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# Effect of the Correlation Properties of One- and Three-Dimensional Inhomogeneities on the High-Frequency Magnetic Susceptibility of Sinusoidal Superlattices

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**Abstract**—The effect of one- (1D) and three-dimensional (3D) inhomogeneities on the high-frequency magnetic susceptibility at the boundary of the first Brillouin zone of a ferromagnetic superlattice is studied. The study is performed with an earlier developed method of random spatial modulation (RSM) of the superlattice period. In this method, structural inhomogeneities are described in terms of the random-phase model, in which the phase depends on three coordinates in the general case. The frequency spacing $\Delta\nu_m$ between two peaks in the imaginary part of the averaged Green's function, which characterizes the gap width in the frequency spectrum at the boundary of the Brillouin zone, is calculated as a function of both the root-mean-square fluctuations $\gamma_i$ and the correlation wavenumbers $\eta_i$ of phase inhomogeneities ($i$ = 1 and 3 for 1D and 3D inhomogeneities, respectively). The function $\Delta\nu_m(\gamma_1, \eta_1)$ for 1D inhomogeneities is shown to be symmetric with respect to interchanging the variables $\gamma_1^2$ and $\eta_1$, whereas the function $\Delta\nu_m(\gamma_3, \eta_3)$ for 3D inhomogeneities is strongly asymmetric with respect to interchanging $\gamma_3^2$ and $\eta_3$. This effect is associated with the difference in form between the correlation functions of 1D and 3D inhomogeneities and can be used to determine the dimensionality of inhomogeneities from the results of spectral studies of such superlattices. *© 2005 Pleiades Publishing, Inc.*

## 1. INTRODUCTION

One-dimensional periodic structures (superlattices) have been extensively applied in various devices. Despite the progress made in the production of such structures, their characteristics are still far from ideal in many cases. This is caused by the fact that the properties of real superlattices depend on technological factors, such as random scatter of the layer thicknesses (one-dimensional structural inhomogeneities) and randomly strained interfaces between layers (two- and three-dimensional inhomogeneities). Therefore, it is challenging to theoretically study the effect of random structural inhomogeneities on the physical properties of superlattices and, in particular, on the characteristics of waves propagating in such materials. Apart from the applied aspects of such studies, it should be noted that partly randomized superlattices are convenient objects for the development of new methods in theoretical physics to investigate media without translational symmetry. At present, different-type models and methods are being used to develop a theory of randomized superlattices. In initially sinusoidal superlattices, one-dimensional randomization is taken into account by introducing a random phase [1, 2]. In superlattices with a rectangular profile of the coordinate dependence of a material parameter along the superlattice axis, randomization is modeled by a disturbance in the sequence of layers of two different materials [3–9] or by random deviations of the layer–layer interfaces from their initial arrangement [10–12]. There are also methods based on the superlattice correlation functions of a postulated shape [13, 14], applications of the geometrical-optics approach [15], and the development of a dynamic theory of elastic composite media [16].

In [17], we proposed another method for investigating the effect of superlattice inhomogeneities on the wave spectrum, which we called the method of random spatial modulation (RSM) of the superlattice period. Let us briefly review this method. The spectral properties of any inhomogeneous medium are known to be best described in terms of averaged Green's functions. The only characteristic that describes a random medium and enters into an expression for an averaged Green's function is the correlation function $K(\mathbf{r})$, which depends on the distance $\mathbf{r} = \mathbf{x} - \mathbf{x}'$ between two points in the medium. Therefore, the first part of the problem reduces to finding the function $K(\mathbf{r})$ for a superlattice that contains certain structural inhomogeneities. The second part of the problem consists in extracting spectral characteristics from the Green's function that contains this correlation function by using standard

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approximate methods. To describe structural inhomogeneities in a sinusoidal superlattice, we used the model of a random phase, which was considered a random function of all three coordinates with an arbitrary correlation radius (the authors of [1, 2] considered only a one-dimensional δ-correlated random function). To find the correlation function $K(\mathbf{r})$ of the superlattice, we have developed a method which is a generalization of the well-known method of determining the time correlation function for a randomly frequency-(phase-)modulated radio signal [18, 19] to the case of spatial (in general, three-dimensional) modulation of the superlattice period (phase). This method is advantageous in that the shape of the correlation function of a superlattice is derived under general assumptions about the character of random spatial modulation of the superlattice period rather than being postulated. It has been shown that, in general, this function has a complex form, which depends on the inhomogeneity dimensionality, the structure of the interface between layers, etc. Knowledge of the correlation functions corresponding to different types and dimensionalities of inhomogeneities allowed us to use the averaged Green's functions to find the eigenfrequencies, damping, and other wave characteristics of superlattices [17, 20–28]. The RSM method allowed us to consider inhomogeneities of different dimensionalities in terms of a single model. The effect of one- (1D) and three-dimensional (3D) inhomogeneities on a wave spectrum has been studied for sinusoidal superlattices and superlattices with zero and arbitrary thicknesses of the interfaces between layers. We have also studied the effect of a mixture of 1D and 3D inhomogeneities [26, 27] and the effect of the anisotropy of a correlation function [28]. However, some important problems have not been solved. For example, the authors of [25–28] showed how the difference in form between correlation functions for 1D and 3D inhomogeneities manifests itself in the characteristics of the wave spectrum of a superlattice when the root-mean-square phase modulation varies in magnitude. In this work, we study the dependences of the wave-spectrum characteristics on both the root-mean-square phase fluctuations and the correlation radii of inhomogeneities and demonstrate radical differences in character between these dependences for 1D and 3D inhomogeneities. We use the true correlation functions obtained by us earlier in [17]. With these functions, we determined the range of applicability of the approximate analytical expressions for the correlation functions of 3D inhomogeneities that were used in [25–28] to simplify computations.

## 2. CALCULATION PROCEDURE

Let us recall, in brief, the main features of the RSM method, which was developed in [17] to find the correlation functions of a superlattice having 1D, 2D, or 3D inhomogeneities of its period. The coordinate dependence of the uniaxial magnetic anisotropy of a ferromagnetic superlattice was taken to be

$$\beta(\mathbf{x}) = \beta_0 + \Delta\beta\rho(\mathbf{x}), \quad (1)$$

where $\beta_0$ is the mean value of the anisotropy, $\Delta\beta$ is its root-mean-square deviation, and $\rho(\mathbf{x})$ is a centered ($\langle\rho\rangle = 0$) and normalized ($\langle\rho^2\rangle = 1$) function. The function $\rho(\mathbf{x})$ describes the periodic variation in the magnetic-anisotropy parameter along the $z$ axis, as well as random spatial modulations of this parameter. Angle brackets mean averaging over an ensemble of random realizations. In [17], this function was taken in the form

$$\rho(\mathbf{x}) = \sqrt{2}\cos[q(z - u(\mathbf{x}) + \psi)], \quad (2)$$

where $q = 2\pi/l$ is the wavenumber of the superlattice and $l$ is its period. Inhomogeneities are characterized by a random spatial phase modulation $u(\mathbf{x})$, which in general is a function of all three coordinates: $\mathbf{x} = \{x, y, z\}$. By introducing the function $\chi(\mathbf{x}, \mathbf{r}) = q[u(\mathbf{x} + \mathbf{r}) - u(\mathbf{x})]$ and averaging the product $\rho(\mathbf{x})\rho(\mathbf{x} + \mathbf{r})$ over $\chi$ with a Gaussian distribution and over the coordinate-independent phase $\psi$ with a uniform distribution (see [17] for more details), we obtain the correlation function of the superlattice in the form

$$K_i(\mathbf{r}) = \cos q r_z \exp\left[-\frac{Q_i(\mathbf{r})}{2}\right], \quad (3)$$

where the structure function $Q_i(\mathbf{r})$ has the form

$$Q_1(r_z) = 2\gamma_1^2[\exp(-k_\parallel r_z) + k_\parallel r_z - 1], \quad (4)$$

$$Q_3(r) = 6\gamma_3^2\left[1 - \frac{2}{k_0 r} + \left(1 + \frac{2}{k_0 r}\right)\exp(-k_0 r)\right], \quad (5)$$

for 1D and 3D inhomogeneities, respectively. Here, $k_\parallel$ and $k_0$ are the correlation wavenumbers of 1D and 3D inhomogeneities, respectively ($r_\parallel = k_\parallel^{-1}$ and $r_0 = k_0^{-1}$ are the correlation radii of inhomogeneities), and

$$\gamma_1 = \sigma_1 q/k_\parallel, \quad \gamma_3 = \sigma_3 q/k_0, \quad (6)$$

where $\sigma_1$ and $\sigma_3$ are the root-mean-square fluctuations of the gradients of the functions $u_1(z)$ and $u_3(\mathbf{x})$.

We consider the situation where an external magnetic field $\mathbf{H}_0$, the static part of the magnetization $\mathbf{M}_0$, and the magnetic-anisotropy axis are directed along the superlattice axis ($z$ axis). By linearizing the Landau–Lifshitz equation for the magnetization ($M_x, M_y \ll M_0$, $M_z \approx M_0$) and introducing circular projections for the resonance (positive) components of the magnetization and the external magnetic field, we obtain an equation for spin waves in the form [20]

$$\nabla^2 m - \left[\nu - \frac{\Lambda}{\sqrt{2}}\rho(\mathbf{x})\right]m = -\frac{h}{\alpha}. \quad (7)$$

Here, $m = M_x + iM_y$, $h = H_x + iH_y$, $\Lambda = \sqrt{2}\,\Delta\beta/\alpha$, and the frequency $\nu$ (measured in wave-vector units) is equal to

$$\nu = \frac{\omega - \omega_0}{\alpha g M_0}, \quad (8)$$

where $\omega_0$ is the frequency of uniform ferromagnetic resonance, $g$ is the gyromagnetic ratio, and $\alpha$ is the exchange constant.

The high-frequency spin-wave susceptibility $\chi(\nu, k)$ is proportional to the averaged Green's function $G(\nu, k)$ of Eq. (7):

$$\chi(\nu, k) = \langle m(\nu, k)\rangle/h_0 = a(k)G(\nu, k), \quad (9)$$

where $h_0$ is the high-frequency field amplitude. The proportionality coefficient $a(k)$ for the case of a spin-wave resonance in a thin magnetic film was analyzed in detail in [20]. The averaged Green's function for Eq. (7) has the form

$$G(\nu, k) = \frac{1}{\nu - k^2 + \frac{1}{2}\Lambda^2 M(\nu, k)}, \quad (10)$$

where $M(\nu, k)$ is the classical analog of the mass operator. It was shown in [23] that, using an approximation similar to the Bourret approximation [29], this quantity can be represented in general in the form

$$M_i(\nu, \mathbf{k}) = -\frac{1}{4\pi}\int \frac{K_i(\mathbf{r})}{|\mathbf{r}|}\exp[-i(\mathbf{kr} + \sqrt{\nu}|\mathbf{r}|)]d\mathbf{r}, \quad (11)$$

where the correlation function $K(\mathbf{r})$ for a sinusoidal superlattice is determined by Eq. (3).

For 1D and isotropic 3D inhomogeneities, integration over angles in Eq. (11) can be preformed exactly. As a result, the following expressions were obtained in [23]:

$$M_1 = -\frac{1}{2i\sqrt{\nu}}\int_0^\infty \exp\left[-\frac{1}{2}Q_1(r_z) - i\sqrt{\nu}r_z\right] \times [\cos(k-q)r_z + \cos(k+q)r_z]dr_z, \quad (12)$$

for 1D inhomogeneities and

$$M_3 = -\frac{1}{2}\int_0^\infty \exp\left[-\frac{1}{2}Q_3(r) - i\sqrt{\nu}r\right] \times \left[\frac{\sin(k-q)r}{k-q} + \frac{\sin(k+q)r}{k+q}\right]dr \quad (13)$$

for 3D inhomogeneities. (Note that in [23] there is a misprint in the expression corresponding to Eq. (12).) Further integration in these expressions with the true functions $Q_1(r_z)$ and $Q_3(r)$ specified by Eqs. (4) and (5) cannot be conducted analytically. Therefore, the dispersion, damping, and susceptibility were studied in our previous papers by using the following approximating correlation functions:

$$K_1(r_z) \approx \cos q r_z \begin{cases} \exp(-\gamma_1^2 k_\parallel r_z), & \gamma_1^2 \ll 1 \\ \exp(-\gamma_1^2 k_\parallel^2 r_z^2/2), & \gamma_1^2 \gg 1, \end{cases} \quad (14)$$

for 1D inhomogeneities and

$$K_3(r) = \cos q r_z[(1-L)\exp(-\gamma_3^2 k_0 r) + L], \quad (15)$$

for 3D inhomogeneities, where $L = \exp(-3\gamma_3^2)$ is the asymptotic expression of $K_3(r)$ at $r \longrightarrow \infty$. The approximate expressions (14) for $K_1(r_z)$ were grounded in [17], whereas the range of applicability of approximation (15) for $K_3(r)$ has not been determined. We will return to this problem later.

## 3. SPIN-WAVE SPECTRUM AND THE HIGH-FREQUENCY SUSCEPTIBILITY OF A SUPERLATTICE

The dispersion and damping of spin waves are specified by a transcendental equation for the complex frequency $\nu = \nu' + i\nu''$; this equation is obtained by equating the denominator of Green's function (10) to zero:

$$\nu - k^2 + \frac{1}{2}\Lambda^2 M(\nu, \mathbf{k}) = 0. \quad (16)$$

The high-frequency susceptibility of a ferromagnet is proportional to the complex Green's function $G(\nu, \mathbf{k}) = G'(\nu, \mathbf{k}) + iG''(\nu, \mathbf{k})$, which depends on the real frequency $\nu$ of the external high-frequency field and the real wave vector $\mathbf{k}$. The wave spectrum $\nu = \nu(k)$ in a superlattice is known to have a band structure. At the value $k = nq/2$ corresponding to the boundary of the $n$th Brillouin zone, a gap (forbidden band) forms in the frequency spectrum. For sinusoidal superlattices, the boundary of the first Brillouin zone is of special interest, since the widths of the subsequent band gaps decrease rapidly with increasing zone number [22]. In superlattices with sharper interfaces between layers, the decrease in the band-gap width with an increase in $n$ is less pronounced (such situations were considered in [22, 24, 25]). Here, we restrict ourselves to the study of the magnetic susceptibility of a sinusoidal superlattice at the boundary of the first Brillouin zone: $k = k_r \equiv q/2$. In the case where there are no inhomogeneities and natural wave attenuation can be neglected, the gap width in the spectrum at $k = k_r$ is equal to $\Lambda$ [this is the distance between the split-spectrum levels $\nu_+(k_r)$ and $\nu_-(k_r)$]. The $G''(\nu)$ dependence at $k = k_r$ exhibits two $\delta$ peaks spaced $\Lambda$ apart. As the root-mean-square fluctuation $\gamma$ of inhomogeneities increases, the spacing $\nu'_+ - \nu'_-$ between the spectrum levels decreases and the gap in the spectrum closes at a certain critical $\gamma$ value. An increase in $\gamma$ is accompanied by an increase in the damping $\nu''(k)$; this function of $k$ reaches a maximum at

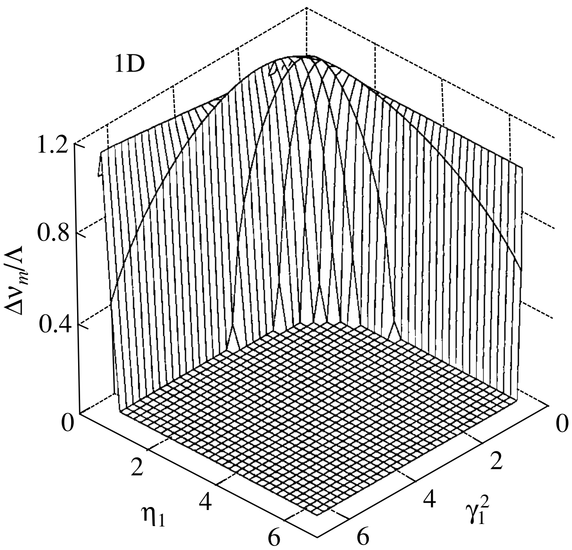


**Fig. 1.** Effect of $\gamma_1^2$ and $\eta_1$ on the spacing between peaks in the imaginary part $G_1''(\nu)$ of the Green's function at the boundary of the first Brillouin zone of a superlattice with 1D inhomogeneities.

$k = k_r$. As $\gamma$ increases, the peaks in the $G''(\nu)$ dependence weaken and approach each other until merging at a certain value of $\gamma$. Qualitatively, the variation in the spacing between the peak maxima $\Delta\nu_m$ corresponds to the variation in the difference $\nu'_+ - \nu'_-$ between the eigenfrequencies; however, there is no exact quantitative relation between these quantities at $\gamma \neq 0$ [20]. This qualitative description of the variation in the spectral gap width and in the spacing between the $G''(\nu)$ maxima with increasing root-mean-square fluctuations is valid for both 1D and 3D inhomogeneities. However, the quantitative differences between the effects of 1D and 3D inhomogeneities are very substantial. For example, in the presence of 1D inhomogeneities, the band gap is closed (or two $G''(\nu)$ maxima merge) at a critical value of $\gamma_1$, which is well below the corresponding critical value $\gamma_3$ for 3D inhomogeneities [23].

As noted above, approximate correlation functions (14) and (15) were used in [17, 20–28] to analyze the eigenfrequencies, damping, and magnetic susceptibility of a superlattice. These approximations make it possible to study the $\nu(k)$ spectrum; otherwise, transcendental equation (16) for $\nu(k)$ cannot be represented in an explicit form without integrating in Eqs. (12) and (13) for the mass operator. However, the susceptibility can be studied without making assumptions regarding $K(\mathbf{r})$, since numerical integration can be performed for each value of $\nu$ in Eqs. (12) and (13) to construct the $G(\nu)$ dependence. Therefore, to calculate the dependences of $G''(\nu)$ on $\gamma_i$ and $\eta_i$, we use both approximate expressions and true expressions (4) and (5) for the structure functions of 1D and 3D inhomogeneities in Eqs. (12) and (13) for the mass operator, respectively.

### *3.1. 1D Inhomogeneities*

With the approximate expression for $K_1(r_z)$ in Eq. (14) corresponding to the condition $\gamma_1^2 \ll 1$, the integral in Eq. (12) for $M_1$ can easily be calculated. At the boundary of the first Brillouin zone ($k = k_r$), we thus obtain a simple expression for the Green's function in the two-wave approximation under the condition $\Lambda$, $k_\parallel^2 \ll \nu$:

$$G_1(\nu) \approx \frac{1}{\Lambda}\frac{X - i\eta_1\gamma_1^2}{X(X - i\eta_1\gamma_1^2) - 1/4}, \qquad (17)$$

where $X = (\nu - k_r^2)/\Lambda$ is the dimensionless frequency detuning from the value $\nu = k_r^2$ and $\eta_1 = k_\parallel q/\Lambda$ is the dimensionless correlation wavenumber. By equating the denominator of this function to zero, we obtain a quadratic equation for the complex frequency $\nu$, from which we find

$$\nu = k_r^2 + \frac{\Lambda}{2}[i\eta_1\gamma_1^2 \pm \sqrt{1 - \eta_1^2\gamma_1^4}]. \qquad (18)$$

As follows from this expression, the spectral gap $\Delta\nu = \nu'_+ - \nu'_-$ is closed at $\eta_1\gamma_1^2 \geq 1$. From Eq. (17) with the real frequency $\nu$, we find that the function $G_1''(\nu)$ has two peaks; the spacing $\Delta\nu_m$ between them decreases with increasing $\gamma_1$ and $\eta_1$, and at $\eta_1\gamma_1^2 \geq 1/\sqrt{2}$, these peaks merge to form one peak.

Figure 1 shows the dependence of $\Delta\nu_m$ on $\gamma_1^2$ and $\eta_1$ calculated by substituting exact structure function (4) into Eq. (12) for the mass operator and performing the integration numerically. As is seen from Fig. 1, the function $\Delta\nu_m$ is symmetric with respect to interchanging the variables $\gamma_1^2$ and $\eta_1$ and is a function of their product with a rather high accuracy. This symmetry is clearly visible for approximate analytical expressions (17) and (18) and is due to the fact that the effective correlation radius of a one-dimensional sinusoidal superlattice at $\gamma_1^2 \ll 1$ is equal to $(\gamma_1^2 k_\parallel)^{-1}$, i.e., is inversely proportional to the product $\gamma_1^2\eta_1$. This symmetry is not so obvious for the $\Delta\nu_m(\eta_1, \gamma_1^2)$ dependence calculated with the exact correlation function. At small values of the product $\gamma_1^2\eta_1$, the spacing $\Delta\nu_m$ between the peaks is slightly in excess of $\Lambda$, which agrees with the analogous effect obtained for the gap width in the wave spectrum in [17]. In that work, this effect was explained in terms of Gaussian correlations, which correspond to the lower line in Eq. (14).

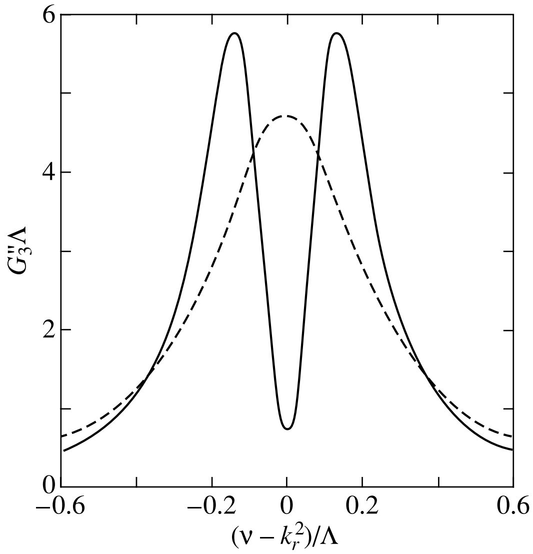


**Fig. 2.** Frequency dependence of the imaginary part $G_3''(\nu)$ of the Green's function at the boundary of the first Brillouin zone of a superlattice with 3D inhomogeneities for $\gamma_3^2 = 1$ and $\eta_3 = 4$ (solid line) and for $\gamma_3^2 = 4$ and $\eta_3 = 1$ (dashed line).

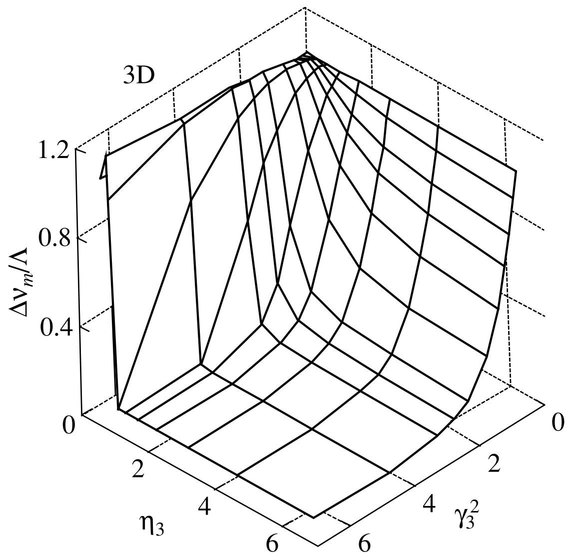


**Fig. 3.** Effect of $\gamma_3^2$ and $\eta_3$ on the spacing between peaks in the imaginary part $G_3''(\nu)$ of the Green's function at the boundary of the first Brillouin zone of a superlattice with 3D inhomogeneities.

### *3.2. 3D Inhomogeneities*

With approximate expression (15) for the correlation function of 3D inhomogeneities, the integral in Eq. (13) for $M_3$ can be calculated exactly. At the boundary of the first Brillouin zone, the Green's function in the two-wave approximation and under the condition $\Lambda$, $k_0^2 \ll \nu$ takes the form

$$G_3 \approx \frac{1}{\Lambda}\frac{X(X - i\eta_3\gamma_3^2)}{X^2(X - i\eta_3\gamma_3^2) - 1/4(X - i\eta_3\gamma_3^2 L)}, \qquad (19)$$

where $\eta_3 = k_0 q/\Lambda$ is the dimensionless wavenumber of 3D inhomogeneities.

By equating the denominator of this function to zero, we obtain a cubic equation for the complex frequency; the dependence of the frequency on $\gamma_3^2$ was analyzed numerically in [25]. As follows from Eq. (19), the Green's function is not symmetric with respect to interchanging the parameters $\gamma_3^2$ and $\eta_3$ in the case of 3D inhomogeneities. Indeed, unlike function (17), which contains only the product $\gamma_1^2\eta_1$, function (19) contains not only the product $\gamma_3^2\eta_3$ but also the asymptotic value $L$ of the correlation function, which depends on $\gamma_3^2$ alone.

Figure 2 shows the frequency dependence of the function $G_3''$ at the boundary of the first Brillouin zone of the superlattice ($k = k_r$) calculated numerically with exact structure function (5) in Eq. (13) for the mass operator. Both curves in Fig. 2 correspond to $\gamma_3^2\eta_3 = 4$. However, the solid line was plotted at $\gamma_3^2 = 1$ and $\eta_3 = 4$, whereas the dashed line was plotted at $\gamma_3^2 = 4$ and $\eta_3 = 1$. In the first case, the function $G_3''(\nu)$ is seen to have two pronounced peaks (i.e., there is a gap in the wave spectrum), whereas in the second case both peaks merge to form one broad peak (the gap in the spectrum is closed). Figure 3 shows the dependence of the spacing between the peaks $\Delta\nu_m$ on $\gamma_3^2$ and $\eta_3$ calculated using exact structure function (5). This dependence differs radically from the $\Delta\nu_m(\gamma_1^2, \eta_1)$ dependence for the 1D inhomogeneities shown in Fig. 1: the function $\Delta\nu_m$ for 3D inhomogeneities is asymmetric with respect to interchanging $\gamma_3^2$ and $\eta_3$. The difference between the spectral characteristics of superlattices with 1D and 3D inhomogeneities is due to the radically different correlation functions of 1D and 3D inhomogeneities. This difference can clearly be illustrated using approximate analytical expressions (14) and (15), whose asymptotic behavior coincides with that of the exact functions $K_1(r_z)$ and $K_3(r)$. For 1D inhomogeneities, the correla-

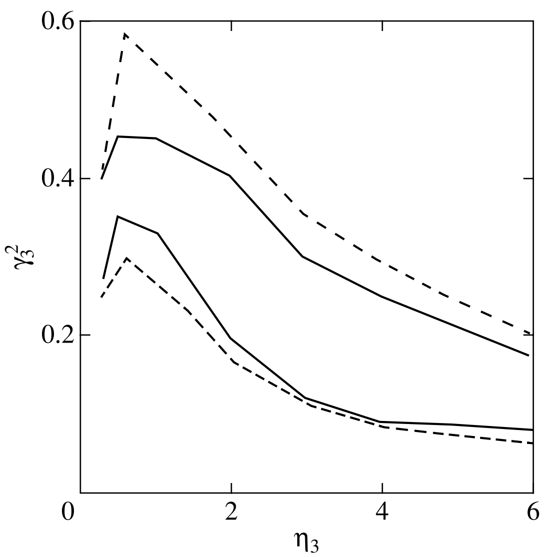


**Fig. 4.** Ranges of applicability of approximate expression (15) for the correlation function and approximate expression (19) for the Green's function. The differences between Eq. (19) and the Green's function as calculated using the exact correlation function do not exceed 10% in the region between the solid lines and 20% in the region between the dashed lines.

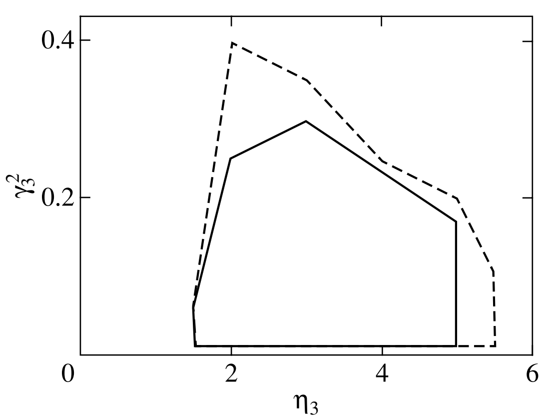


**Fig. 5.** Ranges of applicability of approximate expressions (20) for the correlation function and (21) for the Green's function. The differences between Eq. (21) and the Green's function as calculated using the exact correlation function do not exceed 10% in the region restricted by solid lines and 20% in the region restricted by dashed lines.

tion function tends to zero at $r_z \longrightarrow \infty$, whereas the descending portion of $K_3(r)$ at $r \longrightarrow \infty$, tends to a nonzero asymptotic value $L$, which depends on $\gamma_3^2$ and is independent of $\eta_3$.

In this work, the magnetic susceptibility of a partly randomized sinusoidal superlattice is calculated for the first time using the exact correlation functions $K_1(r_z)$ and $K_3(r)$. This allowed us to compare the exact results and the results calculated with approximate correlation functions (14) and (15) and determine the range of applicability of the latter functions. For 3D inhomogeneities, the range of applicability of approximate expression (15) for the correlation function and approximate expression (19) for the Green's function is shown in Fig. 4. To find this range, we compared both the spacing between the peaks in the imaginary part of the Green's function and the peak widths obtained using the approximate and exact correlation functions. Then, on the ($\gamma_3^2$, $\eta_3$) parametric plane, we determined the region where the difference between these characteristics did not exceed 10% (solid lines) or 20% (dashed lines). It should be noted that the peak width was found to be a critical characteristic in most cases. The spacing between the peaks is described by approximate expression (19) with a much higher accuracy than is the peak width. As is seen from Fig. 4, there is a rather broad region of parameters $\gamma_3^2$ and $\eta_3$ in which the approximate analytical expression for the Green's function (19) is valid. The fact that this expression gives bad results for small values of $\gamma_3^2$ came as a surprise.

For this reason, we calculated the Green's function for 3D inhomogeneities using another approximating correlation function,

$$K_3(r) = \cos qr_z\left[(1-L)\exp\left(-\frac{\gamma_3^2 k_0^2 r^2}{2}\right) + L\right]. \quad (20)$$

This function differs from Eq. (15) in that it falls off as a Gaussian rather than exponentially, as is the case in Eq. (15). With Eq. (20), the integral in Eq. (13) for $M_3$ can also be calculated exactly. In the two-wave approximation and under the conditions used above ($\Lambda$, $k_0^2 \ll \nu$), the Green's function at the boundary of the first Brillouin zone can be found to be

$$G_3 \approx \frac{1}{\Lambda}\left\{X - \frac{1-L}{2\sqrt{2}\eta_3\gamma_3}\left[D\left(\frac{X}{\sqrt{2}\eta_3\gamma_3}\right) - i\frac{\sqrt{\pi}}{2}\exp\left(-\frac{X^2}{2\eta_3^2\gamma_3^2}\right)\right] - \frac{L}{4X}\right\}^{-1}, \quad (21)$$

where $D(s) = e^{-s^2}\int_0^s e^{t^2}dt$ is Dawson's integral. The frequency dependence of the function $G_3''(\nu)$ as described by Eq. (21) is compared with $G_3''(\nu)$ calculated using the exact correlation function. The characteristics for comparison were the same as above, namely, the spacing between the peaks and the peak width. As a result, we obtained the range of applicability of Eq. (21) and approximate correlation function (20) of a superlattice (Fig. 5). It is seen that this region overlaps only partially with the region shown in Fig. 4. A comparison of these regions shows that, e.g., at small values of $\eta_3$ and $\gamma_3^2 \approx$ 0.3–0.5, the approximation of the correlation function by Eq. (15) is more exact as compared to Eq. (20). At small values of $\gamma_3^2$ and $\eta_3 \approx 2$–5, the situation is reversed and Eq. (20) is more exact. Thus, approximations (15) and (20) complement each other.

## 4. CONCLUSIONS

We have studied the effect of the correlation properties of 1D and 3D structural inhomogeneities of an initially sinusoidal ferromagnetic superlattice on its high-frequency magnetic susceptibility. To describe the stochastic properties of inhomogeneities, we used correlation functions derived earlier using the method of random spatial modulation of the superlattice period [17]. In this method, structural inhomogeneities in a superlattice are described in terms of the model of a random phase, which is assumed to depend on the $z$ coordinate in the case of 1D inhomogeneities and on all three coordinates ($x$, $y$, $z$) for 3D inhomogeneities. The random phase is characterized by a monotonically decreasing correlation function with arbitrary values of the relative root-mean-square fluctuations $\gamma_i$ and normalized correlation wavenumbers $\eta_i$, where $i = 1$ and 3 for 1D and 3D inhomogeneities, respectively. As shown earlier in [17], the form of the correlation functions $K_i(r)$ of the superlattice obtained using this model and the RSM method depends only weakly on the form of the correlation functions characterizing the stochastic properties of the random phase. However, this form depends strongly on the dimensionality of inhomogeneities: for 1D inhomogeneities, we have $K_1(r_z) \longrightarrow 0$ as $r_z \longrightarrow \infty$, whereas for 3D inhomogeneities $K_3(r)$ tends to a nonzero asymptotic value $L = \exp(-3\gamma_3^2)$ as $r \longrightarrow \infty$.

These correlation functions have been used to calculate the averaged Green's function $G_i(\nu, k)$ from which the high-frequency susceptibility is determined in the case of 1D and 3D inhomogeneities. The mass operator of the Green's function was found in the Bourret approximation by performing numerical integration of expressions containing the exact correlation functions $K_1(r_z)$ and $K_3(r)$ for the 1D and 3D cases, respectively. The frequency dependence of the imaginary part $G''(\nu)$ of the Green's function was studied at a fixed value of the wave vector $k$ corresponding to the boundary of the first Brillouin zone of the superlattice (experimentally, the wavenumber can be fixed due to the size effect in the situation corresponding to a spin-wave resonance in a superlattice film [20]). In this case, the $G_i''(\nu)$ dependence exhibits two peaks and the spacing between the peaks $\Delta\nu_m$ approximately corresponds to the gap width in the wave spectrum at the boundary of the Brillouin zone. The dependence of $\Delta\nu_m$ on $\gamma_i$ was studied earlier in [20, 23] using approximate expressions for $K_1(r_z)$ and $K_3(r)$. In this work, we have studied the dependences of $\Delta\nu_m$ on both $\gamma_i$ and $\eta_i$ using exact expressions for $K_1(r_z)$ and $K_3(r)$. For 1D inhomogeneities, the two-dimensional function $\Delta\nu_m(\gamma_1, \eta_1)$ was shown to be symmetric with respect to interchanging the variables $\gamma_1^2$ and $\eta_1$ (Fig. 1), whereas in the case of 3D inhomogeneities the function $\Delta\nu_m(\gamma_3, \eta_3)$ is strongly asymmetric with respect to interchanging $\gamma_3^2$ and $\eta_3$ (Fig. 3). This effect is associated with the difference in form between the correlation functions for 1D and 3D inhomogeneities. For the correlation function $K_1(r_z)$, the correlation radius is inversely proportional to the product $\gamma_1^2\eta_1$, which causes the function $\Delta\nu_m$ to be symmetric with respect to interchanging $\gamma_1^2$ and $\eta_1$. The correlation radius of the function $K_3(r)$ is inversely proportional to the analogous product $\gamma_3^2\eta_3$. However, the function $K_3(r)$ differs from $K_1(r_z)$ in terms of the asymptotic value $L$, which divides the entire correlation volume into two parts, one of which is characterized by a finite correlation radius (above the asymptotic value $L$) and the other by an infinite correlation radius (below $L$). This asymptotic value depends on $\gamma_3^2$ and is independent of $\eta_3$, which leads to asymmetry of the function $\Delta\nu_m(\gamma_3, \eta_3)$ with respect to interchanging $\gamma_3^2$ and $\eta_3$. This effect can be used to determine the dimensionality of structural inhomogeneities in a superlattice by spectral methods if independent changes in the values of $\gamma_i^2$ and $\eta_i$ can be controlled technologically.

We have also compared the functions $\Delta\nu_m(\gamma_3, \eta_3)$ as calculated using either the exact correlation function $K_3(r)$ or approximate analytical expressions for this function. This comparison allowed us to construct diagrams in the ($\gamma_3^2$, $\eta_3$) plane (Figs. 4, 5) that determine the range of applicability of the approximate analytical expressions for $K_3(r)$, namely, Eq. (15), which was used earlier in [25–28], and Eq. (20), which was derived in this work. These diagrams also specify the range of applicability of approximate analytical expressions (19) and (21) for the Green's function.

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