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DETERMINING THE REFRACTIVE INDEX OF THE BACKGROUND MEDIUM IN SUBSURFACE TOMOGRAPHY

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We propose a noncontact method for determining the mean refractive index of the medium in problems of subsurface tomography using ultra-wideband multi-position radio sounding in the case of a plane air–ground interface. The method is based on the computer oblique focusing of the radioimage of the sounded half-space containing buried objects. The proposed method does not require any a priori information on the position of the scattering objects in the medium. The feasibility of the proposed approach is verified experimentally.

1. INTRODUCTION

In problems of detection and characterization of land-buried objects, one of promising approaches is based on the use of ultra-wideband radiation along with the application of the synthetic-aperture radar (SAR) technique for data processing. In certain cases, the possibility of noncontact sounding of the ground is of crucial importance. In particular, this is necessary in the problem of detection of land-buried antipersonnel plastic mines [1–3].

The literature describes numerous methods for solving the inverse problem of subsurface tomography (i.e., reconstruction of the distribution of inhomogeneities buried under the ground surface) based on the synthetic-aperture radar technique [1–7]. The most common sounding scheme assumes the translational displacement of the transmitter–receiver system with a fixed step in the horizontal direction parallel to the medium interface. Papers [5–7] describe a fast algorithm whose main idea consists in focusing the scattered ultra-wideband radiation to a near-surface point of the medium, which ensures high resolution in the horizontal plane. Moreover, high resolution in the vertical direction is ensured due to use of the ultra-wideband radiation. In this case, the distance along the vertical direction corresponds to the time delay multiplied by the wave velocity in the medium. However, this velocity is *a priori* unknown since the refractive index of the medium is unknown. Therefore, in order to determine the absolute depth of the object location, one needs to know the mean value of the refractive index. In view of this, along with the spatial reconstruction of inhomogeneities, an important problem is to determine the mean refractive index of the medium using noncontact methods.

The most widespread method for determining the refractive index is the method of diffraction hyperbolas [1–4]. The diffraction hyperbola is the response of a pulse ground-penetrating radar to a point scatterer in the medium. The hyperbola is plotted in the coordinate plane in which the horizontal coordinate corresponds to the antenna displacement and the vertical coordinate is the time delay of the received scattered pulse. The shape of the diffraction hyperbola depends on the refractive index of the medium. Therefore, the hyperbola shape allows one to determine the refractive index, since the pulse propagation time increases with increasing refractive index and the diffraction hyperbolae becomes "stretched" in the

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vertical direction. To realize this method, the presence of a point scatterer in the medium is necessary, although exact information on its position is not required. An important point is that the transmitter–receiver system should be located either directly on the interface or sufficiently close to it, when the wave refraction at the air–ground interface can be neglected. If the antenna is lifted above the ground surface, then the refraction at the interface makes the shapes of the diffraction hyperbolas weakly dependent on the refractive index, which impedes its evaluation.

In the present work, we propose a new method of evaluating the refractive index of the medium in the case of highly lifted antennas using super-wideband radio sounding. The proposed method is workable for almost an arbitrary distribution of scattering inhomogeneities in the medium.

2. THE OBLIQUE FOCUSING

Consider the following measurement scheme. An omnidirectional transceiver antenna is moved with a fixed step in the horizontal plane at a certain height above the studied ground surface. At each point of the antenna location, the ground is sounded by ultra-wideband radiation and the amplitude and phase of the scattered field E are measured at all operating frequencies. The measurement result can be represented in the form of a scalar function $E(x, y, f)$ of three variables, where x and y are the Cartesian coordinates of the antenna in the horizontal plane and f is the sounding-signal frequency.

Using the data on the scattered field $E(x, y, f)$, it is possible to determine the three-dimensional distribution of the scattering inhomogeneities under the ground surface. The algorithm proposed in works [5, 6] ensures the sounding of the medium by a narrow vertical collimated wave beam, which is possible due to the radiation focusing to the near-surface point of the medium. When passing to the time domain by using the inverse Fourier transform with respect to the frequency, the medium is scanned along the beam axis, which is equivalent to the medium sounding by a short pulse propagating along the beam. Therefore, the distance along the vertical axis of the reconstructed three-dimensional image should correspond to one half of the time delay of the pulse in the medium, multiplied by the phase velocity. Since the refractive index of the medium is unknown, we assume it equals unity for definiteness.

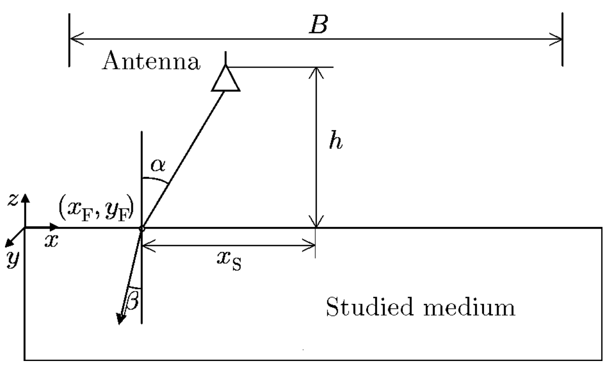

Fig. 1. The sounding scheme with synthesizing the oblique focusing.

To determine the refractive index, we use the oblique-focusing method proposed in [7], which allows one to sound the medium not only in the vertical direction (as in [6]), but also along an oblique path. Using the oblique sounding [7], one obtains a three-dimensional image which is linearly distorted compared with the image obtained by the vertical sounding [6]. Due to the wave refraction at the medium interface, this linear distortion depends on the refractive index.

The main idea of the oblique-focusing method consists in the focusing of the SAR to the near-surface point displaced from the center of the synthetic aperture. The geometry of the oblique-focusing method is shown in Fig. 1, where x_{S} is the displacement of the focus point with coordinates $(x_{\mathrm{F}},\, y_{\mathrm{F}})$ from the aperture center.

Mathematically, the oblique-focusing algorithm can be represented in the form of a convolution integral:

$$F(x_{\mathrm{F}}, y_{\mathrm{F}}, f) = \iint\limits_{S} E(x_0, y_0, f) M_{\mathrm{S}}(x_{\mathrm{F}} - x_0, y_{\mathrm{F}} - y_0, f)\, \mathrm{d}x_0\, \mathrm{d}y_0, \tag{1}$$

where $M_{\mathrm{S}}(x, y, f) = \exp(-i\, 2k\sqrt{(x - x_{\mathrm{S}})^2 + y^2 + h^2}$ is the weight function of the focusing, $k = 2\pi f/c$ is the wave number in free space, h is the height of the transceiver antenna above the medium interface, f is

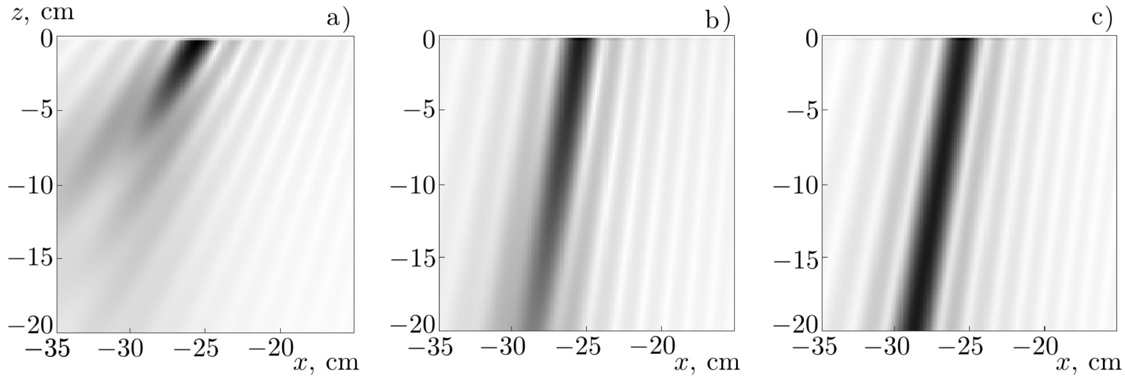

Fig. 2. Instrument function in the case of oblique focusing ($\alpha = 30°$) at an operating frequency of 10 GHz for free space (*a*), the medium with refractive index $n = 3$ (*b*), and the medium with $n = 3$ if the approximate instrument function is used (*c*).

the radiation frequency, c is the speed of light in free space, $F(x, y, f)$ is the scattered field focused to the near-surface point of the medium, and S is the surface over which the transceiver antenna is moved within the base size B.

Using the solution of the direct problem [6], Eq. (1) can be rewritten in the single-scattering approximation in the form:

$$F(x_{\mathrm{F}}, y_{\mathrm{F}}, f) = \iiint\limits_V \Delta\varepsilon(x, y, z) Q\left(x_{\mathrm{F}} - x, y_{\mathrm{F}} - y, z, f\right) \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z, \tag{2}$$

where

$$Q\left(x_{\mathrm{F}} - x, y_{\mathrm{F}} - y, z, f\right) = k_1^2 \iint\limits_S G^2(x - x_0, y - y_0, z, f) M_{\mathrm{S}}(x_{\mathrm{F}} - x_0, y_{\mathrm{F}} - y_0, f)\,\mathrm{d}x_0\,\mathrm{d}y_0$$

is the instrument function of the system in the case of oblique focusing, $G(x, y, z, f)$ is the Green's function taken from [6], and $\Delta\varepsilon(x, y, z)$ is the distribution of the scattering inhomogeneities in the medium.

Using Eq. (1), one obtains that the elongated region of maximum values of the instrument function is oblique with respect to the vertical. Figure 2*a* shows the numerically calculated instrument function $Q(x, y, z, f = 10)$ GHz of the system for free space. Figure 2*b* shows the instrument function of the system for the medium with the refractive index n=3. It is seen in Fig. 2*b* that for a large refractive index, there occurs the vertical "stretching" of the region in which the instrument function is maximum. This effect is related to an increase in the electrical length of the wave path and to the wave refraction at the interface of the media. At the same time, the angle between this elongated region and the vertical axis decreases, but still remains nonzero, which can be used for determining the refractive index. Note that the instrument function $Q(x, y, z, f)$ shown in Fig. 2*b* enters Eq. (2), which describes the operation of convolution and can therefore be applied to all inhomogeneities of the sounded medium by varying the values of x_{F} and y_{F}. This means that for different coordinates x_{F} and y_{F}, the inhomogeneities of the lower half-space are scanned at the same angle.

Let us use the above-mentioned feature of the instrument function and approximate it by the following expression:

$$Q(x, y, z, f) \approx Q(x - z \tan\beta, y, 0, f) \exp[2ik_1\,(x \sin\beta + z \cos\beta)],$$

where $k_1 = kn$ is the wave number in the medium, β is the angle between the vertical and the axis of the localization region of the instrument function in the medium. Figure 2*c* shows the form of the approximated instrument function $Q(x, y, z, f = 10\,GHz)$. It is seen that up to depths of 10–15 cm, the used approximation yields a small error. Using this simplification, we perform the inverse Fourier transform over the frequency in Eq. (2):

$$\tilde{F}(x_{\rm F}, y_{\rm F}, t) \equiv \int\limits_{\Delta f} F(x_{\rm F}, y_{\rm F}, f)\exp(-2\pi i f t)\,{\rm d}f$$
$$= \iiint\limits_{V} \Delta\varepsilon(x,y,z)\tilde{Q}(x_{\rm F} - x - z\tan\beta, y_{\rm F} - y, ct\cos[\beta/(2n)] - z)\,{\rm d}x\,{\rm d}y\,{\rm d}z, \quad (3)$$

where

$$\tilde{Q}(x_{\rm F} - x - z\tan\beta, y_{\rm F} - y, ct\cos[\beta/(2n)] - z)$$
$$\equiv \int\limits_{\Delta f} Q(x_{\rm F} - x - z\tan\beta, y_{\rm F} - y, 0, f)\exp(2ik_1\,[(x_{\rm F} - x)\sin\beta + z\cos\beta] - 2\pi i f t)\,{\rm d}f$$

is the instrument function of the system in the time domain. This function can be highly localized if a sufficiently wide frequency range is used. As an example, Fig. 3 shows the numerically calculated instrument function $\tilde{Q}$ for the frequency band $\Delta f = 20$ GHz. Using the high localization of the instrument function $\tilde{Q}$ and Eq. (3), one can approximately write

$$\tilde{F}(x_{\rm F}, y_{\rm F}, t) \propto \Delta\varepsilon(x_{\rm F} - ct\sin[\beta/(2n)], y_{\rm F}, ct\cos[\beta/(2n)])\,. \quad (4)$$

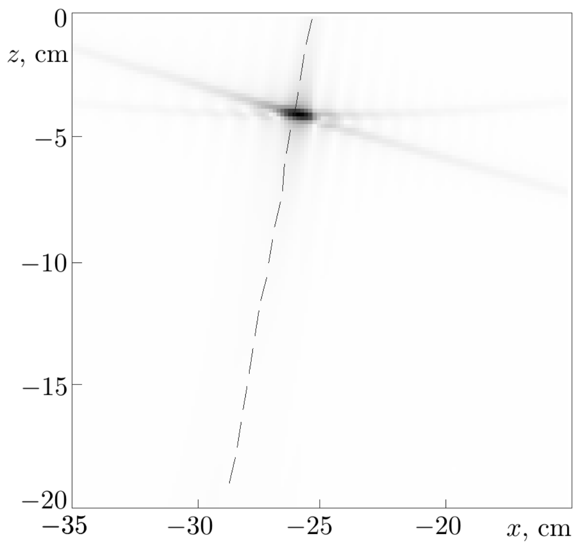

Fig. 3. Localization of the instrument function of the system in the time domain.

Note that the above consideration does not take into account polarization of the radiation. However, this is not really necessary since the transmission coefficient at the medium interface for the oblique focus has the same value for the entire reconstructed image and plays the role of a constant factor which can be eliminated using the normalization procedure. Moreover, we do not discuss the influence of absorption in the medium.

3. USING THE LEAST-SQUARE METHOD FOR DETERMINING THE REFRACTIVE INDEX

The angle β of the oblique scanning of the medium can be related to the incidence angle α which does not depend on the refractive index of the medium and is determined only by the aperture position and the displacement of the focusing point with respect to the aperture center (see Fig. 1). It is reasonable to assume that this angle is determined by the bisectrix of the focal cone whose base is the synthetic aperture and the vertex coincides with the focal point. In what follows, the incidence angle α will be called the oblique-focusing angle.

Let us introduce the variable $z = ct/2$. Then, taking into account the refraction law at the medium interface ($\sin\alpha = \sin\beta$) and approximation (4), one can write

$$\tilde{F}(x, y, 2z/c, \alpha) \propto \Delta\varepsilon(x', y', z'),$$

where

$$x' = x - z\sin(\alpha)/n^2, \qquad y' = y,$$
$$z' = z\sqrt{n^2 - \sin^2(\alpha)}/n^2 \tag{5}$$

are the actual coordinates of the inhomogeneities in the sounded medium. The oblique-focusing angle α is specified in the data-processing procedure and can be chosen arbitrarily. The coordinate transformation

$$x_\alpha = x' + z'\sin\alpha/\sqrt{n^2 - \sin^2\alpha}, \qquad y_\alpha = y',$$
$$z_\alpha = z'n^2/\sqrt{n^2 - \sin^2\alpha}, \tag{6}$$

which is inverse of transformation (5), allows one to compare the data-processing results (4) with the actual distribution $\Delta\varepsilon(x', y', z')$ of the inhomogeneities. The latter should not depend on α and therefore coincides with the result for the vertical focusing

$$\Delta\varepsilon(x', y', z') \propto \tilde{F}(x_\alpha, y_\alpha, 2z_\alpha/c, \alpha) = \tilde{F}(x', y', 2z'n/c, \alpha = 0). \tag{7}$$

Equation (7) becomes identity if the parameter n in this equation coincides with the refractive index of the sounded medium. Therefore, Eq. (7) can be considered an implicit equation for finding n and which should be satisfied for all points (x', y', z') of the lower half-space. Since experimental data always contain measurement noise, estimation of the parameter n by the least-square method reduces to minimization of the functional

$$\Phi[n] = \sum_{(x',y',z')\in V} \left| \tilde{F}(x_\alpha, y_\alpha, 2z_\alpha/c, \alpha) - \tilde{F}(x', y', 2z'n/c, \alpha = 0) \right|^2. \tag{8}$$

In this case, errors of the approximations adopted in the model will also be compensated to some extent.

In order to estimate the potential accuracy of the proposed method, we note that it is determined by the difference between the inhomogeneity distributions reconstructed for $\alpha = 0$ and $\alpha \neq 0$. For fairly small angles α, this difference manifests itself mainly in the horizontal displacement Δx of the images, which depends of the depth z' of the inhomogeneities. The first relation in Eq. (6) indicates that the error Δn of determination of the refractive index leads to the displacement of the reconstructed distributions of the inhomogeneities by the quantity

$$\Delta x \approx z'\, n\, \Delta n \sin(\alpha) \Big/ (n^2 - \sin^2\alpha)^{3/2} \approx z'\, \Delta n/n^2 \sin\alpha.$$

Such a displacement will be physically indistinguishable if its value does not exceed the horizontal size of the focal region, which is determined by the expression

$$R = \lambda h/B,$$

where B is the synthetic aperture base, h is the antenna height above the medium interface, and λ is the wavelength. From this, we write the following approximate estimate for the accuracy of the proposed method of determining the refractive index:

$$\Delta n \approx \frac{n^2}{\sin\alpha}\,\frac{\Delta x}{z'} = \frac{n^2}{\sin\alpha}\,\frac{\lambda h}{z'B}. \tag{9}$$

It is clear that for improvement of the accuracy, one should avoid using small angles α of oblique focusing. Moreover, the oblique focusing is not efficient for media with large refractive indices n. As an example, we give an estimate of the accuracy of determining the refractive index for dry sand with $n = 1.5$

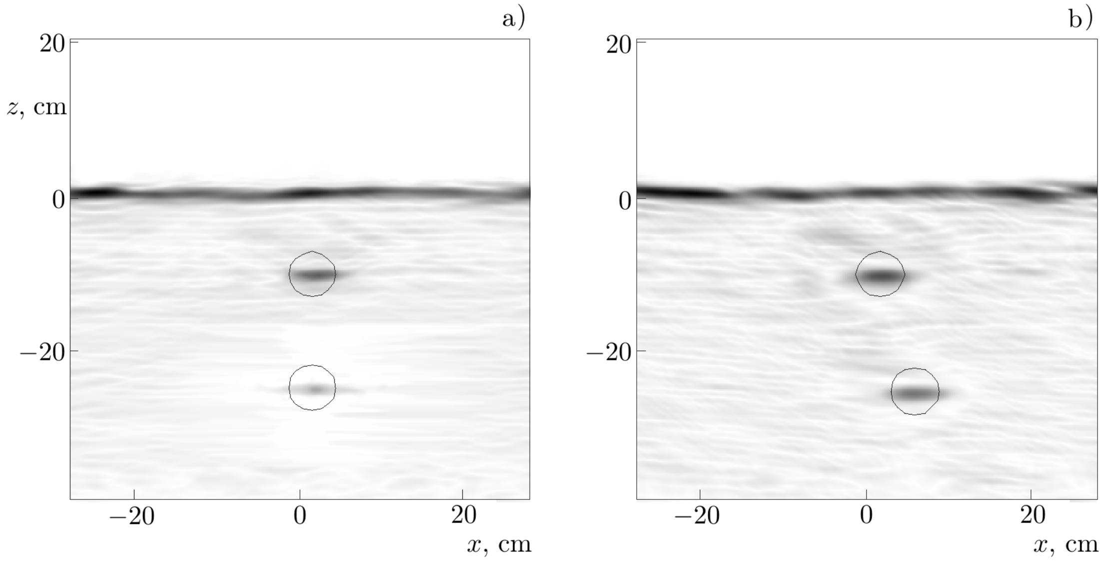

Fig. 4. Comparison between the reconstructed images for (*a*) vertical and (*b*) oblique focusing.

in the case of the angle $\alpha = 30°$ and the sounding depth $z' = 16$ cm. According to Eq. (9), we obtain the estimate $\Delta n \approx 0.23$ for a 56-cm base synthesized at a height of 30 cm above the sand for the radiation frequency $f = 20$ GHz.

On the whole, the accuracy of the method increases with increasing depth of the scattering objects and decreasing refractive index of the medium. Using the averaging in the least-square method, the actual accuracy of the method can be increased compared with the approximate estimate given by Eq. (9).

4. EXPERIMENTAL VERIFICATION OF THE METHOD

To test the proposed method, we performed an experiment on determining the mean refractive index of dry sand. In the experiment, two corner reflectors with a side of 5 cm in length were used as scatterers. The corner reflectors were buried in sand one above another at 6 and 16 cm depths. Further, the measurements were made in a frequency range of 500 MHz to 20 GHz. The transceiver antenna system was moved with 1-cm steps within a 56-cm region, and the antenna height above the sand was equal to 35 cm.

TABLE 1.

The oblique-focusing angle, deg	Determined refractive index
5	1.28
10	1.56
15	1.56
20	1.71
25	1.38
30	1.61

The sounding data were processed using the oblique-focusing algorithm. Figure 4 shows examples of the images obtained for the cases of (*a*) vertical and (*b*) oblique focusing (the inclination angle $\alpha = 30°$). The image processing yielded the refractive index $n = 1.6$. Table 1 shows the results of determining the refractive index from experimental data for various focusing angles up to 30°. The mean refractive index turned out to be equal to 1.5.

The obtained result agrees with direct determination of the refractive index using the vertical focusing, which gives $n = 15/10 = 1.5$ for the apparent distance 15 cm between the scatterers and the distance 10 cm directly measured by a probe. On the whole, the error of determination of n by the oblique focusing method amounts to 0.2, which is close to the estimate obtained above. The achieved accuracy is not very high. However, it is important to emphasize that the proposed method is noncontact and does not require any *a priori* information on the location of the sounded objects.

5. CONCLUSIONS

A new method was proposed for noncontact determination of the refractive index of the medium using the synthetic-aperture radar technique. The noncontact method of determination of the mean refractive index does not require any *a priori* information on the position of scattering objects in the medium. The performed experiments corroborated the workability of the method.

The experimental studies were carried out in the laboratory of the Department of Microwave and Communication Engineering in the Institute of Electronics, Signal Processing, and Communications at the Faculty of Electrical Engineering and Information Technology of the Otto-von-Guericke University of Magdeburg .

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