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Trapping of an Electron in the Transmission through Two Quantum Dots Coupled by a Wire¶

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We consider single-channel transmission through a double quantum dot that consists of two identical single dots coupled by a wire. The numerical solution for the scattering wave function shows that the resonance width of a few of the states may vanish when the width (or length) of the wire and the energy of the incident particle each take a certain value. In such a case, a particle is trapped inside the wire as the numerical visualization of the scattering wave function shows. To understand these numerical results, we explore a simple model with a small number of states, which allows us to consider the problem analytically. If the eigenenergies of the closed system cross the energies of the transmission zeroes, the wire effectively decouples from the rest of the system and traps the particle. *© 2005 Pleiades Publishing, Inc.*

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Dramatic changes of the widths of resonance states may arise by resonance trapping: at strong coupling, a few resonance states of the system align with the channels and become short-lived, while the remaining ones decouple more or less strongly from the continuum of decay channels. This effect, which was first found in nuclear reactions [1], has been observed meanwhile in many different systems [2]. In atoms, it may appear as population trapping in laser-induced continuum structures [3]. In microwave cavities, it is studied theoretically [4] and experimentally [5]. Resonance trapping is caused by the existence of singular points in the complex energy plane [6].

In the transmission through microwave cavities or quantum dots (QDs), an additional parameter for varying the widths of the resonance states is involved. The transmission is determined by the manner the leads are attached to them, and the widths of the resonance states can be changed even without changing the coupling strength between the system and lead [7]. In a double QD, an internal wire couples the two single dots. The coupling and the wire's energy can be controlled. This allows us to even stabilize the system at certain parameter values without varying the coupling strength to the environment [8], at least in the one-channel case. On the one hand, the position of the transmission zeroes through such a system is determined by the spectroscopic properties of the single dots since the leads are attached only to them. On the other hand, the transmission is resonant in any case and related to the spectroscopic properties of the double QD as a whole. These two conflicting facts cause some nontrivial constraint on the system in order to fulfill the unitarity of the *S*-matrix [8]. As a consequence, the widths of the resonance states may be strongly parameter-dependent, and some of them may even *vanish* at certain parameter values. Such a case presents a novel bound state that corresponds to the confinement of an electron in the internal wire as we will show in the following.

The relation between transmission zeroes and resonance states with vanishing width has been studied also by other authors. Firstly, a drastic narrowing of the resonant peak was shown by Shahbazyan and Raikh in a junction of two resonant impurities [9]. In [10], an antibonding state is found to be totally decoupled from the leads and to give rise to a "ghost" Fano peak with zero width in the system of two coupled QDs. According to [11], a dynamic confinement of electrons in time-dependent quantum structures may appear due to the coherent interaction between two Fano resonances. The system studied is a double-well structure in which the electrons are confined in the region between the two wells at some special values of the energy of the incident particle and the length of the region between the wells. It is the aim of the present study to show that a similar phenomenon appears in the system of identical QDs connected by a wire. Localization of electrons in the wire takes place in this system without time-periodic perturbation, as we will show by considering the wave functions of the resonance states with vanishing width.

¶ The text was submitted by the authors in English.

The split gate in the middle of a double QD allows us to control, e.g., the width W of the wire that connects the two single QDs [12–14]. Using the exact correspondence between the quantum-mechanical description of the single-electron transmission through a QD and the transmission of planar electromagnetic waves through a microwave billiard [15], it is convenient to control the length L of the connecting waveguide. In any case, considering the wire as a stripe with the width W and the length L, we present the wire as a third quantum subsystem with the energies $\varepsilon_w \sim m^2W^{-2} + n^2L^{-2}$, where m and n are the quantum numbers of the wire. These modes appear in addition to the eigenenergies $\varepsilon_i \propto R^{-2}$ of the single QDs, where R is the characteristic scale of the single dot. If L, $W \ll R$ and $\varepsilon_w \gg \varepsilon_i$, the role of the wire as a third quantum subsystem is not relevant because the coupling between the dots is of tunneling type. However, for the case $\varepsilon_w \sim \varepsilon_i$, the wire degrees of freedom are important. In this case, the quantum system consists, indeed, of two quantum subsystems coupled by a third quantum subsystem that has its own energy spectrum. In the following, we present the theory for the one-channel case of this system. It is not restricted to the description of double billiards but can be applied also to the transmission through a system consisting of two scattering centers that are connected by a waveguide. Such a system might be, e.g., waveguides with two bends [16–18].

In order to vary smoothly the width of the wire in the numerical computation, we apply an auxiliary potential

$$V(x, y) = V_0\{1 + 0.5[\tanh(C(y - W/2)) - \tanh(C(y + W/2))]\} \quad (1)$$

to squeeze the wire in a similar manner as in a real double QD system [12–14]. In computations, we take $V_0 = 100$, $C = 17$. The transmission probability, presented in the log scale in Fig. 1, clearly shows transmission zeroes appearing at certain energies independently of the width W of the stripe (for details, see the discussion in [8]). The most interesting features appear, however, at the points where the eigenenergies of the double QD system cross the transmission zeroes (two points are marked by open circles in Fig. 1).

Originally, the spectroscopic values such as the positions in energy of states are defined for the discrete eigenstates of Hermitian Hamiltonian H_B that describes the closed quantum system. When embedded into the continuum of scattering states, the discrete eigenstates of the closed system turn over into resonance states with a finite lifetime. The effective Hamiltonian $H_{\rm eff}$ of the open quantum system contains H_B as well as an additional term [2] that describes the coupling of the resonance states to the common environment,

$$H_{\rm eff} = H_B + \sum_C V_{BC}\frac{1}{E^+ - H_C}V_{CB}. \quad (2)$$

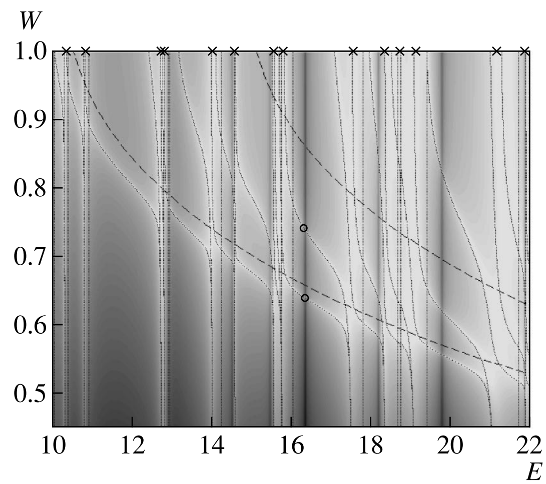

Fig. 1. The probability $\ln(T(E, W))$ for the transmission through the double QD shown in Fig. 3 versus energy and width of the wire. The eigenenergies of each single QD are marked by crosses. The eigenenergies of the wire are shown by dashed curves, and those of the closed double QD system, by thin dotted curves. In the present calculation, the eigenenergies of the closed system are almost the same as the eigenenergies of the open system. The radius of the QD in units of the width d of the input and output leads is $3d$. The length of the wire is $2.5d$. The size of all diaphragms is $0.6d$.

Here, V_{BC} and V_{CB} stand for the coupling matrix elements between the eigenstates of H_B and the environment that may consist of different continua C, e.g., the scattering waves propagating in the left and right leads attached to the closed system. The concept of the effective Hamiltonian appeared first in Feshbach's papers [19] and, independently, in Livshitz's study of open quantum system [20]. $H_{\rm eff}$ is non-Hermitian, its eigenvalues z_k and eigenfunctions are complex and contain the "external" interaction of the resonance states via the continuum. The complex eigenvalues of the effective Hamiltonian determine the positions and widths of the resonance states. They are energy-dependent functions, since the non-Hermitian effective Hamilton operator (2) depends on energy. Nevertheless, spectroscopic values for resonance states can be defined also for the resonance states [2, 21] by solving the fixed-point equations

$$E_k = {\rm Re}(z_k)|_{E = E_k} \quad (3)$$

and defining

$$\Gamma_k = 2{\rm Im}(z_k)|_{E = E_k}. \quad (4)$$

The values E_k and Γ_k characterize a resonance state whose position in energy is E_k and whose decay width is Γ_k. These values coincide approximately with the poles of the S matrix.

The effective Hamiltonian $H_{\rm eff}$ appears in the derivation of the S matrix [2, 21]. Calculations performed

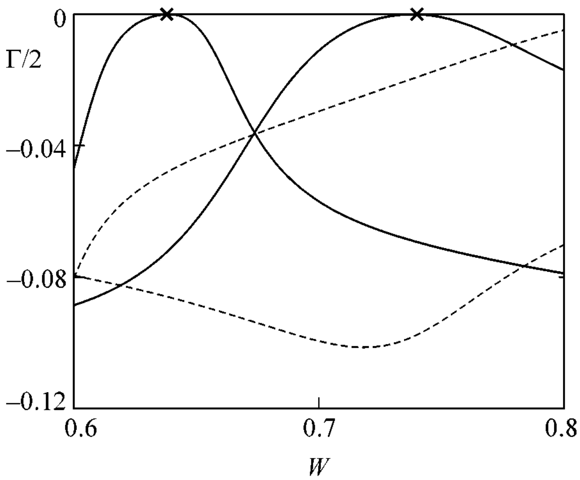

Fig. 2. The imaginary part Γ_k of the first four eigenvalues z_λ as a function of W for E = 16.36. The first two Γ_k which vanish at W = 0.638 and W = 0.74 are shown by the solid curves. Corresponding points are marked by crosses. They exactly correspond to points shown in Fig. 1 by open circles.

with H_{eff} correspond therefore to calculations on the basis of the S matrix, but they contain additionally a unique definition of the spectroscopic values [21]. Explicitly in the tight-binding approach, H_{eff} is given in [22, 23]. The results of a numerical computation for the system consisting of two quantum dots coupled by a wire are shown in Fig. 2 for the imaginary parts of four complex eigenvalues z_k. The points marked by crosses in Fig. 3 are found by self-consistent solution of Eqs. (3) and (4). They exactly correspond to the points shown in Fig. 1.

The numerically computed scattering wave functions (Fig. 3) demonstrate that, at these points, the electrons are localized inside the wire that connects the two single QDs. This effect appears, of course, only in the open double-dot system. In the closed system, a localization in the wire will never occur.

In order to understand this mechanism of electron confinement, we use the periodicity of the transmission picture (Fig. 1), which allows us to restrict the investigation to the transmission properties of a simple model with only a few states in each single QD [8].

The Hamiltonian of the closed system consists of three parts: two parts describe the two single QDs, and a third one is related to the wire. The Hamiltonian of minimal dimension, which can cause a zero in the transmission through a single QD, is two [8]. Then, the total Hamiltonian, which can explain the characteristic features of Figs. 1 and 2, has the following matrix form:

$$H_B = \begin{pmatrix} \varepsilon_1 & 0 & u & 0 & 0 \\ 0 & \varepsilon_2 & u & 0 & 0 \\ u & u & \varepsilon_w & u & u \\ 0 & 0 & u & \varepsilon_2 & 0 \\ 0 & 0 & u & 0 & \varepsilon_1 \end{pmatrix}. \quad (5)$$

For simplicity, it is assumed here that the two single QDs are identical and that all the coupling constants u between the wire and the single QDs are the same. The Hamiltonian (5) differs from those used in the literature [10, 24, 25] for the description of a double QD by taking explicitly into account the wire as a third subsystem. The eigenenergies ε_w of the wire depend on at least two values: on the width W and the length L. Without loss of generality, we can consider the energy ε_w to be the parameter by which the system can be controlled.

The knowledge of the eigenstates of the closed quantum system allows us to formulate the S-matrix

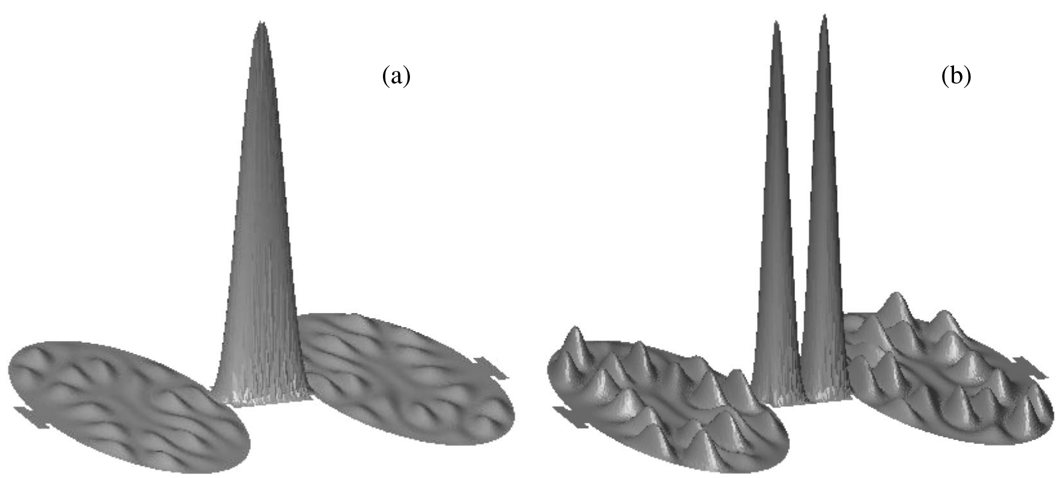

Fig. 3. The density probability for the transmission through the double QD for the two cases shown in Fig. 1 by open circles: (a) E = 16.36, W = 0.638; (b) E = 16.36, W = 0.74.

and the effective Hamiltonian in the manner described in [23, 26] in order to consider the transmission through the system. Let E_m and $|m\rangle$ with $m = 1, \ldots, 5$ denote the five eigenenergies and eigenstates of (5). The amplitudes $\langle j = 1, 2|m\rangle$ describe the left billiard, $\langle j = 3|m\rangle$ the wire, and $\langle j = 4, 5|m\rangle$ the right billiard. Similar to [10], we assume that the left lead is coupled to both states ($j = 1, 2$) of the left billiard with the same strength v. Correspondingly, the right lead is coupled to both states ($j = 4, 5$) of the right billiard with the same strength v. Then, the coupling matrix elements between the closed system and the two leads L and R can be written as

$$\langle E, C|V|m\rangle = v\sqrt{\frac{\sin k}{2\pi}}V_C(m), \tag{6}$$

where $C = L, R$, $V_L(m) = v\sum_{j=1,2}\langle j|m\rangle$, $V_R(m) = v\sum_{j=4,5}\langle j|m\rangle$. The factor $\sqrt{\sin k}$ in (6) results from the one-dimensional leads [8, 23]. The matrix elements of the effective Hamiltonian are [8, 23, 26]

$$\begin{aligned}&\langle m|H_{\mathrm{eff}}|n\rangle \\ &= E_m\delta_{mn} - (V_L(m)V_L(n) + V_R(m)V_R(n))e^{ik}.\end{aligned} \tag{7}$$

Using the S-matrix formalism [2, 23, 26], the amplitude for the transmission through the system reads [23]

$$t = -2\pi i\sum_{\lambda}\frac{\langle L|V|\lambda)(\lambda|V|R\rangle}{E - z_\lambda}. \tag{8}$$

Figure 4a closely reproduces a fragment of the total picture (Fig. 1), including the appearance of transmission zeroes. According to Fig. 4b, the decay widths of the eigenstates 2 and 4 of the effective Hamiltonian are independent of the wire's energy ε_w, while those of the other states depend on it. The state 3, lying in the middle of the spectrum, crosses the transmission zero at

$$\varepsilon_w = \varepsilon_b = \frac{\varepsilon_1 + \varepsilon_2}{2}. \tag{9}$$

At this energy, we have from (5) the following eigenenergies

$$E_{1,5} = \varepsilon_b \pm \eta, \quad E_2 = \varepsilon_1, \quad E_3 = \varepsilon_b, \quad E_4 = \varepsilon_2 \tag{10}$$

and eigenstates

$$\langle 1| = \frac{\sqrt{2}u}{\eta}\left(\frac{u}{\eta - \Delta\varepsilon}, \frac{u}{\eta + \Delta\varepsilon}, -1, \frac{u}{\eta + \Delta\varepsilon}, \frac{u}{\eta - \Delta\varepsilon}\right),$$

$$\langle 2| = \frac{1}{\sqrt{2}}(1, 0, 0, 0, -1),$$

$$\langle 3| = \frac{u}{\eta}\left(1, -1, \frac{\Delta\varepsilon}{u}, -1, 1\right), \tag{11}$$

$$\langle 4| = \frac{1}{\sqrt{2}}(0, 1, 0, -1, 0),$$

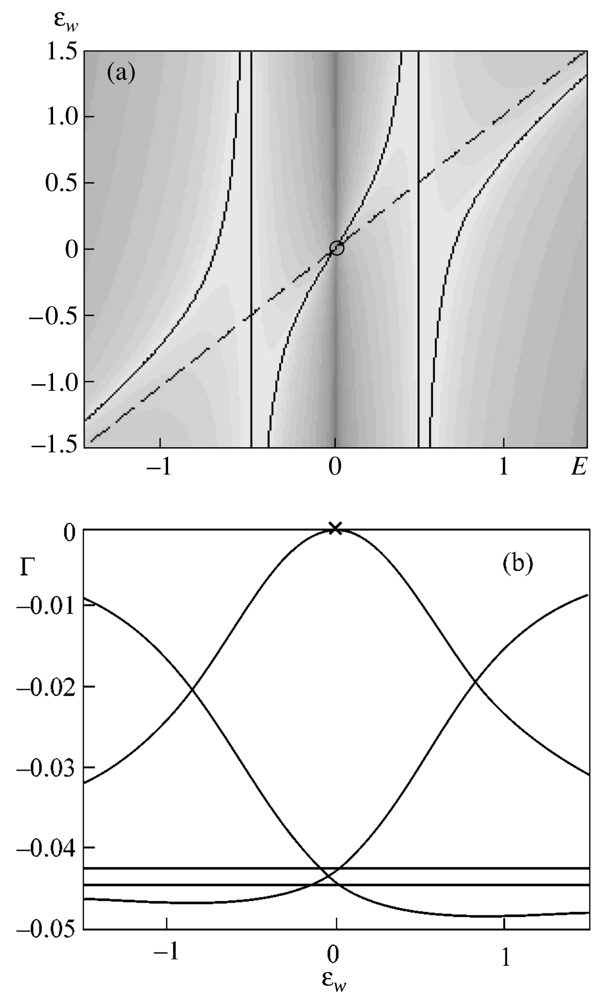

Fig. 4. (a) The transmission through a system with two identical QDs that are connected by a wire versus incident energy E and energy of the wire ε_w. The eigenenergies of (5) are shown by solid curves, and the energy ε_w of the wire is shown by the dashed curve. The QDs have energies $\varepsilon_{1,2} = \pm 1/2$. $v = 1/2$, $u = 1/4$. (b) The imaginary part of the five eigenvalues z_λ of the effective Hamiltonian (2) as a function of ε_w for $E = 0.5$ (at $E = 0$, the widths of the two short-lived states are equal). One of imaginary parts is equal to zero at $\varepsilon_w = \varepsilon_b = 0$ for all energies E marked by cross.

$$\langle 5| = \frac{\sqrt{2}u}{\eta}\left(\frac{u}{\eta + \Delta\varepsilon}, \frac{u}{\eta - \Delta\varepsilon}, 1, \frac{u}{\eta - \Delta\varepsilon}, \frac{u}{\eta + \Delta\varepsilon}\right),$$

where $\eta^2 = \Delta\varepsilon^2 + 4u^2$, $\Delta\varepsilon = (\varepsilon_2 - \varepsilon_1)/2$. Substituting (11) into (6), we obtain

$$\langle m|V|E, C = L, R\rangle = v\sqrt{\frac{\sin k}{8\pi}}(1 \pm 1 \quad 0 \pm 1 \quad 1) \tag{12}$$

for the elements of the coupling matrix. One can see that, under condition (9), the wire decouples from the rest of the system. Substitution of (12) into (2) shows

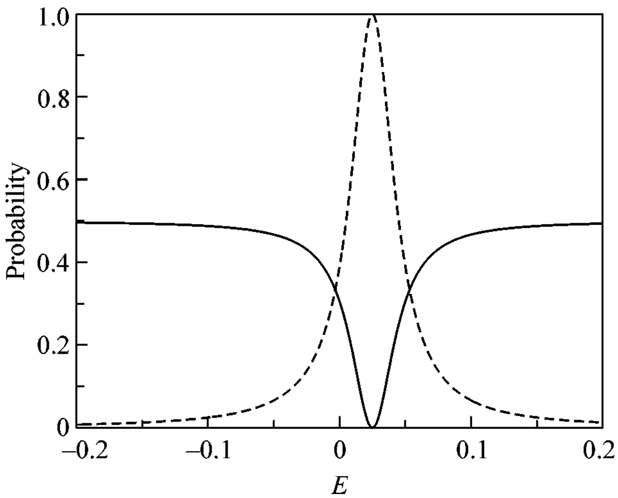

Fig. 5. The probabilities for the electron to be in the wire (dashed curve) and in the right and left QD (full curve) as a function of the wire's energy. $\varepsilon_{1,2} = \pm 1/2$, $v = 1/2$, $E = 0.02$, $u = 0.2$.

that, at $\varepsilon_w = \varepsilon_b$, the effective Hamiltonian matrix 5×5 decomposes as $H_{\text{eff}} = H_1 \oplus E_3 \oplus H_2$, where

$$H_1 = \begin{pmatrix} E_1 + \Gamma & \Gamma \\ \Gamma & E_5 + \Gamma \end{pmatrix}; \qquad H_2 = \begin{pmatrix} E_2 + \Gamma & \Gamma \\ \Gamma & E_4 + \Gamma \end{pmatrix}, \tag{13}$$

and $\Gamma = -v^2 \exp(ik)/2$. Obviously, the imaginary part of the third eigenvalue of H_{eff} is zero, i.e., the width of the third eigenstate vanishes at $\varepsilon_w = \varepsilon_b$. According to formula (105) of [23], the scattering wave function can be decomposed in the set of eigenfunctions of H_{eff},

$$\psi = \sum_{\lambda, C = L, R} \frac{V_\lambda(E, C)}{E^+ - z_\lambda} \psi_\lambda, \tag{14}$$

where the $V_\lambda(E, C)$ are the coupling matrix elements between the leads C and the *open* double QD described by the eigenstates ψ_λ of H_{eff}. Since the eigenvalue z_3 is real, $V_3(E, C) \longrightarrow 0$ if $E \longrightarrow z_3 = E_3 = \varepsilon_b$. Therefore, a particle moving from the left (right) lead to the left (right) billiard is fully reflected. That means the wire is decoupled from the reservoirs and the electrons are trapped inside the wire. According to (8), the contributions from $\lambda = 1$ and 5 as well as those from $\lambda = 2$ and 4 to the transmission cancel each other at $E = \varepsilon_b$ due to the symmetry in relation to ε_b, while the contribution from $\lambda = 3$ vanishes. As a consequence, an electron will not be transmitted at the energy $E = \varepsilon_b$. The probability for the electron to be in the left QD is $\sum_{j=1,2} |\psi(j)|^2$ and in the right one $\sum_{j=4,5} |\psi(j)|^2$. For the case of time reversal symmetry (both leads are equivalent), these probabilities coincide (Fig. 5, full curve). The probability for the electron to be in the wire is given by $|\psi(3)|^2$ (Fig. 5, dashed curve). A localization of the electron in the wire takes place when $\varepsilon_w = \varepsilon_b$.

The physical reason of the localization of electrons inside the wire in the one-channel case is flux conservation expressed by the unitarity of the S-matrix. A full localization takes place only when the two single QDs are identical and both leads are equivalent. The effect exists also when the two single dots are slightly different from one another and (or) time reversal symmetry is broken. It is, however, somewhat reduced in such a case. These features are similar to those observed in laser-induced continuum structures in atoms, which are called population trapping [3]. The mechanism by which the resonance states with vanishing width are created is, however, different in the two cases.

The electron localization in the transmission through a system with two identical QDs can be seen also in the generalized Fabry–Perot approach [11]. We ignore the evanescent modes in the wire, the total transmission amplitude in the single-channel case can be easily calculated as a geometrical sum over all the individual transmitted and reflected elementary processes. This gives the simple expression

$$T = \frac{t_1^2}{1 - r_1^2 \exp(2iqL)} \tag{15}$$

for the transmission probability, where t_1 and r_1 are the complex amplitudes of the transmission and reflection for a single quantum billiard, and q is the wave number of the connecting waveguide related to the energy of the single-channel transmission by $E = q^2 + \pi^2/W^2$. The bound states are defined by the zeroes of the denominator in T, i.e., by $\sin(\phi(E) + q(E)L) = 0$, where $\phi(E) = \arg(r1)$. One obtains, therefore, a quantization rule for the particle trapped in the one-dimensional box of length L. In fact, Fig. 3 shows the first two of these bound states in the wire. However, exponentially small evanescent modes in the internal wire slightly violate formula (15).

In conclusion, controlling the eigenenergies of the wire that connects the two single QDs of a double QD, by means of the gate voltage, the widths of the resonance states of the double QD system and the transmission through this system can be manipulated. When the width of one of the resonance states vanishes, the electrons are trapped in the wire and the transmission is zero. This effect might be used for quantum information storage.

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