assume that the reservoirs are connected to both sites of the single QDs with the same coupling constant v. Then the elements of the coupling matrices (6) are the following:

$$\langle L,E|V|m\rangle = v\sqrt{\frac{\sin k}{2\pi}} \langle\varepsilon_1|m\rangle \tag{7}$$

provided that the energy level ε_1 is the lowest in energy; see Eq. (3).

Knowing the Hamiltonian (1) of the closed system and the coupling matrix elements (6) and (7) to the reservoir, the effective Hamiltonian H_{eff} can be obtained [13]. It is non-Hermitian. Its complex eigenvalues z_i provide the positions in energy, $\text{Re}(z_i)$, as well as the widths, $\Gamma_i/2 \equiv -\text{Im}(z_i)$, of the resonance states. After diagonalizing H_{eff}, the transmission through the double QD reads [13,14]

$$t = -2\pi i \sum_{\lambda} \frac{\langle L|V|\lambda)(\lambda|V|R\rangle}{E - z_\lambda}. \tag{8}$$

Here $|\lambda)$ and $(\lambda|$ are, respectively, the right and left eigenfunctions of H_{eff}. They are bi-orthogonal [15]. The transmission probability is $T=|t|^2$.

In the double dot system, the leads are attached to the single dots. The transmission zeros are determined therefore by the spectroscopic properties of the single dots, and not by those of the double dot as a whole. Due to the unitarity of the S matrix, transmission zeros appear, in the single-channel case, always between every two states. As a consequence, no transmission zeros will appear when the single dots have only one state each. The most important difference between the cases $N=1$ and $N=2$ of the single QDs is therefore that the double dot as a whole is no longer necessarily one dimensional in the $N=2$ case. Therefore zeros in the transmission probability may appear in the $N=2$ case in contrast to the $N=1$ case.

We underline here once more that our approach differs from the standard S matrix theory. Although the resonance part of the S matrix has the standard view, there are differences between the two approaches which are important in the regime of overlapping resonances [15]. In our approach

(i) the coupling matrix elements entering the numerator of Eq. (8) are calculated by means of the eigenfunctions of H_{eff},

(ii) the z_k are the eigenvalues of H_{eff},

(iii) the coupling matrix elements and the z_k depend on the energy E (and other parameters),

(iv) the energy E of the incident particle is real. The positions and widths of the resonances are determined by the complex eigenvalues of H_{eff}, and the poles of the S matrix are not considered.

We underline furthermore that the Hamiltonian H_B of the closed system is diagonalized before it enters into the expression for the effective Hamilton operator H_{eff} and that the S matrix is unitary at all energies E also in the regime of overlapping resonances [16].

III. DOUBLE DOT WITH TWO IDENTICAL SINGLE DOTS COUPLED BY A WIRE

We represent here the results of some calculations for the transmission through double quantum dots with altogether $2N+1$ states: N states in each single QD and one state in the wire that connects the two single dots. The two single dots are assumed here to be identical, i.e., the energies of the two states of the left single dot are the same as those of the right dot. The energy ϵ of the wire is assumed to depend linearly on the length L of the wire. This assumption does not play any role for the study of the origin of transmission zeros in double dots. Similar results are obtained with, e.g., a quadratic dependence of ϵ on the length L [13]. One may consider the dot considered by us (Fig. 1) also as a triple QD. We keep, however, the notation "double QD" in order to express the different nature of the wire and of the two single dots.

In Fig. 2, the transmission probability versus energy E and length L of a double QD is shown for the case that each single dot has two states and that both sites of the single QD are connected to the reservoir with the coupling matrix elements (7). The figure shows a zero in the transmission probability, indeed; see Fig. 2(b). According to Figs. 2(c) and 2(d), the positions and decay widths of the eigenstates 2 and 4 of the effective Hamiltonian are independent of the length L of the wire while those of the other states depend on L. State 3, lying in the middle of the spectrum, crosses the transmission zero at $L=2.75$. Here, the decay width of this state approaches zero for all energies E.

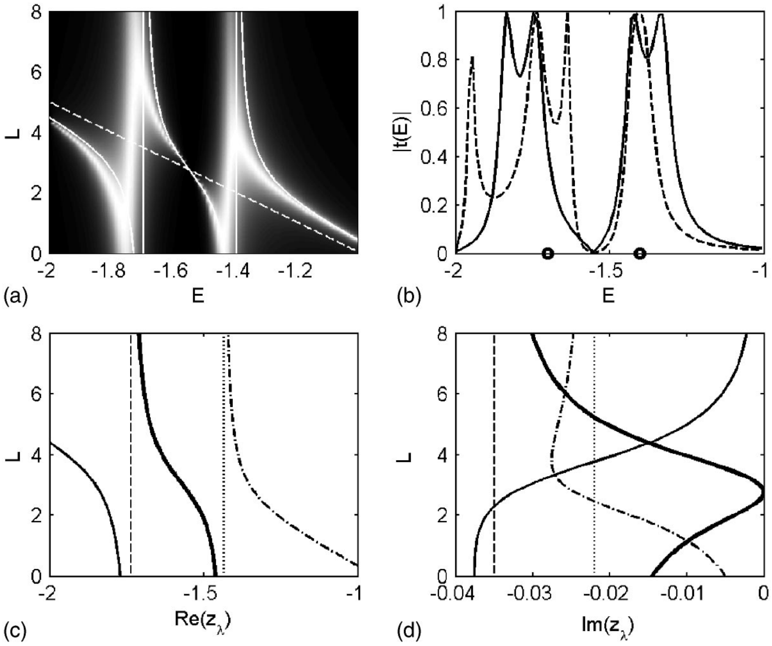

FIG. 2. (a) The transmission through a double QD with two identical single QDs that are connected by a wire according to Fig. 1. The eigenvalues of H_B are shown by full lines. $\varepsilon_1=-1.7$, $\varepsilon_2=-1.4$, and $\epsilon(L)=-1-L/5$ (dashed line), $v=0.3$, $u=0.1$. (b) The modules of the transmission amplitude $|t(E,L)|$ for the same double QD as in (a) for fixed lengths $L=2.75$ (solid line) and $L=4$ (dashed line). The energies of the two single QDs are shown by circles. The real part (c) and imaginary part (d) of the five eigenvalues z_k of the effective Hamiltonian as a function of L for $E=-1.5$. Thin solid line: z_1; dashed line: z_2; thick solid line: z_3; dotted line: z_4; and dash-dotted line: z_5. At $L=2.75$ the imaginary part of the third eigenvalue is equal to zero at all energies E.

In Fig. 3(a), the transmission through a double QD with two identical single QDs is shown, while Fig. 3(b), shows the transmission through one of these single QDs (the lower curves correspond to the modules of the transmission amplitude and the upper curves to their phases). In the double QD, the two single QDs are connected to the leads and to the wire as shown in Fig. 1. Comparing the two figures, we see that the transmission zero of the double QD coincides with that of the single QD. This result is in agreement with the fact that the leads are coupled to the single dots. However, there is a difference between the zeros in both cases as will be explained in the following.

Single QD [Figs. 3(b) and 3(d)]: The transmission zero of the single QD is due to the destructive interference of the two neighboring resonance states [3,6,14] and is located between the energies of the single QD. It is caused by the unitarity of the S matrix [15] with account of the fact that the leads are attached to the single QDs being the constituents of the double dot. Around the energy E_0, the transmission amplitude vanishes, $t(E_0)=0$. Here $\text{Re}[t(E)]\sim(E-E_0)^2$ while $\text{Im}[t(E)]\sim d\{\text{Re}[t(E)]\}/dE\sim(E-E_0)$. It holds therefore for the modulus of the transmission amplitude $|t(E)|\sim|E-E_0|$ near E_0, see Fig. 3(b), and $dt(E)/dE|_{E_0}\neq 0$. Thus the phase of the transmission amplitude $\arg[t(E)]$ jumps by π at E_0, according to Ref. [6]. The geometrical origin of this phase jump can clearly be seen in Fig. 3(d). By pathing through the origin of the coordinates $\text{Re}(t)=0$, $\text{Im}(t)=0$, the value $\arg(t)$ is not defined unambiguously. The phase jumps by $\pm\pi$, and the sign of the phase jump is not observable. We can call such a zero a first order zero.

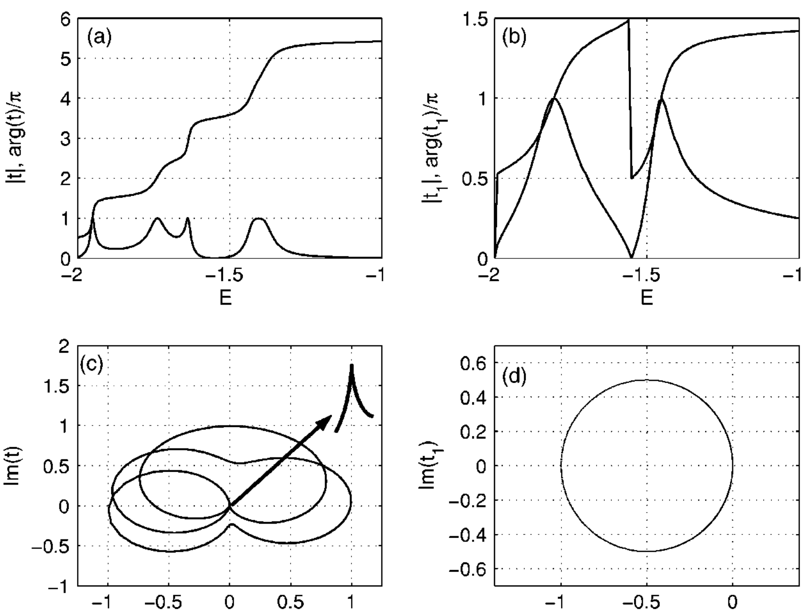

FIG. 3. (a) The modules $|t(E)|$ (lower curve) and the phase $\arg(t)/\pi$ (upper curve) of the transmission amplitude for a double QD with two identical single QDs. The parameters are the same as in Fig. 2, and $L=4$ as in Fig. 2(b), dashed line. (b) The modules $|t_1(E)|$ (lower curve) and the phase $\arg(t_1)/\pi$ (upper curve) of the transmission amplitude for one of the single QDs that is part of the double QD considered in (a). The energy positions of the single QD levels are shown by circles in Fig. 2(b). (c) Evolution of imaginary and real parts of the transmission amplitude t of the double QD with energy. At the upper right corner, a zoomed fragment of the evolution is shown which demonstrates that the evolution has cusplike behavior in the vicinity of the transmission zero. (d) The same as (c) but for the single QD. Here, the evolution is of standard type.

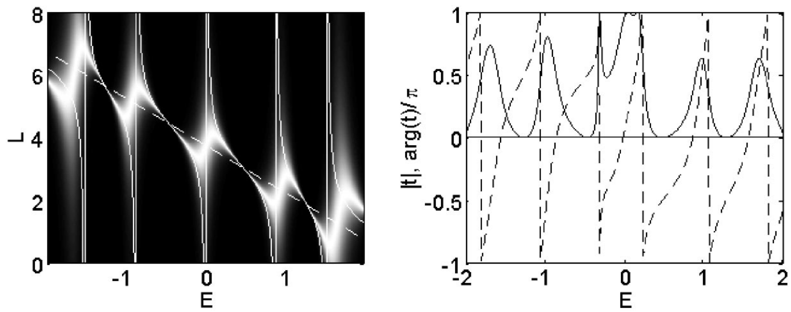

FIG. 4. (a) The transmission through a double QD consisting of two identical single QDs with $d=5$ states that are connected by a wire, versus energy and length of the wire. The eigenvalues of H_B are shown by thin lines while the energy $\epsilon(L)=5/2-2L/3$ of the mode in the wire is shown by the dashed line. (b) Energy dependence of the modules (solid line) and of the phase $\arg(t)/\pi$ (dashed line) of the transmission amplitude for $L=4$. $\varepsilon_n=-1.75\cos(\pi n/6)$, $n=1,2,\ldots,5$, $v=0.5$, $u=0.2$. There are 11 resonance states in the double dot, 4 transmission zeros of second order, and no phase jumps.

Double QD [Figs. 3(a) and 3(c)]: The zeros in the transmission through the double QD with two identical single QDs are of another type. They are of second order since it is $|t(E)|\sim|t_1(E)|^2$ in the vicinity of the energy $E=E_0$. It follows therefore $t(E)\sim(E-E_0)^2$ near E_0; see Fig. 3(a). The energy evolution of the real and imaginary parts of the transmission amplitude is shown in Fig. 3(c). In the inset of the figure, the evolution of Re(t), Im(t) at the origin of the coordinate system is shown in zoomed resolution. It has a cusplike behavior, and there is no phase jump at all. We present in Fig. 3(a) the phase behavior of the transmission amplitude t that is a combination of two jumps with opposite sign, resulting in a zero phase jump at the point $E=E_0$. This result agrees with the general statement given in Ref. [6], that phase jumps do not appear when $dt(E)/dE|_{E_0}=0$ (as in our case) at the energy $E=E_0$.

Thus zeros of second order in the single-channel transmission of a double QD are given by zeros of first order in the transmission of the single QDs (that constitute the double QD) when they are identical. If these two single QDs have each N energy levels then $N-1$ transmission zeros of second order will appear in the double QD. A numerical computation for the particular case $N=5$ confirms this conclusion (Fig. 4). Furthermore, there are no phase jumps at the transmission zeros of second order, as can be seen from Fig. 4(b), in complete agreement with the theory.

We consider now the evolution of the modules of the transmission amplitude and the corresponding phase shifts when the decay width of one of the states approaches zero. The results shown in Fig. 5 are performed for the double QD the transmission of which is shown in Fig. 2 together with the eigenvalues z_k of the effective Hamiltonian H_{eff} as a function of L. The latter ones are related to the poles of the S matrix; see Ref. [12]. At $L=2.75$, the third eigenstate crosses the energy of the transmission zero, and its decay width $\mathrm{Im}(z_3)/2$ approaches zero. As long as $L\neq 2.75$ and $\mathrm{Im}(z_3)\neq 0$, the phase of the transmission amplitude varies by π more or less smoothly, according to the phase shift caused by a resonance state with a finite decay width. When $L\to 2.75$ and $\mathrm{Im}(z_3)\to 0$, the phase jumps by π due to the vanishing decay width of the resonance state. Therefore we have also in this case a phase jump of the transmission amplitude by π. Correspondingly, the transmission zero becomes of first order at $L=2.75$; see Fig. 2(b). That means the resonance state whose decay width vanishes when crossing the energy of the transmission zero, restores the first order of the transmission zero as well as the phase jump by π.

We mention here that resonance states with vanishing decay width are considered also by other authors. In Ref. [7], they are called "ghost" Fano resonances that appear in a double QD attached to leads. In Ref. [17], the appearance of discrete levels in the continuum is shown to correspond to the occurrence of special localized electron states that appear due to a "collapse" of Fano resonances. In atomic physics, the phenomena related to resonance states with vanishing decay width are known as population trapping [18]. They result from the interplay of the direct coupling of the states and their coupling via the continuum under the influence of, e.g., a strong laser field. In the case considered in the present paper, they appear due to some constraint onto the system as

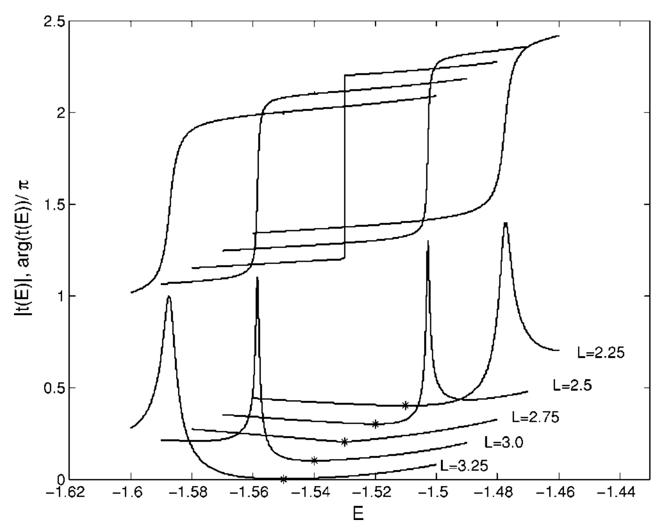

FIG. 5. The energy dependence of the modules $|t(E)|$ (bottom) and of the phase $\arg[t(E)]/\pi$ (top) of the transmission amplitude for $L=2.25, 2.5, 2.75, 3.0, 3.25$. The other parameters are the same as those in Fig. 2. The transmission zeros are denoted by stars. They are of second order. The ordinate is shifted every time by 0.1 when L is changed by 0.25. All phases are shifted by π.

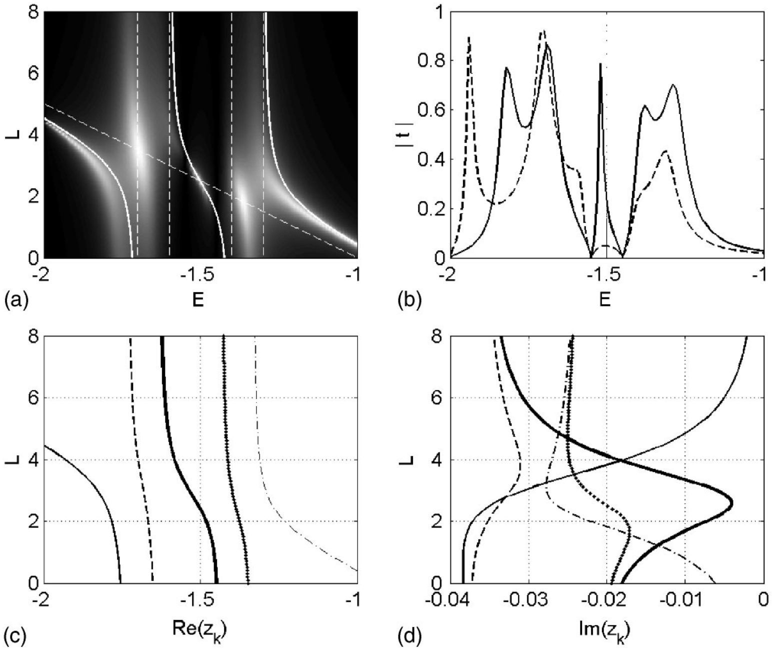

FIG. 6. The same as Fig. 2, but with two different single QDs: $\varepsilon_{1_L}=-1.7$, $\varepsilon_{2_L}=-1.4$, $\varepsilon_{1_R}=-1.6$, $\varepsilon_{2_R}=-1.3$.

a consequence of the unitarity of the S matrix. On the one hand, the position of the transmission zeros is determined by the spectroscopic properties of the single dots. On the other hand, however, the transmission is resonant and related to the spectroscopic properties of the double QD. These two facts cause some nontrivial constraint onto the system in order to fulfill the condition of unitarity of the S matrix for the double QD as a whole.

IV. DOUBLE DOT WITH TWO DIFFERENT SINGLE DOTS COUPLED BY A WIRE

Now we will consider the transmission through a double QD consisting of two different single QDs coupled to the wire and to the leads as shown in Fig. 1. The single QDs have each two states, $\varepsilon_{i_L}\neq\varepsilon_{i_R}$, $i=1,2$; see Fig. 6.

In contrast to the foregoing case, the width of the state in the middle of the spectrum does not approach zero; see Fig. 6(d). It remains different from zero for all L. This result illustrates very nicely that the transmission zeros originate from the interference between neighboring resonances. In Fig. 2, the two single dots have the same spectrum, and the transmission zero appears at the energy $(\varepsilon_1+\varepsilon_2)/2$. At this energy, the interference between the two pairs of outer states of the double dot is destructive and gives a vanishing contribution to the transmission. In order to achieve the transmission zero of the double dot, the width of the state in the middle of the spectrum has to vanish when it crosses the energy of the transmission zero as a function of the length L of the wire; see Fig. 2(d). In Fig. 6, however, the spectra of the two single QDs are different. The constraint onto the middle state is therefore reduced: the width of this state remains different from zero at all L. It is reduced only in such a manner that the state with this value of the width is able to interfere destructively with the other four states in order to achieve the two transmission zeros.

These results show that the interference between neighboring states is basic for the transmission zeros that appear in double QDs, as well as for the corresponding phase jumps

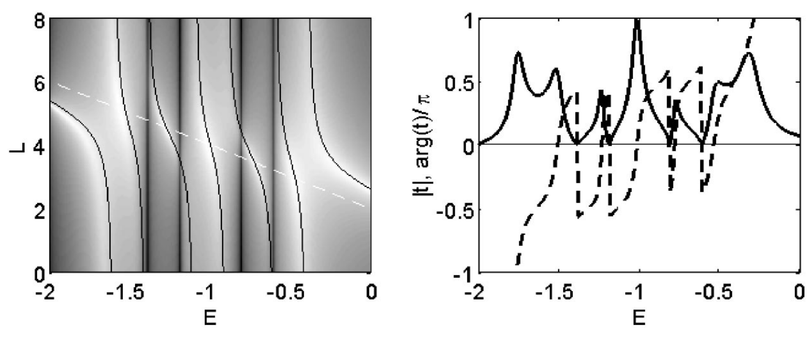

FIG. 7. (a) The transmission $\ln|T(E,L)|$ through a double QD with different single QDs connected by a wire, and (b) the energy dependence of the modules (solid line) and of the phase $\arg(t)/\pi$ (dashed line) of the transmission amplitude for this double QD system at a fixed length $L=4$ of the wire. The energy levels of the left single QD are $\varepsilon_{k_L}=-1.6,-1.1,-0.6$, while those of the right single QD are $\varepsilon_{k_R}=-1.4,-0.9,-0.4$. Further, $\epsilon(L)=1-L/2$, $v=0.3$, $u=0.2$. The full lines in (a) are the eigenvalues of H_B while the dashed line is the energy $\epsilon(L)$. There are seven resonance states of the double dot, four transmission zeros of first order, and the corresponding four phase jumps.

(Fig. 3). Additionally, the system has the freedom to vary the widths of the resonance states in order to satisfy the unitarity of the *S* matrix.

When each single QD has *N* states, the number of zeros in the transmission through the double QD is $2(N-1)$ when the spectra of the two single dots are different from one another. This conclusion is demonstrated by the results of numerical calculations shown in Fig. 7. We see four transmission zeros of first order. At each of these transmission zeros, the phase jumps by $-\pi$ (compare Fig. 3).

V. SUMMARY

The results presented in the present paper have shown that transmission zeros in double QDs show some nontrivial behavior since two conditions for their appearance have to be fulfilled which are independent from one another. On the one hand, the transmission zeros are related to the spectroscopic features of the single dots due to the fact that full reflection is determined by the area of attachment, and the leads are attached to the single dots. On the other hand, however, the resonance states of the system are characteristic of the double QD as a whole. As a consequence, even the number of transmission zeros differs, as a rule, from the number of resonance states. This result does not agree with the simple Fano interference picture where each resonance state creates a zero in the reaction cross section due to its interference with the smooth background. The origin of the zeros in the transmission through a double QD is rather strongly related to the interferences between neighboring resonance states. A similar result has been obtained for single dots in Ref. [6].

Moreover, in double dots it may happen that the interferences between neighboring resonances cannot provide the transmission zero due to some symmetries in the system. In such a case, the width of one of the resonance states approaches zero when crossing, as a function of a certain parameter, the energy at which the transmission vanishes. Due to this constraint, the width of at least one of the resonance states is strongly parameter dependent, and the system as a whole is unstable against parameter variations.

The relation between transmission zeros and phase jumps can be seen in the following manner. When the two single dots have different spectra, the transmission zeros of the double QD are of first order. Transmission zeros of first order cause phase jumps by π as shown by means of numerical examples for both single and double dots. When, however, the two single dots are identical, the transmission zeros of the double dot are of second order and there is no phase jump related to them. When there is a resonance state at this energy the phase of the amplitude of the transmission through the double dot jumps by π also in this case since the width of the resonance state that crosses the transmission zero at a certain length $L=L_0$ of the wire vanishes at L_0. Resonance states with vanishing width may be called ghost Fano resonances according to Ref. [7].

When *M* identical quantum dots are connected to a string, higher-order transmission zeros may arise. An odd number *M* causes a phase jump by π, while no phase jump is related to the transmission zeros for even *M*. The transmission through such a string of quantum dots jumps, however, by π in any case due to the appearance of resonance states with zero width, as discussed above for the double dot.

In our numerical studies, we considered the appearance of transmission zeros in double QDs with the structure shown in Fig. 1 and with the coupling matrix elements (7). Since the leads are attached to the single dots, the transmission zeros are determined by the requirement of the unitarity of the *S* matrix with account of the resonance states of the single dots. This means that transmission zeros in the double dot appear only when at least one of the single dots has at least two levels. Some further examples may illustrate this behavior. When, e.g., one of the single QDs loses its 2D character by the manner it is integrated in the double QD system, only transmission zeros of first order appear in the double QD. When both single QDs are included as 1D dots, the double QD has no transmission zero at all. In this case, the whole system behaves as a 1D chain of sites without any transmission zeros. This last result is in agreement with that obtained in Ref. [3] by using another method.

We underline that the results presented in the present paper are received in the *S* matrix formalism by using the eigenvalues and eigenfunctions of the effective Hamiltonian H_{eff} of the open QD. One of the characteristic features of this approach is the strong parameter (and energy) dependence of the decay widths of the resonance states. This strong parameter dependence ensures the unitarity of the *S* matrix at all energies [16] and causes, e.g., the existence of resonance states with zero width (Figs. 2, 5, and 6).

As a summary of the results obtained in the present paper we state the following. The interferences between neighboring resonance states as well as the parameter dependence of the widths of the states achieve the transmission zeros of double QDs in the single-channel case. At the transmission zeros the phases jump by mostly (but not always) π.

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