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Transport of cold atoms in optical lattices

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Abstract. The work discusses transport of cold atoms in optical lattices. Two related but different problems are considered: interacting Bose atoms subject to a static field (i.e., the atoms in a tilted lattice); and non-interacting atoms in a tilted lattice in the presence of a buffer gas. For these two systems we found, respectively: periodic, quasiperiodic, or decaying Bloch oscillations, as it depends on the strength of atom-atom interactions and the magnitude of the static field; diffusive directed current of atoms, similar to the electron current in ordinary conductors.

1 Introduction

This work is devoted to the transport properties of cold atoms in optical lattices. Needless to say that this system mimics electrons in crystal lattices and, in this sense, presents an artificial crystal. As compared to the natural crystals this artificial crystal has a number of advantages where, to mention few of them, (i) instead of the long-range Coulomb interaction between electrons we have the short-range collision interaction between neutral atoms, which is much easier to treat analytically; (ii) the optical lattices are rigid (no phonons) and free from defects; (iii) the system parameters, like tunnelling rate, strength of atom-atom interactions, lattice geometry, etc. can be continuously varied and one may choose these parameters practically on his own will; (iv) depending on the atomic spices (or isotope of the same spices) the carriers obey either Bose or Fermi statistics. With all these in mind, it is not surprising that currently we observe an ever increasing activity in experimental and theoretical studies of the transport of cold atoms in a lattice (see [1–8], only the experimental works are cited), where one of the aims of these studies is to revisit and, if required, to revise the traditional theory of conductivity developed in the field of solid-state physics.

At this point we would like to mention one more important in the context of the present work advantage of the discussed system. As know, most of analytical methods in the solid-state physics (like, for example, the linear response theory) refer to the thermodynamic limit $N \to \infty$. The system of neutral atoms in a lattice offers an unique opportunity to approach this limit by gradually increasing the number of carriers. Indeed, nowadays the experimentalists routinely work with coherent atomic ensembles consisting of only $N \sim 10^3$ atoms and incoherent ensembles consisting of less than 10 atoms [9]. Thus one may expect that in the nearest future it will be possible to create a coherent ensemble with the given number of atoms. Our theoretical analysis of atomic transport follows this line of progress in the experimental physics. Namely, we begin with a small ensemble of interacting atoms, analyze this system beyond the standard approaches, and then try to extrapolate the obtained results to the thermodynamic limit.

The structure of the paper is as follows. In introductory section 2, aimed to recall the reader some essentials of the solid-state theory, we discuss the dynamics of a single atom in a tilted optical lattice. The dynamics of an ensemble of interacting atoms is considered in section 4, where we restrict ourselves by the case of Bose statistics. This section is preceded by section 3

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devoted to some not very well known properties of the Bose-Hubbard model, which provides the theoretical framework for our analysis of the multi-particle dynamics. Finally, in section 5, we introduce and analyze a simple model for ordinary conductivity with cold atoms. The main results of the work are summarized in the concluding section 6.

2 Single-particle dynamics

Let us consider a single atom in the 1D optical lattice, subject to a static (for example, gravitational, if one think about the vertically oriented optical lattices) field. The system Hamiltonian reads,

$$\widehat{H} = \widehat{H}_0 + Fx, \qquad \widehat{H}_0 = \frac{\hat{p}^2}{2M} + V_0 \cos\left(\frac{2\pi x}{d}\right). \tag{1}$$

where M is the atomic mass, F the magnitude of a static field, d the lattice period, given by one half of the laser wave length, and V_0 is proportional to laser intensity. We are interested in solutions of the time-dependent Schrödinger equation $i\hbar\partial\psi(x,t)/\partial t = \widehat{H}\psi(x,t)$. The common approach to this problem is to look for the solution in the form,

$$\psi(x,t) = \sum_\alpha c_\alpha(t)\phi_{\alpha,\kappa'}(x), \tag{2}$$

where $\phi_{\alpha,\kappa'}(x)$ are the Bloch states with the quasimomentum κ' evolving according to the classical equation of motion $\dot{p} = -F$, i.e $\kappa' = \kappa_0 - Ft/\hbar$. If the energy gap separating the ground Bloch band $E_0(\kappa)$ of $\widehat{H}_0$ from the rest of the spectrum is larger than the Stark energy Fd, one can neglect the Landau-Zenner transitions to higher energy bands and the solution reads as $c_\alpha(t) = \exp\left(-\frac{i}{\hbar}\int_0^t \mathrm{d}t' E_\alpha(\kappa')\right)\delta_{\alpha,0}$. Keeping in mind that the Bloch functions $\phi_{\alpha,\kappa}(x)$ and their dispersion relations $E_\alpha(\kappa)$ are periodic functions of the quasimomentum, this result implies periodic oscillations of any observable (like, for example, the mean atomic momentum) with the Bloch period $T_B = 2\pi\hbar/Fd$.

Note that one can come to the same conclusion in a different way, by using the tight-binding approximation (which will be our main approximation through the paper). Indeed, denoting by $|l\rangle$ the Wannier functions associated with the l-th well of the optical potential, the tight-binding counterpart of the Hamiltonian (1) reads

$$\widehat{H}_{TB} = E_0\sum_l |l\rangle\langle l| - \frac{J}{2}\sum_l \big(\,|l+1\rangle\langle l| + |l-1\rangle\langle l|\,\big) + Fd\sum_l l\,|l\rangle\langle l|, \tag{3}$$

where J is the hopping matrix elements. The Hamiltonian (3) can be easily diagonalized, giving the spectrum $E(\kappa) = E_0 - J\cos(d\kappa)$ if $F = 0$ (note, in passing, that the tight-binding model approximates the ground Bloch band by a cosine function), and

$$E_l = E_0 + Fdl\ , \quad \text{if} \quad F \neq 0. \tag{4}$$

The discrete linear spectrum (4) again implies periodic evolution of the wave function and, hence, periodic oscillations of observables. The eigenfunctions of (3), known as the Wannier-Stark states, are given by

$$|\Psi_l\rangle = \sum_m \mathcal{J}_{m-l}\left(\frac{J}{2Fd}\right)|m\rangle, \tag{5}$$

where $\mathcal{J}_n(z)$ is the ordinary Bessel function. Because the Bessel functions $\mathcal{J}_n(z)$ are exponentially small for $|n| > |z|$, the Wannier-Stark states $|\Psi_l\rangle \approx |l\rangle$ for $Fd > J$. This phenomenon, often referred to as the Wannier-Stark localization by a strong static field, will be shown in section 4 to have an important consequence for the multi-particle dynamics.

3 Multi-particle Hamiltonians

We proceed with the multi-particle problem, where we shall focus on the case of Bose atoms. Then a multi-particle generalization of the tight-binding Hamiltonian (3) is given by

$$\widehat{H} = \widehat{H}_{BH} + Fd\sum_{l} l\hat{n}_l \tag{6}$$

where $\widehat{H}_{BH}$ is the celebrated Bose-Hubbard model,

$$\widehat{H}_{BH} = -\frac{J}{2}\left(\sum_{l} \hat{a}^{\dagger}_{l+1}\hat{a}_l + h.c.\right) + \frac{U}{2}\sum_{l} \hat{n}_l(\hat{n}_l - 1) \tag{7}$$

[from now on we set $E_0 = 0$ in (3) and (7)]. The last term in the Hamiltonian (7) describes the short-range interactions between neutral atoms, where the interaction constant U is defined by the atomic s-wave scattering length and the spatial extension of the Wannier functions.

Let us first discuss the spectral properties of the Bose-Hubbard model, which are crucial for understanding the dynamics of Bose atoms. To simplify the analysis we impose the periodic boundary conditions, i.e., $\hat{a}^{\dagger}_{L+1} \equiv \hat{a}^{\dagger}_1$. In fact these boundary conditions can be realized in practice by using the ring optical lattices [10]. In what follows, however, we shall mainly focus on linear lattices and, hence, the imposed periodic boundary conditions should be considered as a mathematical trick which facilitates the convergence of the limit $L \to \infty$.

3.1 Weak interactions

For $U = 0$ the spectrum of (7) is obviously given by

$$E = -J\sum_{\kappa} \cos(d\kappa)n_\kappa, \qquad \sum_{\kappa} n_\kappa = N\ , \tag{8}$$

where n_κ is the number of atoms in the single-particle Bloch state with the quasimomentum κ. (Note that for a finite L the quasimomentum κ takes discrete values $\kappa = 2\pi k/Ld$ and, hence, there are only L allowed values for the quasimomentum.) The minimal (ground) and the maximal energies of the system, $E = \pm JN$, correspond to situations where all atoms occupy either the center, $\kappa = 0$, or the edge, $\kappa = \pi/d$, of the Brillouin zone. In between, $-JN \le E \le JN$, the density of states $f(E)$ can be shown to be approximated by the Gaussian with the width $\sim J\sqrt{N}$ (see figure 1 below). The eigenfunctions $|\Psi_E\rangle$ with energies falling in this central part of the spectrum are given by the symmetrized product of the Bloch states with different quasimomenta.

As U is increased the eigenfunctions $|\Psi_E\rangle$ undergo a structural change, reflected in statistical properties of the spectrum. Namely, for a given U there is a finite energy interval $E_{min}(U) < E < E_{max}(U)$ where the spectrum obeys the universal Wigner-Dyson statistics for the Gaussian ensemble of random matrices, i.e., is chaotic in the sense of Quantum Chaos [11,12]. Moreover, for the relevant to laboratory experiments parameters $\bar{n} = N/L \sim 1$ and $U > 0.1J$ this energy interval includes more than 80 percents of the energy levels. As an illustration to these statements, the lower panel in figure 1 depicts the results of the statistical analysis of the cental part of the spectrum [13]. The integrated distribution, $I(s) = \int_0^s P(s)\mathrm{d}s$, of the normalized distances between the nearest levels, $s = (E_{i+1} - E_i)f[(E_{i+1} + E_i)/2]$ is shown for $U = 0.02J$ and $U = 0.2J$. It is seen that for $U = 0.2J$ the level spacing distribution perfectly follows the Wigner-Dyson distribution $P(s) = \frac{\pi}{2}s\exp\left(-\frac{\pi}{4}s^2\right)$, which is a hallmark of Quantum Chaos. The statistical analysis of the eigenfunctions, not presented here, is done in the recent works [14,15]. In section 5 we shall utilize the chaotic properties of the Bose-Hubbard model to calculate the atomic current induced by a static field.

For the sake of completeness we briefly discuss the ground and low-energy states belonging to the tail of $f(E)$, which are of particular interest with respect to the problem of superfluidity

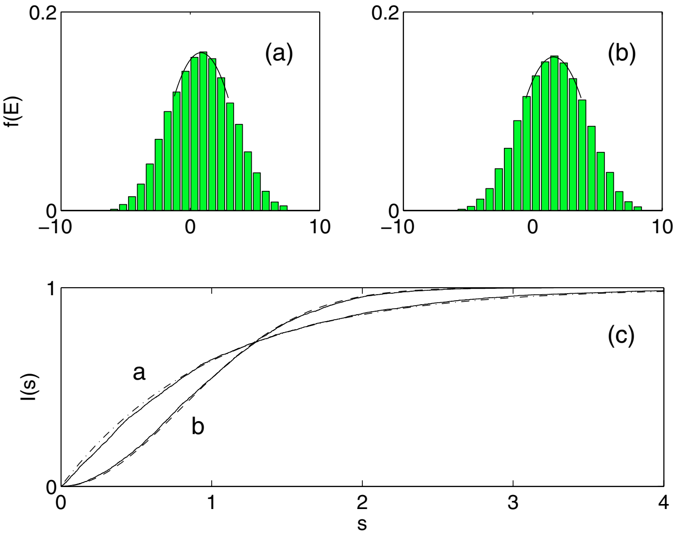

Fig. 1. (a,b) – Density of states of $N = 7$ atoms in a lattice with $L = 9$ sites for the interaction constant $U = 0.02J$ and $U = 0.2J$, respectively. The magnitude of the random potential $\epsilon = 0.2$. (c) – integrated level spacing distributions for the central part of the spectrum. It is seen that for $U = 0.02J$ the distribution follows the Poissonian statistics (dash-dotted line), while for $U = 0.2J$ the spectral statistics is Wigner-Dyson (dashed line).

(superconductivity) of Bose atoms. With increase of U these states also undergo a structural change but remain regular. Moreover, one can find them analytically by adopting the Bogoliubov theory for the case of finite N and L [15,16]. It is interesting to note that in the ring optical lattice some of the Bogoliubov states can support a persistent atomic current (i.e., not decaying circulation of atoms along the ring) even in the presence of an external scattering potential $\hat{V} = \sum_l \epsilon_l \hat{n}_l$ [15].

3.2 Strong interactions

To avoid a possible misunderstanding it is worth mentioning that 'weak' and 'strong' interactions are relative notion and, when distinguishing these two cases, only the ratio between the interaction constant and the hopping matrix element plays the role but not the value of U itself. Since the hopping matrix element J exponentially decreases with increasing depth of an optical lattice one can easily realize the case of strong interactions in a laboratory experiment by simply increasing the laser power.

For a large U the multi-particle energy band (given for a small U by the Gaussian density of states) splits into the Hubbard sub-bands separated by the gaps of the order of U. Our particular interest is the lowest band, which in the limit $U \to \infty$ is described by the hard-core bosons model

$$\widehat{H}_{HC} = -\frac{J}{2} \sum_l (\tilde{a}^\dagger_{l+1} \tilde{a}_l + h.c.) \ . \qquad (9)$$

In the Hamiltonian (9) the creation and annihilation operators satisfy an additional condition $(\tilde{a}^\dagger_l)^2 = (\tilde{a}_l)^2 = 0$ which prohibits the double occupancy of a single well. As know, the hard-core bosons model can be solved analytically by mapping it to the system of spinless fermions (see [17] and references therein) The problem, however, is that the model (9) is valid only in the limit $U \to \infty$ and for any finite U (which is obviously the case realized in practice) there are residual interactions between the atoms. These interactions modify the hard-core bosons Hamiltonian as [18]

$$\widehat{H} = \widehat{H}_{HC} - \frac{J^2}{U}\sum_l \tilde{n}_{l+1}\tilde{n}_l - \frac{J^2}{2U}\left(\sum_l \tilde{a}^\dagger_{l+1}\tilde{n}_l\tilde{a}_{l-1} + h.c.\right) \tag{10}$$

where the condition $U \gg J$ is implicitly assumed. We have found that either of two additional terms in (10) can break an integrability of the model. Nevertheless, when taking together, they effectively cancel each other and the system remains (quasi)regular until some critical $J^2/U \sim 0.1$.

We conclude this section by summarizing our studies of the finite size Bose-Hubbard model. Starting from non-interacting atoms, an initial increase of the interaction constant causes the onset of chaos in the system. The further increase of the interaction constant, however, convert it back to integrable by approaching the hard-core bosons limit. For the filling factor $\bar{n} = N/L$ of the order of unity the interval on U, where the system is chaotic, is approximately given by $0.1 < U/J < 10$. To identify this interval for arbitrary atomic densities remains an open problem.

4 Bloch oscillations of interacting atoms

We come back to the problem formulated in the beginning of section 3, – the dynamics of interacting Bose atoms subject to a static force F. First of all we note that the translation symmetry of the system, broken by the static term in (6), can be actually recovered by using the gauge transformation,

$$\widehat{H} \to \widehat{H}(t) = -\frac{J}{2}\left(\sum_l e^{-i\omega_B t}\hat{a}^\dagger_{l+1}\hat{a}_l + h.c.\right) + \frac{U}{2}\sum_l \hat{n}_l(\hat{n}_l - 1), \tag{11}$$

where $\omega_B = Fd/\hbar$. This allows us to impose periodic boundary conditions also in the considered here case $F \neq 0$. The second remarks concerns the magnitude of the static force. Namely, considering the system dynamics one should distinguish between the cases of weak forcing, $Fd < J$, and strong forcing, $Fd > J$. This is a particular property of the multi-particle Bloch oscillations, – for a single atom the Bloch dynamics are periodic oscillations of the atomic momentum, $p(t) = p_0 \sin(\omega_B t)$, independent of the magnitude of the static force. (As was mentioned above, in this work we use the single-band approximation, where F may formally take an arbitrary large value.)

4.1 Weak forcing

The characteristic feature of the weak forcing regime is a rapid decay of Bloch oscillations due to the decoherence effect of atom-atom interactions. In a qualitative level this phenomenon can be understood by analyzing the instantaneous spectrum of the Hamiltonian (11) where $\theta = \omega_B t$ is considered as a parameter. An example of this spectrum for a very small system $N = L = 5$ and $U = 0.1J$ is given in figure 2(a). The thin line in the figure shows the diabatic continuation of the ground state, by following which the atoms would oscillate coherently. However, due to Landau-Zener transitions at avoided crossings (which substitute the real crossing at $U = 0$), the other quasimomentum Fock states become populated when the static force drag the system along the thin line. This leads to a thermalization of the atoms and, as a consequence, to the decay of Bloch oscillations.

In principle, one can try to describe the thermalization process by thoroughly analyzing the Landau-Zener tunnelling at the avoided crossings. An alternative (and, actually, more constructive) approach is to study the properties of the Floquet-Bloch operator, which we define

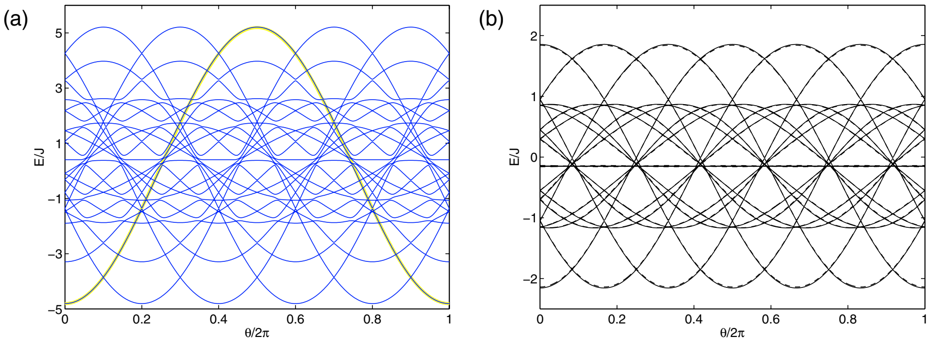

Fig. 2. (a) Instantaneous spectrum of the Hamiltonian (11) for $U = 0.1J$ ($L = 5$, $N = 5$). The thin line shows the diabatic continuation of the ground state. (b) Low-energy instantaneous spectrum of (11) for $U = 10J$ ($L = 6$, $N = 3$). Dashed lines show the spectrum of the effective Hamiltonian (10) parameterized by the phase θ.

as the evolution operator over one Bloch period:

$$\widehat{W} = \widehat{\exp}\left[-\frac{i}{\hbar}\int_0^{T_B} \widehat{H}(t)dt\right]. \tag{12}$$

It has been found that the matrix of the Floquet-Bloch operator (12) can be well identified with a random matrix of the circular orthogonal ensemble [19]. It is intuitively clear that this property of the Floquet-Bloch operator implies an irreversible decay of Bloch oscillations.

The above result refers to the case of weak-to-moderate atom-atom interactions $U \sim J$. As was shown in section 3, for strong interactions $U \gg J$ the system approaches the hard-core bosons limit. Figure 2(b) shows the instantaneous spectrum of the lowest Hubbard band for $N = 3$, $L = 6$ and $U = 10J$. As compared to figure 2(a), the gaps of avoided crossings are now exponentially small. This insures the diabatic transitions at the crossings and, hence, non-decaying Bloch oscillations. A transition from periodic Bloch oscillations for vanishing atom-atom interactions to rapid decay for $U \sim J$ and back to periodic oscillations for strong interactions is illustrated in figure 3 for $N = 5$ atoms in $L = 25$ wells. We also would like to note that the depicted time evolution of the mean momentum is converged in the thermodynamic sense, i.e., the further increase of the system size does not affect the displayed curves.

4.2 Strong forcing

For $Fd \gg J$ the Floquet-Bloch operator (12) can be calculated analytically by using the result on the Wannier-Stark localization by a strong field (see section 2). Namely, in the Fock basis the matrix of the operator (12) is diagonal with the elements $\sim \exp\left[-i(U/Fd)\sum_l n_l^2\right]$. This leads to periodic modulations of Bloch oscillations with the modulation frequency $\omega_U = U/\hbar$ [20]. In particular, for the mean atomic momentum we have

$$p(t) = p_0 \sin(\omega_B t)\exp\{-2\bar{n}[1 - \cos(\omega_U t)]\}. \tag{13}$$

The similar to (13) result also holds in the case of hard-core bosons where, however, the modulation frequency is given by $\omega_U = J^2/U\hbar$ [18].

5 Atomic current

It was shown in the previous section that the static field either thermalize the system during few Bloch cycles or induces the quasiperiodic Bloch oscillations. Can we get in the system a

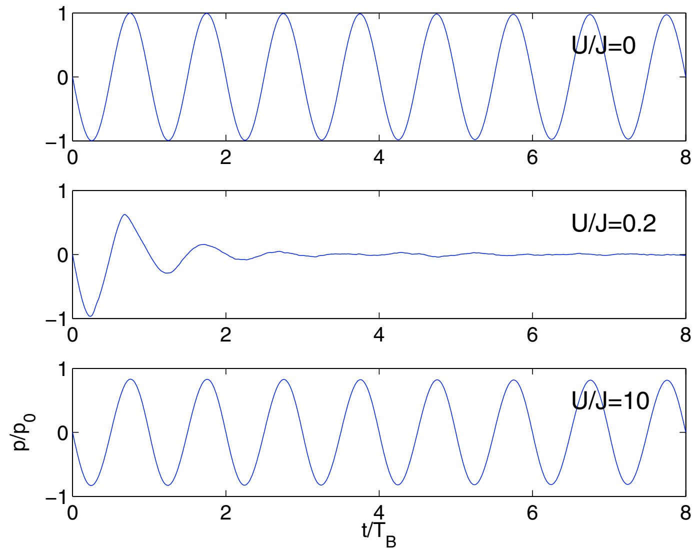

Fig. 3. Bloch oscillations of Bose atoms for different ratio of the interaction constant to the hopping matrix elements ($L = 25$, $N = 5$). The initial conditions are given by the ground states of the system.

directed current of atoms, similar to the electron current in ordinary conductors? The answer is 'yes' if we satisfy a necessary condition for the directed current, which one obtains by drawing an analogy with the electron system. Namely, the Stark energy Fx, realized by moving carriers, should be absorbed by some reservoir. (For electrons in the solid crystal this is obviously the bath of phonons.) To this end we introduce a simple model for directed atomic current in optical lattices [23].

We consider spin-polarized Fermi atoms subject to a static force F with an admixture of Bose atoms. (To some extent this mimics the experiment [21]). To simplify the problem we shall assume that Bose atoms are not affected by the static field [22]. Moreover, since Pauli's exclusion principle prohibits interactions between the spin-polarized Fermi atoms, we shall use the single-particle approach for Fermi atoms. Thus we have

$$\widehat{H} = \widehat{H}_F + \widehat{H}_B + \widehat{H}_{int}, \tag{14}$$

where $\widehat{H}_F$ is the tight-binding Hamiltonian (3) of Fermi atoms, $\widehat{H}_B$ the Hamiltonian of Bose atoms given by the Bose-Hubbard model (7), and $\widehat{H}_{int} = \epsilon \sum_l \hat{n}_l |l\rangle\langle l|$ interactions between Bose and Fermi atoms with the interaction constant ϵ.

The key feature of the model (14) is that the Bose-Hubbard model is generally a chaotic system and, because of this, it possesses the properties of a reservoir [24]. In particular, the cross-correlation function for the atom number fluctuations,

$$R_{l,m}(t,t') = Tr[\rho_B(k_B T)\Delta\hat{n}_l(t)\Delta\hat{n}_m(t')],$$
$$\Delta\hat{n}_l(t) = \exp(i\widehat{H}_B t/\hbar)\hat{n}_l \exp(-i\widehat{H}_B t/\hbar) - \bar{n}, \tag{15}$$

rapidly decays in time and can be approximated by δ-function,

$$R_{l,m}(t,t') = \bar{n}(\bar{n}+1)\delta\left(\frac{t-t'}{\tau}\right)\delta_{l,m}, \tag{16}$$

where $\tau \approx 3J_B/\hbar$ [23]. Using (16) we derive the master equation for the reduced density matrix of the Fermi subsystem,

$$\frac{\partial \rho^{(F)}_{l,m}}{\partial t} = -\frac{i}{\hbar}\left[\widehat{H}_F, \rho^{(F)}\right]_{l,m} - \gamma(1-\delta_{l,m})\rho^{(F)}_{l,m}\ , \tag{17}$$

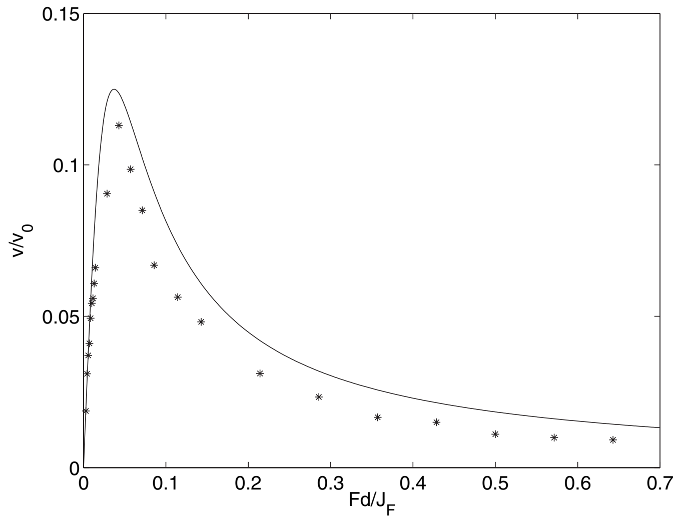

Fig. 4. Stars: drift velocity (or Fermionic current) $\bar{v}$ vs. static tilt (or voltage) Fd. Parameters of the Bose subsystem $N = 7$, $L = 9$, $U_B/J_B = 3/7$, where $J_B = J_F$. Interaction constant between Fermi and Bose atoms $\epsilon = 0.143 J_F$. Solid line shows the Isaki-Tsu dependence (19) for the relaxation constant γ given in (18).

which gives the following result for the mean atomic velocity,

$$v(t) = v_0 e^{-\gamma t} \sin(\omega_B t), \qquad \gamma = \frac{\tau \bar{n}(\bar{n}+1)\epsilon^2}{\hbar^2}. \tag{18}$$

Although the exponential decay of Bloch oscillations of Fermi atoms could be expected from the very beginning, the master equation formalism provides an analytical expression for the relaxation constant γ which, as shown below, enters the expression for the directed diffusive current [25].

Up to now we got the decay of oscillations but did not get the directed current. The reason for this is that the result (16) assumes an infinite temperature of the Bose subsystem, where $\rho_B(t = 0)$ is proportional to the identity matrix. If we consider Bose atoms at a finite temperature, i.e., if $\rho_B(t = 0) \sim \exp(-E/k_B T)$, we observe the decay of oscillations to some nonzero $\bar{v}$, $v(t) = v_0 \exp(-\gamma t) \sin(\omega_B t) + \bar{v}$. This decay is accompanied by heating of the Bose subsystem, where the rate of heating (which can be conveniently characterized by increase of the mean energy of Bose atoms) relates to the drift velocity $\bar{v}$ of Fermi atoms as $\partial E_B/\partial t = \bar{v} F$, – in full analogy with heating of ordinary conductors by the electron current.

We also studied the dependence of the directed current $\bar{v}$ on the magnitude of the static force. Remarkably, that our simple model (14) is capable to reproduce the famous Isaki-Tsu dependence for the electron current across the semiconductor superlattices,

$$\bar{v} = \frac{v_0}{4} \frac{\omega_B/\gamma}{1 + (\omega_B/\gamma)^2}. \tag{19}$$

According to equation (19) (and keeping in mind that $\omega_B = Fd/\hbar$) the current initially increases linearly with F (Ohmic regime), takes maximal value for $\omega_B = \gamma$, and then decreases as $1/F$ (so-called negative differential conductivity regime). Figure 4 shows the results on directed current obtained by simulating the dynamics of one Fermi atom and $N = 7$ Bose atoms in a lattice with $L = 9$ sites (periodic boundary conditions). In spite of a relatively small system size, a reasonable agreement with the Isaki-Tsu dependence is noticed.

6 Conclusion

In this work we addressed the problem of conductivity with cold atoms in optical lattices. Naively one might expect that the atomic current can be induced by tilting the lattice, i.e., when the atoms are subject to some external static field. However, it is known for decades that in the case of non-interacting atoms, where the single-particle approach is justified, the static field induces Bloch oscillations but not the directed current. We thoroughly investigated the effect of atom-atom interactions on Bloch oscillations and show that neither weak nor strong interactions cannot cause the directed atomic current. In the present paper we restricted ourselves by considering the case of Bose atoms but the same conclusion holds for Fermi atoms [26]. We also would like to note that our studies of the multipaticle Bloch oscillations reveal the importance of Quantum Chaos for understanding the different regimes of the system dynamics.

Although there are no directed current for a pure ensemble of Bose or Fermi atoms, one can get it by using a mixture of atoms. We introduced a simple model for atomic conductivity, the crucial ingredient of which is separation of the system into two subsystem, – the Fermi atoms, which are subject to a static field, and Bose atoms, which play the role of a bath for Fermi atoms. Utilizing the chaotic property of interacting Bose atoms, we obtained an analytical expression for the overall relaxation constant in the system. Moreover, substituting this relaxation constant into the Isaki-Tsu equation we reproduced numerically observed dependence of the fermionic current on the magnitude of the static field.

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