

# Computation of current–voltage characteristics of the SNS junctions

Denis Gokhfeld

*L.V. Kirensky Institute of Physics SD RAS, Akademgorodok 50/38, Krasnoyarsk 660036, Russia*

Available online 14 April 2007

## Abstract

Simplified model for current–voltage characteristics of weak links (superconductor – normal metal – superconductor junctions, microbridges, superconducting nanowires) is suggested. It is based on approach which considers Andreev reflections as responsible for the transfer of dissipative current through the metallic Josephson junction. The current–voltage characteristics of tin microbridges at different temperatures were computed.

© 2007 Elsevier B.V. All rights reserved.

PACS: 74.25.Fy; 74.45.+c; 85.25.Am

Keywords: Weak link; Andreev scattering; Subharmonic gap structure

## 1. Introduction

Current–voltage characteristics (CVCs) of superconductor – normal metal – superconductor (SNS) junctions demonstrate the rich peculiarities that is promising for different applications. The Resistively-Shunted-Junction model is commonly used for the description of CVCs of SNS junctions instead poor possibilities of this phenomenological model. It is due to simplicity of RSJ in comparison with the microscopic-based theories of SNS junctions. Pereira and Nicolisky suggested the simple modification [1] of the theory [2] where the time dependent Bogolubov–de Gennes equations were solved for the Andreev reflected quasiparticles. The model [1] simulates the general form of the experimental CVCs of SNS junctions [3] and reproduces the negative differential resistance region on CVC [4]. However, the description of CVCs by [1] is qualitative only. New simple modification of KGN theory is proposed in this article that may lead to more extensive using of the KGN-based approach to the calculation of weak link characteristics (see Fig. 1).

## 2. Model for the dissipative current through SNS junction

The detailed description of the model is presented in [5]. A voltage-biased SNS junction is considered with a constant electric field which is in the N layer in negative  $z$ -direction perpendicular to the NS interfaces. The normal layer N has the thickness  $2a$  more smaller than the thickness of the superconducting bank. The approximated expression for the energy of Andreev bound states is suggested:

$$E_r(k_{zF}) = \frac{\hbar^2 k_{zF}}{2am^*} \pi \left( r + \frac{1}{2} \right) / \left( 1 + C \frac{\hbar^2 k_{zF}}{2am^* \Delta} \right), \quad (1)$$

where  $m^*$  is the effective mass of electron,  $\Delta$  the value of energy gap of superconductor at the temperature  $T$ ,  $k_{zF}$  the  $z$  component of Fermi wave vector of quasiparticles,  $r = 0, 1, 2, \dots$ ,  $C$  is the fitting multiplier. Then the density of bound states [6] has been derived:

$$g(E) = \frac{A}{\pi} \left( \frac{4am^*}{\pi \hbar^2} \right)^2 \frac{E}{(1 - C \frac{2E}{\pi \Delta})^3}, \quad (2)$$

where  $A$  is the normal layer area.

The best agreement of the energy spectrum (1) with the semiclassical approximation [2,6] and the density of bound

*E-mail address:* [gokhfeld@yandex.ru](mailto:gokhfeld@yandex.ru)

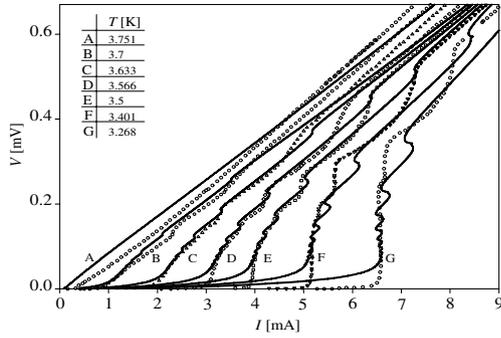


Fig. 1. The current–voltage characteristic of Sn microbridges. Experiment [7] (points) and calculations (solid lines).

states (2) with one of an SNS junction with thick superconducting banks resulted by [6] is for  $C = \pi/2(1 - am^* \Delta / \hbar^2 k_F)$  for  $C > 1$  and  $C = 1$  otherwise. After transformations and simplifications [5] the total current density is resulted:

$$j(V) = \frac{V}{R_N A} + \sum_n \exp\left(-\frac{2a}{l}n\right) \times \left\{ \frac{2em^* a^2}{\pi^3 \hbar^5} * \int_{-\Delta+neV}^{\Delta} dE \frac{|E| \sqrt{\Delta^2 - E^2}}{\left(1 - C \frac{2|E|}{\pi \Delta}\right)^3} \tanh\left(\frac{E}{2k_B T}\right) + \frac{ek_F^2}{4\pi^2 \hbar} \int_{E_1}^{\Delta+eV} dE \frac{E}{\sqrt{E^2 - \Delta^2}} \tanh\left(\frac{E}{2k_B T}\right) \right\}, \quad (3)$$

where  $l$  is the inelastic mean free path and  $R_N$  is the resistance of the  $N$  region,  $E_1 = -\Delta + neV$  for  $-\Delta + neV \geq \Delta$  and  $E_1 = \Delta$  otherwise.

The expression (3) allows to calculate CVCs of weak links for different thicknesses of the normal layer at different temperatures. The subharmonic gap structure, the negative differential resistance, the excess current and the current peak at the small voltage are reproduced on CVCs.

### 3. Fitting of the experimental current–voltage characteristics

The obtained model was used to compute CVCs of the current biased tin microbridges [7,8] (Figs. 1 and 2). Both

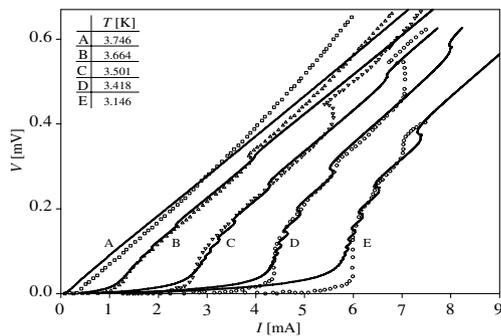


Fig. 2. The current–voltage characteristic of Sn microbridges. Experiment [8] (points) and calculations (solid lines).

sets of CVCs have the similar peculiarities: the subharmonic gap structure, the current peak at the small voltage and the excess current.

Comparison of the computed  $I(V)$  curves and the experimental  $V$  vs.  $I$  dependencies displays satisfactory agreement at temperatures smaller than  $0.99T_c$ . The  $I(V)$  curves were calculated with the reasonable parameters: the critical temperature, the energy gap at zero temperature, the Fermi wave vector of Sn ( $T_c = 3.77$  K,  $\Delta_0 = 0.57$  meV,  $k_F = 1.62 \text{ \AA}^{-1}$ ). The length of microbridges  $2a$  is  $5000 \text{ \AA}$  and  $l = 15a$ . The BCS dependence of  $\Delta$  on  $T$  was used. The high voltages regions on experimental CVCs are close by the computed curves at higher temperatures. It is possibly reasoned by self-heating occurred at high voltages in these experiments [7,8]. Some discrepancy of the computed curves and the experimental points at low voltages is because there were the current-biased measurements of CVCs instead voltage-biased one.

### 4. Conclusion

Simplified model for calculation of current–voltage characteristics of the weak links (SNS junctions, microbridges, superconducting nanowires) was developed. This model makes the KGN approach [2] more convenient for description of experiments. The model was applied for computation of the current–voltage characteristics of tin microbridges at different temperatures.

### Acknowledgements

I am thankful to D.A. Balaev, R. Kümmel and M.I. Petrov for fruitful discussions. This work is supported by program of President of Russian Federation for support of young scientists (Grant MK 7414.2006.2), Krasnoyarsk Regional Scientific Foundation (Grant 16G065), program of Russian academy of science “Quantum macrophysics” 3.4, Lavrent’ev competition of young scientist projects (Project 52).

### References

- [1] L.A.A. Pereira, R. Nicolsky, Physica C 282–287 (1997) 2411.
- [2] R. Kümmel, U. Günsenheimer, R. Nicolsky, Phys. Rev. B 42 (1990) 3992.
- [3] Y.A. Gorelov, L.A.A. Pereira, A.M. Luiz, R. Nicolsky, Physica C 282–287 (1997) 2491.
- [4] M.I. Petrov, D.A. Balaev, D.M. Gokhfeld, S.V. Ospishchev, K.A. Shaihtudinov, K.S. Aleksandrov, Physica C 314 (1999) 51.
- [5] D.M. Gokhfeld, cond-mat/0605427, 2006.
- [6] H. Plehn, U. Günsenheimer, R. Kümmel, J. Low Temp. Phys. 83 (1991) 71.
- [7] V.N. Gubankov, V.P. Kosheletz, G.A. Ovsyannikov, Fiz. Nizk. Temp. 7 (1981) 277.
- [8] M. Octavio, W.J. Skocpol, M. Tinkham, Phys. Rev. B 17 (1978) 159.