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Dipole and Bloch Oscillations of Cold Atoms in a Parabolic Lattice

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Abstract—The paper studies the dynamics of a Bose–Einstein condensate loaded in a 1D parabolic optical lattice and excited by a sudden shifting of the lattice center. Depending on the initial shift, this dynamics is either the dipole or Bloch oscillations of the atoms. The effects of the dephasing and atom–atom interactions on the above atomic oscillations are discussed.

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1. Bloch oscillation (BO) of a quantum particle in a periodic potential are one of the most fascinating phenomena of quantum physics [1]. Since the pioneering experiment [2] in 1996, this phenomenon has been intensively studied for the cold atoms in optical lattices [3], with recent emphasis on the effect of the quantum statistics (Fermi or Bose) and the atom–atom interactions. In particular, the dynamics of the degenerate Bose gases, on which we will focus here, was studied experimentally in the works [4–6]. It should be stressed from the very beginning that, when addressing this problem theoretically, one has to distinguish between the quasi-one-dimensional lattices (created by two counterpropagating laser beams) and truly 1D lattices (or so-called modulated quantum tubes). Indeed, in the former case, the number of the atoms per one well of the optical lattice can be as large as 10^3–10^4 and the mean field approach (based on Gross–Pitaevskii or nonlinear Schrödinger equation) is generally justified. This is not the case of the truly 1D lattices, where only a few atoms occupy each well, and, hence, a microscopic analysis is required. For a tilted infinite lattice, such an analysis, based on the Bose–Hubbard model, was presented in our recent works [7–9], where two regimes of BO—quasiperiodic and irreversibly decaying—were identified.

When referring to the typical laboratory experiment, an additional complication comes from the presence of harmonic confinement. Clearly, harmonic confinement should modify BO of Bose atoms, and the aim of this work is to estimate its effect. At the same time, the parabolic lattices are of their own interest, because they also allow the dipole oscillations of BEC to be studied. The recent experiment [10] has shown that there is a fundamental difference between the dipole oscillations in the quasi- and truly 1D lattices. While in the former case the main effect of the periodic potential can be taken into account by simply replacing the atomic mass with its effective mass in the ground Bloch band [11], one observes a rapid dissipative decay of oscillations in the latter case. In the present paper, we also briefly discuss the dipole oscillations of BEC in truly 1D lattices, partially overlapping in this part with recent theoretical work [12].

2. We consider atoms in the parabolic lattice $V(x) = M\omega^2x^2/2 - V\cos^2(2\pi x/d)$, where the atoms are set into motion by a sudden shift of the trap origin. The relevant parameters of the system are the hopping matrix element J, the "parabolicity" $\nu = M\omega^2d^2$, and the initial shift $l_0 = \Delta x/d$. Neglecting the atom–atom interactions, the dynamics of the system is described by the pendulum model [3, 13],

$$i\hbar\dot{a}_l = \frac{\nu}{2}l^2a_l - \frac{J}{2}(a_{l+1} + a_{l-1}), \qquad (1)$$

where $a_l(t)$ is the complex amplitude of the atoms in the lth well of the optical lattice. The separatrix of the pendulum corresponds to the shift

$$l^* = 2(J/\nu)^{1/2}. \qquad (2)$$

If the initial shift $l_0 < l^*$, the pendulum shows oscillations around the trap origin and, referring to the original system, this regime is regarded as the dipole oscillations of the atoms. If $l_0 > l^*$, the pendulum is in the rotational regime, and the dynamics of the atoms can be regarded as BO in a local static field $F = \nu l_0/d$. We begin with BO and shall assume for a moment the absence of any atom–atom interactions.

Because the local static force F is not homogeneous, we have an additional process of dephasing of BO in a parabolic lattice, as compared to the paradigm case of a tilted infinite lattice. When discussing the mean atomic

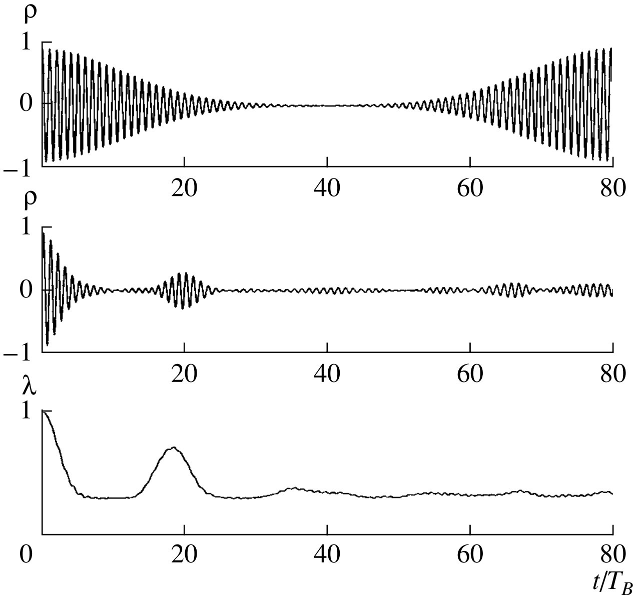

Fig. 1. Bloch oscillations of $N = 5$ atoms in the parabolic lattice with parabolicity $\nu = 0.04J$: (a) the mean momentum of noninteracting atoms; (b) the mean momentum of interacting ($W = 0.2J$) atoms; (c) macroscopic coherence of the system of interacting atoms. The initial shift $l_0 = 8l^* = 80$.

momentum, one can estimate this effect of dephasing by evaluating the sum

$$p(t) \sim \sum_m \exp(-m^2/2\gamma^2)\sin[(\omega_B + \nu m)/\hbar t], \qquad (3)$$

where $m = l - l_0$ and γ is the width of the atomic wave packet (measured in the lattice period). Replacing the sum with the integral, we obtain

$$p(t) \sim \exp(-t^2/2\tau_\gamma^2)\sin(\omega_B t), \qquad (4)$$

where

$$\tau_\gamma = \hbar/\gamma\nu. \qquad (5)$$

It is seen in Eq. (5) that the dephasing time τ_γ is defined by both the wave packet width and by the trap frequency. Based on this result, one might conclude that a narrow wave packet is preferable for studying BO in the parabolic lattices. This is, however, not exactly true, because a narrow wave packet implies a lower contrast of the interference pattern measured in the laboratory experiments. Thus, one has to find a compromise between the contrast and dephasing when preparing the initial wave packet.

The irreversible decay of BO according to Eq. (4) is a consequence of our approximation of the sum by the integral. Without this approximation, the decay of oscillations is followed by periodic revivals with the period $T_\nu = 2\pi\hbar/\nu$ [14]. One of these revivals is illustrated in the upper panel of Fig. 1, which shows the dynamics of the mean momentum of the noninteracting atoms in the parabolic lattice with parabolicity $\nu = 0.04J$. As the initial state of the system, we choose here the ground state of the atoms in the parabolic lattice with a slightly tighter confinement $\nu' = 4\nu$, which was then shifted by the distance $l_0 = 8l^* = 80$. Note that, by changing ν', we change only the dephasing time (through the change of the wave packet width $\gamma = \gamma(\nu')$), while the revival time is defined exclusively by the parameter ν.

3. Next we address the effect of atom–atom interactions. In the case of quasi-one-dimensional parabolic lattices, Bloch and dipole oscillations of the interacting atoms were studied in number of the papers by using the mean-field approach [13, 15–19]. As known, the mean-field approach is justified in the limit of large occupation number $\bar{n} \longrightarrow \infty$ and vanishing interaction $W \longrightarrow 0$. This limit leads (in the simplest case) to the nonlinear discrete Schrödinger equation,

$$i\hbar\dot{a}_l = \frac{\nu}{2}l^2 a_l - \frac{J}{2}(a_{l+1} + a_{l-1}) + g|a_l|^2 a_l, \qquad (6)$$

where $g = WN$ is the macroscopic interaction constant. In the present work, we focus on the case of truly one-dimensional lattices, where the mean occupation number $\bar{n} \sim 1$. Clearly, the mean field approach is not applicable here and one has to treat the system microscopically by using, for example, the Bose–Hubbard model,

$$H = \sum_l \frac{\nu}{2}l^2\hat{n}_l - \frac{J}{2}\left(\sum_l \hat{a}^\dagger_{l+1}\hat{a}_l + \text{h.c.}\right) + \frac{W}{2}\sum_l \hat{n}_l(\hat{n}_l - 1). \qquad (7)$$

The main question we address below is the effect of atom–atom interactions on the Bloch dynamics depicted in the upper panel of Fig. 1.

First we shall discuss the initial conditions in some more detail. Throughout the paper we shall consider the ground many-body state of the atoms in a parabolic lattice as the initial wave packet (which is shifted then by the distance l_0). Clearly, along with the ratio J/ν, this state is also defined by the ratio of the interaction constant to the hopping matrix element. Namely, it is essentially given by the symmetrized product of the single-particle atomic state for $W < J$, while it is close to the Mott insulator state for $W \gg J$. In what follows, we restrict ourselves to a relatively weak interaction. Then the ground state of the system can be well approximated by the many-body wave function

$$|\tilde{\Psi}_0\rangle = \sqrt{N!}\prod_l \frac{a_l^{n_l}}{\sqrt{n_l!}}|\mathbf{n}\rangle, \qquad (8)$$

where $|\mathbf{n}\rangle = |\ldots, n_{-1}, n_0, n_1, \ldots\rangle$ is the Fock basis and a_l satisfy the stationary Gross–Pitaevskii equation

$$\frac{\nu}{2}l^2 a_l - \frac{J}{2}(a_{l+1} + a_{l-1}) + g|a_l|^2 a_l = E_0 a_l. \qquad (9)$$

For example, for $N = 5$, $\nu = 0.04J$, and $W = 0.2J$, the overlap of the state (8) with the exact ground state $|\Psi_0\rangle$ is $|\langle\Psi_0|\tilde{\Psi}_0\rangle|^2 = 0.97$. We note that the state (8) is completely coherent and is analogous to the super-fluid state in a homogeneous lattice. We shall characterize the macroscopic coherence of the given many-body state $|\Psi\rangle$ by the maximal eigenvalue λ of the single-particle density matrix

$$\rho_{l,m} = \langle\Psi|\hat{a}_l^+\hat{a}_m|\Psi\rangle. \qquad (10)$$

Then the macroscopic coherence of the state (8) is $\lambda = 1$.

We proceed with the dynamics. The middle panel in Fig. 1 shows the mean momentum of $N = 5$ interacting atoms ($W = 0.2J$). In comparison with the noninteracting case (upper panel), a qualitative change is noticed. This change can be understood by analyzing the macroscopic coherence of the system, shown in the lower panel. It is seen that the macroscopic coherence oscillates with some characteristic period T_W. In the case of an infinite tilted lattice, these oscillations were studied in [7]. The origin of the oscillations was shown to be the Stark localization of the single-particle wave functions which, together with discreetness of the atom number, leads to the following expression for the macroscopic coherence:

$$\lambda = \exp(-2\bar{n}[1 - \cos(Wt/\hbar)]). \qquad (11)$$

In Eq. (11), $\bar{n}$ is the mean number of atoms per lattice site [20] and the limit $Fd \gg J$ is implicitly assumed. Since for the considered local static force $Fd = \nu l_0 = 3.2J$ the Stark localization is not complete, the oscillations of the macroscopic coherence irreversibly decay in time. Nevertheless, if this irreversible decay of coherence is slow on the time scale of the dephasing time, one can observe the revival of BO of the interacting atoms—the effect which attracts much attention because it provides an independent and accurate method for measuring the microscopic interaction constant W.

4. Let us now turn to the case $l_0 < l^*$. In this case, one deals with the dipole oscillations of BEC, where the characteristic frequency is obviously given by the frequency of small pendulum oscillations $\omega_0 = (\nu J)^{1/2}/\hbar$. (We recall in passing that the frequency of BO was given by $\omega_B = \nu l_0/\hbar \approx 2\omega_0 l_0/l^*$, $l_0 \gg l^*$.) For vanishing atom–atom interactions, these dipole oscillations are shown in the upper panel of Fig. 2, where $l_0 = l^*/2 = 5$ and the other parameters are the same as in Fig. 1. The dephasing time τ_γ is again given by Eq. (5) but with the parameter ν replaced by the nonlinearity parameter $\tilde{\nu} = \nu/8$ [21] (the latter parameter also defines the revival time). The middle and lower panels in Fig. 2 refer to the interacting atoms. The exponential decay of the macroscopic coherence is noticed. The other point to which we want to draw the attention of the reader is that a moderate interaction stabilizes the dipole oscillations against the dephasing. In the mean-field approach (which reduces the Bose–Hubbard model to the discrete nonlinear Schrödinger equation), this phenomenon was predicted in [13].

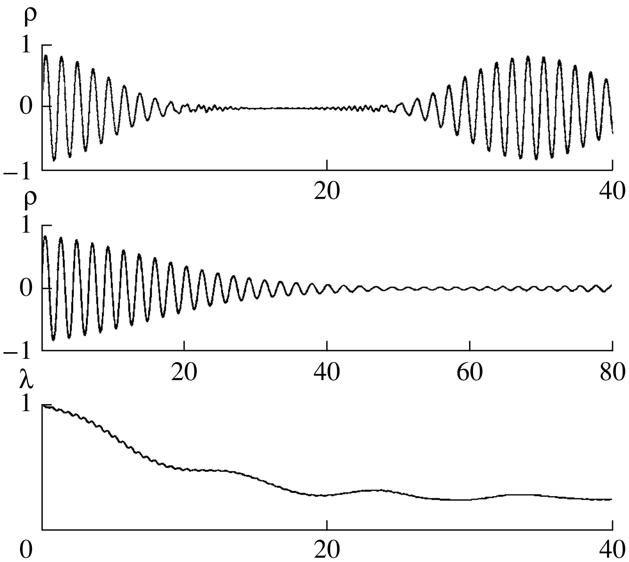

Fig. 2. Dipole oscillations of $N = 4$ atoms in the parabolic lattice with parabolicity $\nu = 0.04J$: (a) the mean momentum of noninteracting atoms; (b) the mean momentum of interacting ($W = 0.2J$) atoms; (c) the macroscopic coherence of interacting atoms. The initial shift $l_0 = l^*/2 = 5$.

5. In conclusion, we have shown that the dynamics of cold atoms in parabolic lattices is governed by the relation between two characteristic times—the dephasing time τ_γ and the decoherence time τ_W.

The dephasing time is inverse proportional to the widths γ of the initial wave packet and the nonlinearity $\tilde{\nu}$ which, in turn, is defined by the initial shift l_0 of the wave packet relative to the separatrix l^*. Namely, $\tilde{\nu} = \nu/8$ for $l_0 \ll l^*$ and $\tilde{\nu} = \nu$ for $l_0 \gg l^*$, where $\nu = M\omega^2 d^2$ is the parabolicity of the lattice. It is interesting to estimate the dephasing time in the typical laboratory experiment. Taking, as an example, the recent experiment [10] with Rubidium atoms in the array of axially modulated quantum tubes, we have $\nu = 0.0014E_R$ and $J = 0.38E_R$ for the modulation amplitude (depth of the optical lattice) of one recoil energy. This gives the separatrix $l^* = 33$ and the period of small dipole oscillations $T_0 = 12.1$ ms. If we assume a dilute gas (which, in fact, is not the case realized in the cited experiment), the width of the initial wave packet is $\gamma \approx (J/4\nu)^{1/4} = \sqrt{l^*}/2 \approx 3$ and, hence, the dephasing time $\tau_\gamma = 85$ ms for dipole oscillations and $\tau_\gamma = 10.6$ ms for BO. Note that these are the upper estimates for the dephasing time, and for an initial shift l_0 close to the separtrix, the dephasing times are substantially smaller. It is also worth noting that there is a maximal shift l_0 above which the single-band approximation (used throughout the paper) might be questioned. The crucial parameter here is the energy gap between the Bloch bands ($\Delta = 0.5E_R$ for the specified parameters). The analysis of BO in a parabolic lattice beyond the single-band approximation will be a subject of a separate paper.

The decoherence time τ_W is defined by the characteristic density of atomic gas $\bar{n}$ and by the value of the macroscopic interaction constant W [22]. The latter, in turn, is defined by the *s*-wave scattering length and by the degree of confinement of the atoms in the wells of the optical potential. In particular, in experiment [10], quantum tubes were created by two crossing quasi-1D optical lattices with the amplitude $V = 30E_R$. For axial modulation with $V = E_R$, this gives $W = 0.73E_R$. For this relatively high value of the interaction constant, a few atoms per tube is enough to destroy the dipole/Bloch oscillations on a very short time scale. This qualitatively explains the results of the experiment [10], where the number of atoms per quantum tube was around 20. To observe the effects discussed in the paper, one has to decrease both the atomic density and the transverse confinement (for example, for the transverse confinement $V = 3E_R$, the interaction constant $W = 0.17E_R$).

We would also like to stress that in this work we restrict ourselves to considering the initial state of the system to be a Bose–Einstein condensate (i.e., the macroscopic coherence $\lambda \approx 1$ at $t = 0$). This is in no way a general case, and for different initial states (realized, for example, for deeper optical lattices) the dynamics of the atoms may fundamentally differ from the discussed Bloch and dipole oscillations.

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