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ORDER, DISORDER, AND PHASE TRANSITIONS IN CONDENSED SYSTEMS

Effects of Cross Correlations between 1D and 3D Inhomogeneities on the High-Frequency Susceptibility of Superlattices

V. A. Ignatchenko and Yu. I. Mankov

Kirenskiĭ Institute of Physics, Siberian Division, Russian Academy of Sciences, Akademgorodok, Krasnoyarsk, 660036 Russia

e-mail: vignatch@iph.krasn.ru

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Abstract—Cross correlations between components of a mixture of one-dimensional (1D) and three-dimensional (3D) inhomogeneities are described by introducing a distribution function taking into account correlations between absolute values of two random variables in the absence of correlations between the variables themselves. This distribution function is used for derivation and analysis of the superlattice correlation function containing a mixture of cross-correlated 1D and 3D inhomogeneities. The effect of such inhomogeneities on the high-frequency susceptibility at the edge of the first Brillouin zone of the superlattice is investigated. It is shown that positive cross correlations partly suppress the effect of a mixture of 1D and 3D inhomogeneities on the wave spectrum: the gap at the boundary of the Brillouin zone increases, and wave damping decreases as compared to the effect produced by a mixture of 1D and 3D inhomogeneities in the absence of cross correlations. Negative cross correlations lead to the opposite effect: the gap decreases and wave damping increases. Cross correlations also lead to the emergence of new resonance effects: a narrow dip or a narrow peak at the center of the band gap (depending on the sign of the correlation factor).

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1. INTRODUCTION

Wave spectra in partly stochastic superlattices have been intensely studied in recent years in view of wide application of such materials in various high-tech devices. In addition, these materials are convenient models for developing new methods in theoretical physics for studying media without translational symmetry. Several theoretical approaches have been developed to investigate such superlattices, including the introduction of a 1D random phase [1, 2], modeling of violation of ordering in the sequence of layers of two different materials [3–9], numerical simulation of random deviations of the interfaces between the layers from their initial periodic arrangement [10–12], postulation of the form of the correlation function of the superlattice with inhomogeneities [13, 14], application of geometrical optics approximations [15], and development of a dynamic theory of composite elastic media [16].

One more method for investigating the effect of inhomogeneities in a superlattice on the wave spectrum, which was proposed in our earlier publication [17], is known as the method of random spatial modulation (RSM) of the superlattice period. It can be briefly described as follows. The method of averaged Green functions is known to be the most consistent approach for describing the spectral properties of any inhomogeneous medium. The only parameter describing the random medium and appearing in the expression for the averaged Green function is the correlation function $K(\mathbf{r})$ that depends on the distance $\mathbf{r}$ between two points of the medium ($\mathbf{r} = \mathbf{x} - \mathbf{x}'$). Consequently, the first stage of solving the problem is reduced to determining function $K(\mathbf{r})$ for the superlattice containing some structural inhomogeneities; the second stage involves the extraction of the spectral parameters from the expression for the Green function containing this correlation function using the standard approximate methods. We used the random phase model for describing inhomogeneities in the structure of a sinusoidal superlattice, which was assumed to be a random function of all three coordinates with an arbitrary correlation radius. To determine the superlattice correlation function $K(\mathbf{r})$, we developed an approach, which is a generalization of the well-known method for determining the time correlation function for a random frequency (phase) modulation of a radio signal [18, 19] to the case of a spatial (3D in the general case) modulation of the superlattice period (phase). The advantage of this method is that the form of the superlattice correlation function is not postulated in it, but is derived from the most general assumptions concerning the nature of random spatial modulation of the superlattice period. It was shown that this function in the general case has a quite complicated form, which is determined to a considerable extent by the dimensionality of inhomogeneities, the structure of the interface between the layers, and so on. Knowledge of the correlation functions corresponding to various types and dimensionalities of inhomogeneities allowed us to apply the methods to studying the averaged Green functions for determining the eigenfrequencies, damping factor, and other wave parameters in superlattices [17, 20–26]. We demonstrated how the resultant differences in the form of the correlation function for 1D and 3D inhomogeneities are manifested in the spectrum of waves propagating in the superlattice [21, 22, 26]. For the first time, we investigated the effect of 3D inhomogeneities with anisotropic correlation properties [25] and a mixture of 1D and 3D inhomogeneities [23, 24] on the wave spectrum. We demonstrated nonadditivity of the contributions from the mixture components to the resultant modification of the wave spectrum parameters associated with these inhomogeneities. Analyzing the mixture of inhomogeneities in [23, 24], we assumed that cross correlations between inhomogeneities with different dimensionalities are absent. The present study is devoted to analysis of the effect of cross correlations between 1D inhomogeneities simulating random deviations of interfaces between the layers from their periodic arrangement and 3D inhomogeneities simulating random deformations of the surface of layer boundaries on the high-frequency susceptibility of the superlattice.

The article has the following structure. In Section 2, we introduce the distribution function for two random quantities, which describes cross correlation between the absolute values of these quantities and derive the correlation function of the superlattice, which takes into account the cross correlation between 1D and 3D inhomogeneities. In Section 3, the high-frequency susceptibility at the boundary of the first Brillouin zone is calculated and the dependence of the imaginary part of the susceptibility on mean square fluctuations of inhomogeneities is analyzed. In Section 4, the results are summarized and discussed.

2. DISTRIBUTION FUNCTION AND CORRELATION FUNCTION

A superlattice is characterized by the dependence of a material parameter A on spatial coordinates $\mathbf{x} = \{x, y, z\}$. Parameter $A(\mathbf{x})$ can be of any physical origin. It can be the density of the material or a force constant for the elastic system of the medium, magnetic anisotropy, magnetization, or exchange constant for the magnetic system. We write parameter $A(\mathbf{x})$ in the form

$$A(\mathbf{x}) = A + \Delta A \rho(\mathbf{x}), \qquad (1)$$

where A is the mean value of the parameter, ΔA is its rms deviation, and $\rho(\mathbf{x})$ is a centered ($\langle\rho(\mathbf{x})\rangle = 0$) or normalized ($\langle\rho^2(\mathbf{x})\rangle = 1$) function. Function $\rho(\mathbf{x})$ describes both the periodic dependence of the parameter along the z axis of the superlattice and random spatial modulation of this parameter, which can generally be a function of all three coordinates x, y, and z.

In this study, we consider a superlattice with a sinusoidal dependence of the material parameter on the z coordinate in the initial state (in the absence of inhomogeneities). Following [23, 24], we present function $\rho(\mathbf{x})$ in the form

$$\rho(\mathbf{x}) = \sqrt{2}\cos[q(z - u_1(z) - u_3(\mathbf{x})) + \psi], \qquad (2)$$

where $q = 2\pi/l$ is the wave number of the superlattice and l is the superlattice period.

Function $u_1(z)$ describes 1D phase inhomogeneities of function $\rho(\mathbf{x})$. The sensitivity of the profile of function $\rho(\mathbf{x})$ to the action of modulation $u_1(z)$ is different for different points of function $\rho(\mathbf{x})$. The slightest changes of the profile occur in the vicinity of the minimum or maximum of function $\cos(qz)$. On the contrary, the displacement of zero points of $\cos(qz)$ under the action of function $u_1(z)$ leads to strong variations of the profile. Zeros of function $\rho(\mathbf{x})$ correspond to the boundaries of the layers of the superlattice. For this reason, in the RSM method we assume that function $u_1(z)$ simulates 1D displacements of the layer boundaries from their initial periodic arrangement.

Function $u_3(\mathbf{x})$ was introduced into Eq. (2) to simulate random deformations of the boundary surfaces of the layers. It might seem at first sight that this function must depend only on two coordinates x and y. However, function $u(x, y)$ in the RSM method describes only 2D deformations, which are homogeneous for all boundaries of the layers in the superlattice and, hence, have an infinitely long correlation radius along the z coordinate. In actual practice, we are interested in the opposite case, when deformations of two nearest boundaries of the layers are either uncorrelated (the correlation radii along the z axis are much smaller than $l/2$), or only the boundaries of several neighboring layers are correlated. For this reason, $u_3(\mathbf{x})$ must be a random function of all three coordinates x, y, and z.

In the general case, this function exhibits anisotropy in correlation properties since the lengths of the correlation radii in the xy plane and along the z axis are determined by different physical factors. However, we will confine our analysis to the simplest case and assume that $u_3(\mathbf{x})$ is a random 3D function with isotropic correlation properties. The coordinate-independent phase ψ is introduced into Eq. (2) to ensure the fulfillment of ergodicity of function $\rho(\mathbf{x})$ (see [17]); this phase is characterized by a uniform distribution on the interval $(-\pi, \pi)$. After averaging of the product of functions $\rho(\mathbf{x})$ and $\rho(\mathbf{x} + \mathbf{r})$ over phase ψ, we obtain

$$\langle\rho(\mathbf{x})\rho(\mathbf{x}+\mathbf{r})\rangle_\psi = \cos(qr_z - \chi_1 - \chi_3), \qquad (3)$$

where

$$\begin{aligned} \chi_1 &= q[u_1(z + r_z) - u_1(z)], \\ \chi_3 &= q[u_3(\mathbf{x} + \mathbf{r}) - u_3(\mathbf{x})]. \end{aligned} \qquad (4)$$

In [23, 24], we assumed that random functions χ_1 and χ_3 are mutually uncorrelated and each of these functions obeys a Gaussian distribution.

This study is aimed at an analysis of the situation when 1D and 3D inhomogeneities are cross-correlated. In the standard theory, the distribution function of two random centered cross-correlated quantities χ_1 and χ_3 can be represented in the form [see, for example, [27]]

$$f(\chi_1, \chi_3) = \frac{1}{2\pi\sigma_1\sigma_3\sqrt{(1-\kappa^2)}} \times \exp\left\{-\frac{1}{2(1-\kappa^2)}\left[\frac{\chi_1^2}{\sigma_1^2} - 2\kappa\frac{\chi_1\chi_3}{\sigma_1\sigma_3} + \frac{\chi_3^2}{\sigma_3^2}\right]\right\}, \tag{5}$$

where σ_1 and σ_3 are rms fluctuations of quantities χ_1 and χ_3, respectively, and κ is the cross-correlation factor for these quantities ($-1 < \kappa < 1$). Such a form of the distribution function describes the situation when (for example, for positive correlations with $\kappa > 0$) the emergence of a positive fluctuation in quantity χ_1 causes an increase in the probability of a positive fluctuation in quantity χ_3 and, accordingly, a decrease in the probability of negative fluctuation of χ_3. In our case, variables χ_1 and χ_3 are functions of one and three spatial coordinates, respectively. As applied to our model, function (5) describes the situation when a positive displacement $u_1(z)$ of the boundary between two layers of the superlattice at a point $z = z_0$ causes an asymmetric (i.e., positive amplitudes are on average larger than negative amplitudes) deformation of boundary $u_3(x, y, z_0)$ in the entire (x, y) plane, which corresponds to point $z = z_0$. A physical mechanism leading to such correlations for 1D and 3D inhomogeneities under investigation is difficult to imagine.

Our task is to analyze the effect of another type of correlations with a clear physical meaning on the spectrum of the system. We are speaking of those correlations when an enhancement of the intensity fluctuations for inhomogeneities of some dimensionality for positive correlations leads to enhancement of intensity fluctuations for inhomogeneities of the other dimensionality and vice versa, irrespective of the sign of these fluctuations. In our model, this means that the increase in the displacement of the boundary at point $z = z_0$ irrespective of its sign must lead to an increase in the deformation amplitudes of this boundary for all values of x and y in the plane $z = z_0$ irrespective of the sign of these amplitudes. The reason for the emergence of such correlations in the most general form may lie in the reasonable assumption that any random instability in the setup for obtaining superlattices which causes an increase in the deviation of the thickness of a layer from the preset value might also increase the probability that the deformation of the surface of this layer increases. To simulate such cross correlations, we introduce here a distribution function, which, in contrast to function (5), describes correlation between absolute values $|\chi_1|$ and $|\chi_3|$ of random functions χ_1 and χ_3, leaving the functions themselves uncorrelated:

$$f(\chi_1, \chi_3) = C\exp\left\{-\frac{1}{2}\left[\frac{\chi_1^2}{A^2} - \frac{2\kappa(|\chi_1| - a_1)(|\chi_3| - a_3)}{AB} + \frac{\chi_3^2}{B^2}\right]\right\}. \tag{6}$$

Coefficients A, B, and C, as well as the expectations $a_i \equiv \langle|\chi_i|\rangle$ of the absolute values, can be found with the help of the normalization conditions

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\chi_1, \chi_3)d\chi_1 d\chi_3 = 1,$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\chi_i| f(\chi_1, \chi_3)d\chi_1 d\chi_3 = a_i, \tag{7}$$
$$\int_{-\infty}^{\infty}\int_{\infty}^{-\infty} \chi_i^2 f(\chi_1, \chi_3)d\chi_1 d\chi_3 = Q_i, \quad i = 1, 3.$$

Here, we are using notation $Q_i^{1/2}$ for rms deviations instead of generally accepted notation σ_i to emphasize that not constants, but structural function $Q_1 = Q_1(r_z)$ and $Q_3 = Q_3(\mathbf{r})$ play the role of rms deviations in the situation considered here.

It is impossible to exactly evaluate the integrals in Eqs. (7) for probability function (6) in contrast to function (5). For this reason, we use the following procedure. The integral with respect to one variable (say, χ_1) can be evaluated exactly for each condition in (7). Using the integral representation of error function erf(x) in the remaining integral with respect to χ_3 and changing the integration order, we can evaluate the integral with respect to χ_3 in infinitely large limits and thus reduce Eqs. (7) to integrals with respect to a certain variable between 0 and 1. Assuming that the correlation factor κ is small, we expanded the integrands in the normalization integrals obtained in this way into a power series up to the first order in κ, after which these functions could be integrated exactly. As a result, we derived from Eqs. (7) the algebraic equations, which were used for determining the constants appearing in function (6) (in this approximation, these constants are independent of κ):

$$A = Q_1^{1/2}, \quad B = Q_3^{1/2},$$
$$C = \frac{1}{2\pi}(Q_1Q_3)^{-1/2}, \quad a_i = \left(\frac{2Q_i}{\pi}\right)^{1/2}. \tag{8}$$

The mixed central second-order moment, which has the form $\langle\chi_1\chi_3\rangle = \kappa\sigma_1\sigma_3$ for function (5), is zero for func-

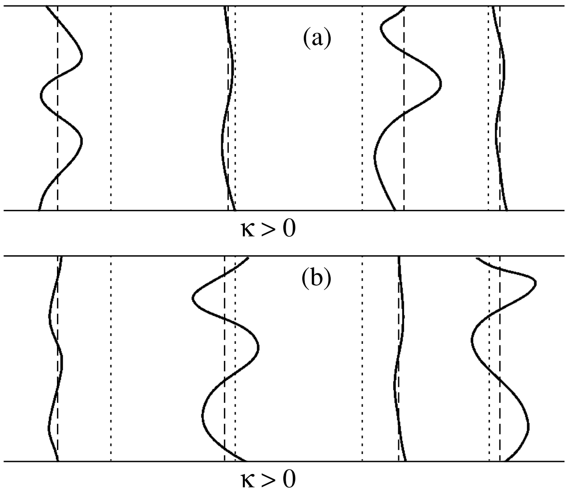

Fig. 1. Schematic diagram of (a) positive and (b) negative spatial synchronization of 1D and 3D inhomogeneities in a superlattice, associated with their cross correlations. The dotted lines correspond to a periodic arrangement of the boundaries between layers in a perfect superlattice, the dashed lines correspond to random 1D displacements of the boundaries, and solid curves correspond to the total effect of 1D displacements and 3D deformations of the boundary surfaces.

tion (6). However, the mixed central moments of absolute values $|\chi_1|$ and $|\chi_3|$ for function (6) is proportional to κ:

$$\begin{aligned}&\langle(|\chi_1| - \langle|\chi_1|\rangle)(|\chi_3| - \langle|\chi_3|\rangle)\rangle \\ &= \kappa\left(1 - \frac{2}{\pi}\right)^2 (Q_1 Q_3)^{1/2}.\end{aligned} \tag{9}$$

It should be emphasized that the distribution function (6) of two variables χ_1 and χ_3, which was introduced to take into account the correlation of the absolute values of these quantities, as well as the standard function (5) taking into account the correlation between these quantities themselves, does not lead to a change in the root-mean-square values of these quantities. The inclusion of correlations only indicates the emergence of stochastic synchronization between χ_1 and χ_3 for function (5) and between $|\chi_1|$ and $|\chi_3|$ for function (6) in the configuration space of variables χ_1 and χ_3. Since χ_1 and χ_3 are functions of spatial coordinates, this in turn leads to stochastic spatial synchronization between intensity fluctuations of 1D and 3D inhomogeneities. For inhomogeneities in a superlattice, this means that, for example, for $\kappa > 0$, strong 3D deformations $u_3(\mathbf{x})$ of the surfaces between the layers are concentrated in the regions of strong 1D deviations $u_1(z)$ of the surfaces from the initial periodic distribution, while weak deformations $u_3(\mathbf{x})$ are concentrated in the regions of weak deviations $u_1(z)$. Accordingly, for $\kappa < 0$, strong 3D fluctuations $u_3(\mathbf{x})$ are concentrated in the regions of weak 1D deviations $u_1(z)$ of the surfaces between the layers. Such a redistribution of deformation-induced 3D fluctuations in space occurs without a change in the values of $\langle\chi_3^2\rangle$ and $\langle\chi_1^2\rangle$. Figures 1a and 1b schematically illustrate the situations corresponding to $\kappa > 0$ and $\kappa < 0$. For simplicity, the case of short correlation radii $r_\parallel$ and r_0 of 1D and 3D inhomogeneities is depicted in the figures, respectively ($r_\parallel$, $r_0 \ll l/2$); in this case, the deviation of neighboring boundaries between the layers (or their deformations) can be regarded as mutually independent. In this case, the effects of positive (Fig. 1a) or negative (Fig. 1b) spatial synchronization for $\kappa \neq 0$ are manifested even at adjacent boundaries.

For real superlattices, the probabilities of the emergence of positive or negative correlations between 1D and 3D inhomogeneities are substantially different. It was noted above that positive cross correlations between inhomogeneities with different dimensionalities may naturally emerge in the standard conditions for obtaining superlattices, while negative correlations emerge only under rather specific conditions (for example, such that an increase in the deviation of the boundary between two layers from equilibrium leads to an increase in the surface tension, suppressing random deformations of the boundary surface).

It should be noted that constant coefficients in function (6) can also be determined in a simpler way; namely, we can use a formal expansion of the correlation part of function (6) into a power series in κ up to the first power of κ,

$$\begin{aligned}f(\chi_1, \chi_3) &\approx C\exp\left\{-\frac{1}{2}\left[\frac{\chi_1^2}{A^2} + \frac{\chi_3^2}{B^2}\right]\right\} \\ &\times\left[1 - \kappa\frac{(|\chi_1| - a_1)(|\chi_3| - a_3)}{AB}\right],\end{aligned} \tag{10}$$

and evaluate the normalization integrals (7) (which can be evaluated exactly in the present case) using approximate expression (10). After integration of Eqs. (7), the terms proportional to κ satisfy the requirements of perturbation theory for $\kappa \ll 1$ in spite of the fact that the term in the square brackets in relation (10), which is proportional to κ, can be infinitely large in the range of χ_i. The algebraic equations derived in this way lead to the same values of parameters A, B, C, a_1, and a_3, as those defined by formulas (8).

The superlattice correlation function $K(\mathbf{r})$ with cross correlations of 1D and 3D inhomogeneities are defined by the formula

$$\begin{aligned}K(\mathbf{r}) &\equiv \langle\rho(\mathbf{x})\rho(\mathbf{x}+\mathbf{r})\rangle_{\psi\chi_1\chi_3} \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\chi_1 d\chi_3 f(\chi_1, \chi_3)\cos(qr_z - \chi_1 - \chi_3),\end{aligned} \tag{11}$$

where function (6) introduced above must appear in the form of distribution function $f(\chi_1, \chi_3)$. In the approximation $\kappa \ll 1$, the coefficients in this function are defined by formulas (8).

In the evaluation of the integrals in formula (11), the same difficulties arise as in the evaluation of normalization integrals in formulas (7). In both cases, these difficulties can be overcome for small values of κ in two ways: (i) by evaluating the integral with respect to one variable exactly, transforming the integral with respect to the other variable to an integral between 0 and 1, expanding the resultant expression into a power series in κ up to the first power of κ and integrating exactly the resultant approximate expression or (ii) by substituting directly the approximate expression for distribution function (10) into formula (11), which makes it possible to take exactly the integrals with respect to both variables χ_1 and χ_3. Both approaches lead to the same expression for the correlation functions of the superlattice in terms of structural functions $Q_1(r_z)$ and $Q_3(r)$,

$$K(\mathbf{r}) = \cos q r_z \{K_1(r_z)K_3(r) + K_{13}(\mathbf{r})\}, \qquad (12)$$

where the expression in the braces is the decreasing part of the correlation function, which consists of the sum of the product of the decreasing parts of the correlation functions for the components of a mixture of 1D and 3D inhomogeneities,

$$K_1(r_z) = \exp\left[-\frac{1}{2}Q_1(r_z)\right], \qquad (13)$$

$$K_3(r) = \exp\left[-\frac{1}{2}Q_3(r)\right], \qquad (14)$$

and the cross-correlation function

$$K_{13}(\mathbf{r}) = \frac{2\kappa}{\pi}\left[1 - \sqrt{2Q_1(r_z)}D\left(\sqrt{\frac{Q_1(r_z)}{2}}\right) - \exp\left(-\frac{Q_1(r_z)}{2}\right)\right]\left[1 - \sqrt{2Q_3(r)} \times D\left(\sqrt{\frac{Q_3(r)}{2}}\right) - \exp\left(-\frac{Q_3(r)}{2}\right)\right]. \qquad (15)$$

Here,

$$D(x) = \exp(-x^2)\int_0^x \exp(t^2)dt$$

is the Dawson integral.

To find structural functions $Q_1(r_z)$ and $Q_3(r)$, we must simulate the correlation properties of modeling functions $u_1(z)$ and $u_3(\mathbf{x})$ or, to be more precise, the correlation properties of their gradients, which, in contrast to $u_1(z)$ and $u_3(\mathbf{x})$, are homogeneous random functions. Both functions $Q_1(r_z)$ and $Q_3(r)$ were determined in [17] (see also the refinement of coefficients in these expressions in [21]) with the help of various forms of model correlation functions for a random modulation. It was shown that the form of functions Q_i is asymptotically independent of the modeling correlation functions (both for small and large values of r), but strongly depends on the dimensionality of inhomogeneities. In the case of exponential modeling correlation functions for $u_1(z)$ and $u_3(\mathbf{x})$, the structural functions were obtained in the form

$$Q_1(r_z) = 2\gamma_1^2[\exp(-k_\parallel r_z) + k_\parallel r_z - 1], \qquad (16)$$

$$Q_3(r) = 6\gamma_3^2\left[1 - \frac{2}{k_0 r} + \left(1 + \frac{2}{k_0 r}\right)\exp(-k_0 r)\right]. \qquad (17)$$

Here, $k_\parallel = r_\parallel^{-1}$ and $k_0 = r_0^{-1}$ are the correlation wavenumbers for 1D and 3D inhomogeneities, respectively, and

$$\gamma_1 = \frac{s_1 q}{k_\parallel}, \quad \gamma_3 = \frac{s_3 q}{k_0}, \qquad (18)$$

where s_1 and s_3 are the rms fluctuations of gradients of functions $u_1(z)$ and $u_3(\mathbf{x})$.

The form of the correlation functions of the superlattice for $\kappa = 0$ has been thoroughly studied for 1D and 3D inhomogeneities as well as for their mixture [24]. In view of the complexity of expressions (13) and (14), we proposed approximating expressions (see, [17] and [22], respectively)

$$K_1(r_z) = \exp(-\gamma_1^2 k_\parallel r_z), \qquad (19)$$

$$K_3(r) = (1 - L)\exp(-\gamma_3^2 k_0 r) + L, \qquad (20)$$

where $L = \exp(-3\gamma_3^2)$ is the asymptotic form of $K_3(r)$ for $r \longrightarrow \infty$.

These expressions clearly show a basic difference in the form of correlation functions for 1D and 3D inhomogeneities: the decreasing part of the correlation function for 1D inhomogeneities exponentially tends to zero as $r_z \longrightarrow \infty$, while the decreasing part of the correlation function for 3D inhomogeneities tends to a nonzero asymptote L for $r \longrightarrow \infty$. For this reason, in addition to volume regions with a finite correlation radius r_0 in the 3D case, there also exist regions with an infinitely long correlation radius. This leads to a sharp difference in the form of the spectra and in the damping of waves in a medium with 1D and 3D inhomogeneities. For a mixture of uncorrelated 1D and 3D inhomogeneities, the decreasing part of the correlation function

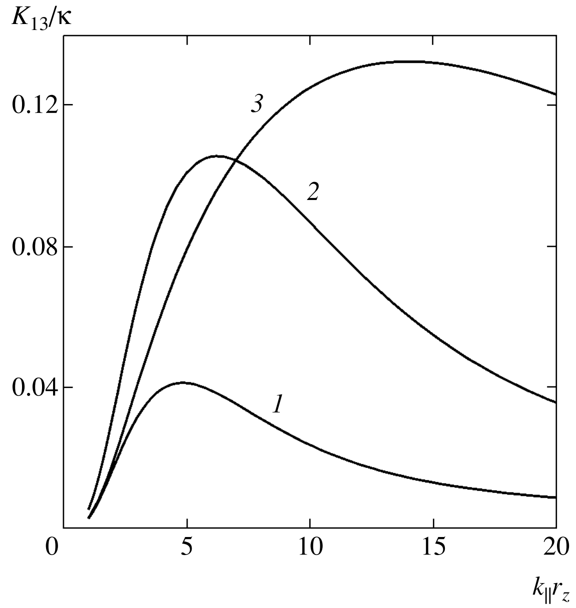

Fig. 2. Dependence of the cross correlation functions K_{13} on r_z for $r_x = r_y = 0$ and $k_0 = k_{\|}$ for $\gamma_1^2 = 0.5$, $\gamma_3^2 = 0.1$ (curve *1*), $\gamma_1^2 = 0.3$, $\gamma_3^2 = 0.3$ (curve *2*), and $\gamma_1^2 = 0.1$, $\gamma_3^2 = 0.5$ (curve *3*).

is equal to the product $K_1(r_z)K_3(r)$ of the decreasing parts of the correlation functions of the mixture components; for this reason, it has an asymptotic form corresponding to 1D inhomogeneities (i.e., exponentially tends to zero as $r \longrightarrow \infty$).

The dependence of the cross-correlation function K_{13} on r_z for $r_x = r_y = 0$ is shown in Fig. 2 for several values of γ_1^2 and γ_3^2, satisfying the condition $\gamma_1^2 + \gamma_3^2 = 0.6$. Function $K_{13}(r_z)$ vanishes for $r_z = 0$, attains its maximal value for a certain value of r_z depending on γ_i^2, and decreases as r_z^{-1} for $r_z \longrightarrow \infty$. Depending on the sign of κ, function K_{13} is either added to the decreasing part of the correlation function of a mixture of inhomogeneities, or is subtracted from it (in the latter case, the decreasing part of $K(\mathbf{r})$ becomes negative in the range of large values of r_z). In any case, when cross correlations between 1D and 3D inhomogeneities are taken into account, the decrease in the correlations in the superlattice obeys a power law, while the decay of correlations in the superlattice is exponential in the absence of such correlations ($\kappa = 0$), when we have a mixture of independent 1D and 3D inhomogeneities. Thus, the decreasing part of the correlation function (12) of the superlattice for $\kappa > 0$ occupies an intermediate position between the decreasing part of the correlation function $K_3(r)$ of a superlattice with 3D inhomogeneities, which tends to constant L as $r \longrightarrow \infty$, and correlation function $K_1(r_z)$ of a superlattice with 1D inhomogeneities or with a mixture of uncorrelated 1D and 3D inhomogeneities, $K_1(r_z)K_3(\mathbf{r})$, which decay according to an exponential law for $r \longrightarrow \infty$.

In view of the complexity of expression (15), we approximate the cross-correlation function by a simpler formula

$$K_{13}(\mathbf{r}) = \frac{2\kappa}{\pi}\frac{\gamma_1^2\gamma_3^2 N(1-L)k_{\|}k_0|r_z|r}{N + 2\gamma_1^4\gamma_3^2(1-L)k_{\|}^2k_0r_z^2r}, \tag{21}$$

where

$$N = L + 2\sqrt{3}\gamma_3 D(\sqrt{3}\gamma_3) - 1. \tag{22}$$

Here, L is the same asymptote $K_3(r)$ as in formula (20) and $D(x)$ is the Dawson integral. The asymptotic forms of approximate expression (21) and exact expression (15) coincide both for small and large values of r_z:

$$K_{13}(\mathbf{r}) \approx \frac{2\kappa}{\pi} \times \begin{cases} \gamma_1^2\gamma_3^2(1-L)k_{\|}k_0|r_z|r, & k_{\|}|r_z| \ll 1, \\ (N/2)\gamma_1^2 k_{\|}|r_z|, & k_{\|}|r_z| \gg 1. \end{cases} \tag{23}$$

3. HIGH-FREQUENCY SUSCEPTIBILITY OF A SUPERLATTICE

Let us consider, for example, spin waves in a superlattice with an inhomogeneous uniaxial magnetic anisotropy parameter $\beta(\mathbf{x})$ (we assume that the direction of the magnetic anisotropy axis is homogeneous). In this case, parameters A and ΔA in expression (1) are, respectively, the mean value of anisotropy β and its rms deviation $\Delta\beta$.

Let us consider the situation when the directions of the external magnetic field $\mathbf{H}_0$, constant magnetization component $\mathbf{M}_0$, and the magnetic anisotropy axis coincide with the direction of the z axis of the superlattice. Carrying out conventional linearization of the Landau–Lifshitz equation for magnetization (M_x, $M_y \ll M_0$, $M_z \approx M_0$) and introducing circular projections for the resonant component of the magnetization and the external varying magnetic field, we obtain the equation for spin waves in the form

$$\nabla^2 m + \left[\nu - \frac{\Lambda}{\sqrt{2}}\rho(\mathbf{x})\right]m = -\frac{h}{\alpha}. \tag{24}$$

Here, $m = M_x + iM_y$, $h = H_x + iH_y$, $\nu = (\omega - \omega_0)/\alpha g M_0$, $\Lambda = \sqrt{2}\Delta\beta/\alpha$, ω is the frequency, $\omega_0 = g[H_0 + (\beta - 4\pi)M_0]$ is the homogeneous ferromagnetic resonance frequency, g is the gyromagnetic ratio, and α is the exchange constant.

The high-frequency spin-wave susceptibility $\chi(\nu, k)$ is proportional to the averaged Green function $G(\nu, k)$ of Eq. (24),

$$\chi(\nu, \mathbf{k}) = \frac{\langle m(\nu, \mathbf{k})\rangle}{h_0} = a(k)G(\nu, \mathbf{k}), \quad (25)$$

where h_0 is the rf field amplitude. The form of the proportionality factor $a(k)$ for spin-wave resonance in a thin magnetic film is analyzed in detail in [20]. The averaged Green function for Eq. (24) has the form

$$G(\nu, \mathbf{k}) = \frac{1}{\nu - k^2 - M(\nu, \mathbf{k})}, \quad (26)$$

where $M(\nu, \mathbf{k})$ is the classical analog of the mass operator, which can be represented in the Bourret approximation [28] in the form [21]

$$M(\nu, \mathbf{k}) = -\frac{\Lambda^2}{8\pi}\int \frac{K(\mathbf{r})}{|\mathbf{r}|}\exp[-i(\mathbf{k}\cdot\mathbf{r} + \sqrt{\nu}|\mathbf{r}|)]d\mathbf{r}. \quad (27)$$

Here, the correlation function $K(\mathbf{r})$ for a sinusoidal superlattice is defined by expression (12).

The left-hand side of wave equation (24) (and, hence, of expression (26) for the average Green function and (27) for the mass operator) remain valid for waves of other physical origin also after appropriate redesignation of the parameters appearing in these expressions. For example, in the scalar approximation, for elastic waves in a superlattice with an inhomogeneous density $p(\mathbf{x})$ of the medium ($A = p$, $\Delta A = \Delta p$), we obtain $\nu = (\omega/v)^2$ and $\Lambda = \sqrt{2}\,\Delta p\omega^2/pv^2$, where v is the velocity of the elastic waves; in the same approximation, for electromagnetic waves in a medium with inhomogeneous permittivity $\varepsilon(\mathbf{x})$ ($A = \varepsilon$, $\Delta A = \Delta\varepsilon$), we obtain $\nu = \varepsilon(\omega/c)^2$ and $\Lambda = \sqrt{2}\,\Delta\varepsilon\omega^2/\varepsilon c^2$, where c is the velocity of light. Thus, all results that will be obtained below for spin waves can easily be generalized for waves of other physical origin.

It is well known that the spectrum $\nu = \nu(k)$ of waves in a superlattice has a band structure. Gaps (forbidden bands) are formed in the spectrum for values of $k = nq/2$ corresponding to the edges of the nth Brillouin zones in the extended zone scheme. We will confine our analysis to the magnetic susceptibility at the edge of the first Brillouin zone ($k = k_r \equiv q/2$). In the absence of inhomogeneities and intrinsic damping of waves, the gap width in the spectrum for $k = k_r$ (which corresponds to the spacing between levels $\nu_+(k_r)$ and $\nu_-(k_r)$ of the split spectrum) is equal to Λ. In this case, two δ-shaped peaks spaced by Λ will be observed on the $G''(\nu)$ dependence for $k = k_r$. With increasing rms fluctuations γ_i of inhomogeneities, the gap $\Delta\nu = \nu'_+ - \nu'_-$ (where $\nu'_\pm = \mathrm{Re}\,\nu_\pm(k_r)$) between the spectral levels decreases and ultimately vanishes for a certain critical value of γ_i. The increase in γ_i is accompanied by damping of $\nu''(k)$, which, being a function of k, has a peak at $k = k_r$. The peaks on the $G''(\nu)$ dependence decrease and become closer with increasing γ_i, while their half-widths Γ increase until these peaks merge into one at a certain value of γ_i. The mode of variation of the spacing $\Delta\nu_m$ between the tops of the peaks corresponds to the change in the difference $\nu'_+ - \nu'_-$ between the eigenfrequencies; however, there is no exact quantitative correspondence between these quantities for $\gamma_i \neq 0$.

Integration in formula (27) was performed using approximate formulas (19)–(21). At the boundary of the first Brillouin zone of a superlattice, we obtained the following expression for the Green function in the two-wave approximation for Λ, $k_\parallel^2$, $k_0^2 \ll (q/2)^2$:

$$G(\nu) = \frac{1}{\Lambda}\Bigg\{X - \frac{1}{4} \times\left[\frac{1-L}{X - i\eta_1\gamma_1^2 - i\eta_3\gamma_3^2} + \frac{L}{X - i\eta_1\gamma_1^2}\right] - \frac{i\kappa N\eta_1}{12\pi\eta_3^2\gamma_1^2} \times[e^{v}E_1(v) + e^{u_-}E_1(u_-) + e^{u_+}E_1(u_+)]\Bigg\}^{-1}. \quad (28)$$

Here,

$$E_1(z) = \int_z^\infty \frac{e^{-t}}{t}dt$$

is the integral exponential function, $X = (\nu - k_r^2)/\Lambda$ is the dimensionless frequency detuning from the value of $\nu = k_r^2$, $\eta_1 = k_\parallel q/\Lambda$ and $\eta_3 = k_0 q/\Lambda$ are the dimensionless correlation wavenumbers for 1D and 3D inhomogeneities, respectively, and

$$v = iX\left[\frac{N}{2\gamma_1^4\eta_3^3\gamma_3^2(1-L)}\right]^{1/3}, \quad u_\pm = -\frac{1}{2}iX(1 \pm i\sqrt{3})\left[\frac{N}{2\gamma_1^4\eta_3^3\gamma_3^2(1-L)}\right]^{1/3}. \quad (29)$$

For $\kappa = 0$, formula (28) can be reduced to the expression for a mixture of uncorrelated 1D and 3D inhomogeneities, which was studied earlier [23, 24], while for $\gamma_3 = 0$ or $\gamma_1 = 0$, formula (28) can be reduced to the expressions for 1D and 3D inhomogeneities, respectively [26]. (It should be noted that a misprint appeared in the Russian version of [26] in the expression for $G(\nu)$ corresponding to 1D inhomogeneities.)

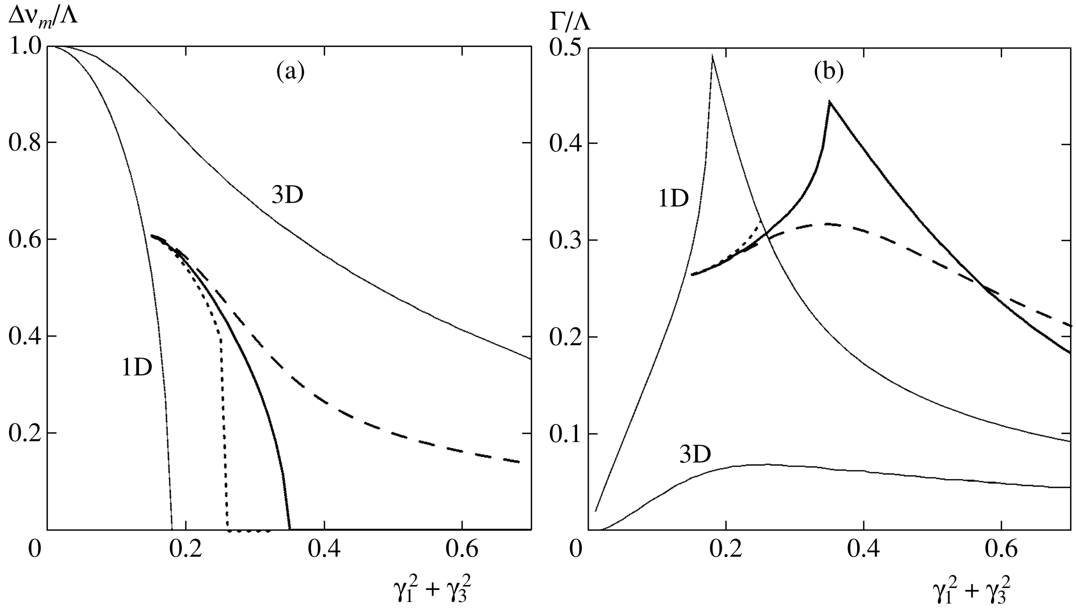

Fig. 3. (a) Spacing between the peaks on the imaginary part of the Green function and (b) half-width of these peaks as functions of the sum $\gamma_1^2 + \gamma_3^2$ for $\gamma_1^2 \neq 0$, $\gamma_3^2 = 0$ (curves 1D), $\gamma_1^2 = 0$, $\gamma_3^2 \neq 0$ (curves 3D), and the curves for a mixture of 1D and 3D inhomogeneities ($\gamma_1^2 = 0.14$ and $\gamma_3^2 \neq 0$) for various values of κ: κ = 0 (solid curves), κ = 0.6 (dashed curves), and κ = –0.6 (dotted curves).

Figures 3a and 3b show the changes in the dependences of $\Delta\nu_m$ and Γ on γ_i^2 resulting from allowance for cross correlations. Since the shape of the peaks on the $G''(\nu)$ dependence becomes asymmetric with increasing γ_i, these graphs depict the values of Γ corresponding to the right half-width of the right peak (and, accordingly, the left half-width of the left peak). Fine solid curves in these figures show the dependences of $\Delta\nu_m$ and Γ for 1D and 3D inhomogeneities; bold solid curves describe these dependences for a mixture of uncorrelated (κ = 0) 1D and 3D inhomogeneities and correspond to the situation analogous to that considered in [24]: further growth of 1D inhomogeneities ceases for $\gamma_1^2 = 0.14$, while rms fluctuation γ_3 of 3D inhomogeneities begins to increase. The dashed and dotted curves in these figures describe the same dependences for a mixture of 1D and 3D inhomogeneities in the presence of positive or negative cross correlations.

Figure 3a shows that positive correlations smoothen the curve describing the dependence of $\Delta\nu_m$ on γ_3^2 as compared to that for a mixture of uncorrelated 1D and 3D inhomogeneities; the former curve lies between those describing the dependence of $\Delta\nu_m$ on γ_3^2 for an uncorrelated mixture and for a medium with 3D inhomogeneities. Negative correlations lead to a steeper descent of the function $\Delta\nu_m(\gamma_3^2)$ as compared to that for κ = 0 and, hence, to the closure of the gap for smaller values of γ_3^2. These results match the behavior of the $\Gamma(\gamma_3^2)$ curves for a mixture of inhomogeneities (see Fig. 3b): the line half-width decreases for κ > 0 and increases for κ < 0 as compared to the $\Gamma(\gamma_3^2)$ curve for an uncorrelated mixture of 1D and 3D inhomogeneities. Thus, positive cross correlations for which stochastic spatial synchronization of intensity fluctuations of 1D and 3D inhomogeneities takes place (see Fig. 1a) partly suppress the effect of a mixture of 1D and 3D inhomogeneities on the wave spectrum: the gap width at the boundary of the Brillouin zone increases and wave damping decreases as compared to the effect observed in a mixture of 1D and 3D inhomogeneities for κ = 0. Negative cross correlations for which the intensity fluctuations of 1D and 3D inhomogeneities have a tendency to lie in different spatial regions (see Fig. 1b) lead to the opposite effect: the gap decreases and the damping increases as compared to those for κ = 0.

The $\Gamma(\gamma_3^2)$ curve for κ < 0 in Fig. 3b is plotted only up to the value of $\gamma_1^2 + \gamma_3^2$ corresponding to the closure of the gap in Fig. 3a since subsequent description cannot be carried out in terms of the line half-width alone. This is due to a peculiar resonance effect emerging at the midpoint of the gap. This effect was observed at first in our study of the shape of the imaginary part of the Green function $G''(\nu)$ corresponding to expression (28), which was derived using approximate expressions (20) and (21) for $K_3(r)$ and $K_{13}(\mathbf{r})$. In view of qualitative novelty of the effect, we calculated function $G''(\nu)$ more rigorously to verify that the observed effect is not a result of approximations used by us. For this purpose, we substituted into expression (27) for the mass operator the correlation function containing expressions (14) and (15) for $K_3(r)$ and $K_{13}(\mathbf{r})$ containing the exact structural function $Q_3(r)$ defined by formula (17). After the integration over angular variables, we evaluated the integral appearing in expression (27) numerically. As a result, we obtained the curves describing the dependence $G''(\nu)$ (Fig. 4), which differ only slightly from the dependences $G''(\nu)$ corresponding to approximate formula (28). Figure 4 shows the shape of the $G''(\nu)$ curve for $\kappa = 0$ (solid curve), $\kappa = 0.6$ (dashed curve), and $\kappa = -0.6$ (dotted curve). It can be seen that $\kappa = 0$ corresponds to a conventional curve with two peaks at frequencies ν_m^+ and ν_m^-, which approximately correspond to eigenfrequencies ν^+ and ν^- for given values of parameters η_1, γ_1 and η_3, γ_3 for a mixture of inhomogeneities. In addition to smooth modification of the $G''(\nu)$ curve (an increase in the peak heights and a decrease in peak widths), positive correlations also lead to the emergence of a resonance singularity on this curve. A narrow dip (gap) appears at the middle of the forbidden gap in the vicinity of point $\nu = k_r^2$, which becomes narrower, but does not disappear upon a further increase in γ_3. When κ changes its sign, this dip also reverses sign, becoming a narrow peak.

In our opinion, the physical mechanism for the emergence of these new resonance effects is as follows. Let us first consider the Green function for 3D inhomogeneities (expression (28) for $\gamma_1 = 0$, $\kappa = 0$). For $X = 0$, the second term in the brackets diverges and both the real and imaginary part of function G vanish. Thus, in contrast to the case of 1D inhomogeneities, for which an increase in γ_1 leads to the closure of the gap, the narrow gap $\Delta\omega$ for 3D inhomogeneities must be observed for any γ_3, decreasing upon an increase in this parameter,

$$\frac{\Delta\omega}{\Delta\omega_0} \approx L^{1/2} \equiv \exp\left(-\frac{3\gamma_3^2}{2}\right), \qquad (30)$$

where $\Delta\omega_0$ is the gap width for an ideal superlattice.

The effect of the narrow gap is due to the singularity in the shape of the correlation function for 3D inhomogeneities (namely, the existence of correlation regions with a finite and an infinite correlation length; the relation between these regions is determined by asymptote L). With increasing γ_3, the volume of the region with an infinite correlation length decreases in proportion to L, and the narrow gap in the wave spectrum of the superlattice, which is associated with this region, decreases in proportion to $L^{1/2}$. The effect of the narrow gap can in fact be observed as long as the value of $\Delta\omega$ exceeds the damping associated with any other processes of wave scattering, except their scattering at 3D inhomogeneities in the structure of the superlattice. The narrow-gap effect is observed neither for 1D inhomogeneities nor for a mixture of uncorrelated 1D and 3D inhomogeneities since the decreasing part of the correlation function decreases exponentially to zero upon an increase in r_z in both cases (i.e., the correlation region with an infinite correlation length is absent).

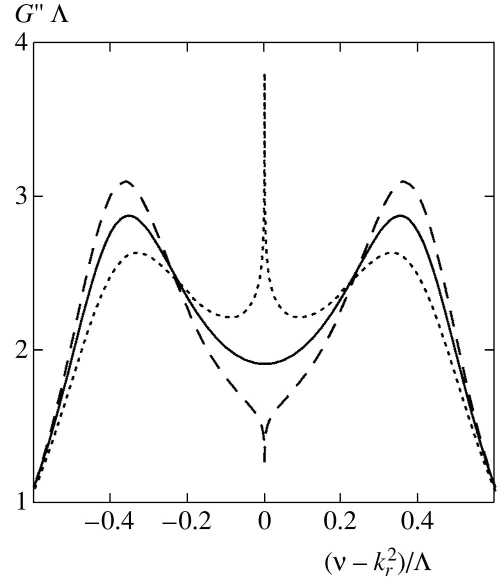

Fig. 4. The shape of the imaginary part of the Green function $G''(\nu)$ for a mixture of 1D and 3D inhomogeneities with $\gamma_1^2 = \gamma_3^2 = 0.1$ and $\eta_1 = \eta_3 = 4$ for $\kappa = 0$ (solid curve), $\kappa = 0.6$ (dashed curve), and $\kappa = -0.6$ (dotted curve).

The actuation of positive correlations in a mixture of inhomogeneities does not lead to the emergence of a region with infinite-length correlations. However, this leads to the emergence of a correlation region with long, but weakly decaying (in proportion to r^{-1}) correlations. This effect precisely causes partial restoration of a narrow gap in the wave spectrum of a superlattice with a mixture of 1D and 3D inhomogeneities for $\kappa > 0$ (dashed curve in Fig. 4). For $\kappa < 0$, this effect of a partially restored narrow gap reverses sign and leads to the narrow-peak effect (dotted curve in Fig. 4). This resonance looks impressive; however, in contrast to positive cross correlations, the emergence of negative correlations requires the fulfillment of rather specific conditions (see Section 2).

4. CONCLUSIONS

This study is devoted to the effect of cross correlations between 1D inhomogeneities simulating random deviations of the boundaries between the layers from their periodic arrangement and 3D inhomogeneities simulating random deformations of the surfaces of these boundaries on the rf susceptibility of the superlattice (Green function). In this study, we take into account a type of correlations such that, for a correlation factor $\kappa > 0$, the buildup of intensity fluctuations for inhomogeneities of one dimensionality leads to the buildup of intensity fluctuations for inhomogeneities of the other dimensionality and vice versa, irrespective of the polarity of such fluctuations. In the most general form, the reason for the emergence of such correlations might lie in the assumption that any random instability in the setup for obtaining superlattices which causes an increase in deviation of the thickness of a layer from a preset value, may also increase the probability that the deformation of the surface of such a layer increases. To simulate this type of cross correlations in a mixture of 1D and 3D inhomogeneities, we introduce the distribution function describing correlation between absolute values $|\chi_1|$ and $|\chi_3|$ of random functions χ_1 and χ_3, leaving the functions themselves uncorrelated. The distribution function introduced in this way leads to the emergence of stochastic synchronization between $|\chi_1|$ and $|\chi_3|$ in the configuration space of variables χ_1 and χ_3. Both χ_1 and χ_3 are functional of space coordinates; this in turn leads to the emergence of stochastic spatial synchronization between intensity fluctuations of 1D and 3D inhomogeneities. This synchronization can be in principle positive or negative depending on the sign of the cross correlation factor κ. However, the probabilities of the emergence of positive or negative cross correlations between 1D and 3D inhomogeneities are substantially different for actual superlattices. Positive cross correlations may emerge in a quite natural way under standard conditions for obtaining superlattices (e.g., due to random instability in the system, leading to synchronous deviation of the position of the boundary between the layers as well as to deformation of the surface of this boundary). However, the emergence of negative cross correlations requires rather specific conditions (e.g., an increase in the deviation of the boundary between the layers from the equilibrium position must lead to an increase in the surface tension quenching random deformations of the surface of this boundary). Using the distribution function introduced here, we derived the correlation function of the superlattice containing a mixture of cross-correlated 1D and 3D inhomogeneities. It was shown earlier [24] that the effect of inhomogeneities on the wave spectrum strongly depends on the asymptotic behavior of the superlattice correlation function for $r \longrightarrow \infty$. The descending part of the correlation function for 1D inhomogeneities exponentially tends to zero for $r_z \longrightarrow \infty$, while the descending part of the correlation function for 3D inhomogeneities tends to nonzero asymptote $L = \exp(-3\gamma_3^2)$ as $r \longrightarrow \infty$. For this reason, in addition to regions with a finite correlation length k_0^{-1}, regions with an infinite correlation length also exist in the 3D case. This leads to a sharp decrease in wave damping and an increase in the effective gap width at the boundary of the Brillouin zone in the 3D case as compared to the 1D case. For a mixture of uncorrelated 1D and 3D inhomogeneities, the descending part of the correlation function has an asymptotic form corresponding to 1D inhomogeneities; i.e., it exponentially tends to zero as $r \longrightarrow \infty$. In this study, we proved that the decreasing part of the correlation function for a mixture of 1D and 3D inhomogeneities with a nonzero cross-correlation factor tends to zero in accordance with a power law (r^{-1}) as $r \longrightarrow \infty$. Thus, as regards its asymptotic properties, this function occupies an intermediate position between the correlation function of a superlattice with 3D inhomogeneities and the correlation function of a superlattice with 1D inhomogeneities or a mixture of uncorrelated 1D and 3D inhomogeneities. In the next part of this study, we analyzed the spacing $\Delta\nu_m$ between the peaks on the imaginary part of the Green function at the boundary of the first Brillouin zone of the superlattice and the half-widths Γ of these peaks for a mixture of 1D and 3D inhomogeneities both for $\kappa > 0$ as for $\kappa < 0$. The spacing $\Delta\nu_m$ between the peaks approximately describes the width of the first forbidden gap in the spectrum of the superlattice, while Γ describes the damping of waves in the superlattice, which is associated with inhomogeneities (for simplicity, we assume that intrinsic damping is zero). It was found that positive cross correlations for which stochastic spatial synchronization of intensity fluctuations of 1D and 3D inhomogeneities takes place lead to partial suppression of the effect of a mixture of 1D and 3D inhomogeneities on the wave spectrum. Indeed, the gap width at the Brillouin zone boundary increases, while damping decreases as compared to the effects observed in a mixture of 1D and 3D inhomogeneities for $\kappa = 0$. Negative cross correlations for which the intensity fluctuations of 1D and 3D inhomogeneities are mainly located in different spatial regions lead to the opposite effect: the gap width decreases and damping increases as compared to the case when $\kappa = 0$. We demonstrated that, in addition to a considerable change in the functional dependences of $\Delta\nu_m$ and Γ on γ_1^2 and γ_3^2, the activation of cross correlations between 1D and 3D inhomogeneities leads to the emergence of a new resonance singularity on the imaginary part of the Green function $G''(\nu)$. For $\kappa > 0$, a narrow dip (gap) appears on the curve describing function $G''(\nu)$ at the center of the forbidden gap in the vicinity of point $\nu = k_r^2$; this dip becomes narrower, but does not disappear upon a further increase in γ_3. Upon a change in the sign of κ, this dip also changes its sign and becomes a narrow peak. The physical mechanism for the formation of these new resonance effects can be described as follows. It follows from the analytic expressions derived for $G''(\nu)$ that a narrow-gap effect should also be observed for 3D inhomogeneities. This effect is associated with the singularity of the correlation function for 3D inhomogeneities (namely, with the existence of correlation regions with finite as well as infinite correlation lengths, whose ratio is determined by asymptote L). With increasing γ_3, the volume of the region with infinite correlation lengths decreases in proportion to L, while the narrow gap in the wave spectrum, which is associated with this region, decreases in proportion to $L^{1/2}$. If we supplement 3D inhomogeneities with 1D inhomogeneities, the volume with infinitely long correlations vanishes together with the narrow gap effect, since the correlation function of the mixture of inhomogeneities decreases exponentially for $r \longrightarrow \infty$. The activation of positive cross correlations in a mixture of inhomogeneities does not lead to the emergence of a region with infinitely long correlation. However, this leads to the emergence of a region with long correlations decreasing in proportion to r^{-1}. Precisely this effect is responsible for partial restoration of a narrow gap in the spectrum of a mixture of 1D and 3D inhomogeneities for $\kappa > 0$. For $\kappa < 0$, the narrow-gap effect changes its sign and leads to a narrow-peak effect.

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REFERENCES

1. J. B. Shellan, P. Agmon, and P. Yariv, J. Opt. Soc. Am. **68**, 18 (1978).
2. Yu. Ya. Platonov, N. I. Polushkin, N. N. Salashchenko, and A. A. Fraerman, Zh. Tekh. Fiz. **57**, 2192 (1987) [Sov. Phys. Tech. Phys. **32**, 1324 (1987)].
3. J. M. Luck, Phys. Rev. B **39**, 5834 (1989).
4. S. Tamura and F. Nori, Phys. Rev. B **41**, 7941 (1990).
5. N. Nishiguchi, S. Tamura, and F. Nori, Phys. Rev. B **48**, 2515 (1993).
6. G. Pang and F. Pu, Phys. Rev. B **38**, 12649 (1988).
7. J. Yang and G. Pang, J. Magn. Magn. Mater. **87**, 157 (1990).
8. D. H. A. L. Anselmo, M. G. Cottam, and E. L. Albuquerque, J. Appl. Phys. **87**, 5774 (1999).
9. L. I. Deych, D. Zaslavsky, and A. A. Lisyansky, Phys. Rev. E **56**, 4780 (1997).
10. B. A. Van Tiggelen and A. Tip, J. Phys. I **1**, 1145 (1991).
11. A. R. McGurn, K. T. Christensen, F. M. Mueller, and A. A. Maradudin, Phys. Rev. B **47**, 13120 (1993).
12. M. M. Sigalas, C. M. Soukoulis, C.-T. Chan, and D. Turner, Phys. Rev. B **53**, 8340 (1996).
13. V. A. Ignatchenko, R. S. Iskhakov, and Yu. I. Mankov, J. Magn. Magn. Mater. **140–144**, 1947 (1995).
14. A. G. Fokin and T. D. Shermergor, Zh. Éksp. Teor. Fiz. **107**, 111 (1995) [JETP **80**, 58 (1995)].
15. A. V. Belinskiĭ, Usp. Fiz. Nauk **165**, 691 (1995) [Phys. Usp. **38**, 653 (1995)].
16. B. Kaelin and L. R. Johnson, J. Appl. Phys., Part 1 **84**, 5451, Part 2 **84**, 5458 (1998).
17. V. A. Ignatchenko and Yu. I. Mankov, Phys. Rev. B **56**, 194 (1997).
18. A. N. Malakhov, Zh. Éksp. Teor. Fiz. **30**, 884 (1956) [Sov. Phys. JETP **3**, 701 (1956)].
19. S. M. Rytov, *Introduction to Statistical Radiophysics,* 2nd ed. (Nauka, Moscow, 1976), Part 1 [in Russian].
20. V. A. Ignatchenko, Yu. I. Mankov, and A. V. Pozdnyakov, Zh. Éksp. Teor. Fiz. **116**, 1335 (1999) [JETP **89**, 717 (1999)].
21. V. A. Ignatchenko, A. A. Maradudin, and A. V. Pozdnyakov, Phys. Met. Metallogr. **91**, S69 (2001).
22. V. A. Ignatchenko, Yu. I. Mankov, and A. A. Maradudin, Phys. Rev. B **65**, 024207 (2002).
23. V. A. Ignatchenko, Yu. I. Mankov, and A. A. Maradudin, Pis'ma Zh. Éksp. Teor. Fiz. **77**, 335 (2003) [JETP Lett. **77**, 285 (2003)].
24. V. A. Ignatchenko, Yu. I. Mankov, and A. A. Maradudin, Phys. Rev. B **68**, 024209 (2003).
25. V. A. Ignatchenko, A. A. Maradudin, and A. V. Pozdnyakov, Pis'ma Zh. Éksp. Teor. Fiz. **78**, 1082 (2003) [JETP Lett. **78**, 592 (2003)].
26. V. A. Ignatchenko and Yu. I. Mankov, Fiz. Tverd. Tela (St. Petersburg) **47**, 565 (2005) [Phys. Solid State **47**, 587 (2005)].
27. G. A. Korn and T. M. Korn, *Mathematical Handbook for Scientists and Engineers* (McGraw-Hill, New York, 1961; Nauka, Moscow, 1968).
28. R. C. Bourret, Nuovo Cimento **26**, 1 (1962); Can. J. Phys. **40**, 782 (1962).

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