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Selective control of states of a three-level quadrupolar nucleus using non-selective radio-frequency pulses

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Abstract

The scheme for obtaining the composite operator of the selective rotation from nonselective RF pulses separated by intervals of free evolution has been proposed for quadrupole nuclei. On an example of three levels, it has been shown that the rotation by this operator is performed with accuracy comparable with the accuracy of a simple selective RF pulse, but in shorter time.

1 Introduction

The expected computational power of the quantum computer uncloses huge possibilities for its practical application, starting from simulation of large quantum systems and ending some mathematical problems which are considered exponentially complex for conventional computers. Principles of operation of quantum computers are investigated rather well, and search of ways of physical realization of computations on quantum systems is now carried on. The great number of works on quantum computations is carried out now on two-level basic elements - qubits [1]. Quantum computations on multilevel quantum systems [2-11] - qudits - are investigated insufficiently though such systems meet in the nature more often, and in works under the theory of computations are made conclusions about possible advantages of the computer on qudits, for example, in stability to noise and in growth rate of dimensionality of Hilbert space at increase of number of basic elements. An example of a quantum system which can be considered as qudit, is the nucleus with spin $I > \frac{1}{2}$ in the strong magnetic field. Interaction of a nucleus quadrupole moment with a gradient of a crystalline field results in formation $2I + 1$ non-equidistant energy levels [2-5] which can be taken for computational basis of qudit. The system control is realized by methods of nuclear magnetic resonance (NMR) and prime experiments are already implemented for spin $I = 1$ [4], $I = \frac{3}{2}$ [5] and $I = \frac{7}{2}$ [3]. As contrasted to spins $I = \frac{1}{2}$, quadrupole nucleus have qualitatively new

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physical properties (for example, sensitivity not only to magnetic, but also to an electric field) which can appear deciding at a choice of element base of the quantum computer.

A key element in any implementation of quantum computing is the ability to selective excites one transition while leaving the rest of the levels unaffected [1]. Thus frequency selective excitation could in principle be achieved by using a long low power radio-frequency (RF) pulse (a selective or "soft" pulse) [4, 5], but an experimental time may be limited by relaxation processes. It is known that a faster scheme based on high-power RF pulses (nonselective or "hard" pulses) and delays can be applied to a system of two spin $I = \frac{1}{2}$(qubits) with different Larmor frequencies [12]. However, such method cannot be used for multilevel systems with quantum transitions separated by a quadrupolar interaction [2-5]. In the present work for a quadrupolar nucleus with three levels (spin $I = 1$, qutrit) we propose a scheme to obtain the composite operator of selective rotation using only hard RF pulses and periods of free evolution. We shown by numerical simulation that fidelity of such operator is comparable to fidelity of the standard selective RF pulse, but its duration is considerably shorter. Our results are important for implementation of quantum algorithms on systems of nuclei with a small quadrupolar interaction.

2 Operators of rotation for qubit anf qutrit

As is known, the state of a quantum logic element is described by superposition of basic states.

$$|\Psi\rangle_{qubit} = c_1|0\rangle + c_2|1\rangle = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}; |\Psi\rangle_{qubit} = c_1|0\rangle + c_2|1\rangle + c_3|2\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}. \tag{1}$$

The change of state (1) is implemented an action of unitary operators (matrices). Operator of the selection rotation is one of the major such operators. Its action in a case of qubits it is possible to present as rotation of spin $I = \frac{1}{2}$ on Bloch sphere:

$$R_\alpha(\theta) = e^{-iu\frac{1}{2}\sigma_\alpha}, \alpha = x, y, z \tag{2}$$

$$R_x(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}; R_y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}; R_z(\theta) = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}. \tag{3}$$

Index α determines a direction of rotation axis; σ_α is Pauli matrices and θ is an angle of rotation. In a case of qutrit, operator R has more complex view and has not a simple geometrical interpretation. Let us define a selective rotation between qutrit basis states m and n as follows:

$$R_\alpha^{m-n}(\theta) = e^{-iuB_\alpha^{m-n}}, \alpha = x, y, \tag{4}$$

$$\begin{gathered} B_x^{0-1} = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; B_x^{1-2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \\ B_y^{0-1} = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; B_y^{1-2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}; \end{gathered} \tag{5}$$

For example, the selective rotation about Y axis between states of $|1\rangle - |2\rangle$ is

$$R_y^{1-2}(\theta) = e^{-i\theta B_y^{1-2}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ 0 & \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}; \tag{6}$$

3 1. Nuclear spin I=1 in the magnetic field

The Hamiltonian of the nucleus with spin $I = 1$ in the high magnetic field has the form [13]:

$$H_0 = \omega_0 I_z + H_q, \qquad H_q = q(I_z^2 - \frac{2}{3}) \tag{7}$$

where $\omega_0 = \gamma B_0$ is the Larmor spin precession frequency, q is the coupling constant of the quadrupole moment of the nucleus with the gradient of the crystal field, and I_z is the spin projection operator onto the direction of the constant external field (the Z axis). The energy is measured in frequency units; i.e., we set $\hbar = 1$. The quadrupole interaction H_q gives rise to the formation of three nonequidistant levels with different I_z values (Fig. 1):

$$|I_z = 1\rangle = |0\rangle; |I_z = 0\rangle = |1\rangle; |I_z = -1\rangle = |2\rangle \tag{8}$$

These states are taken as basis states of the qutrit.

Control of the system state performs by RF magnetic field with amplitude B_1 and frequency ω during for finite time $t_r \gg \frac{1}{\omega}$, in other words this is RF pulses. Interaction with this field is

$$V = -\frac{1}{2}\Omega(e^{-i(\omega t+\varphi)} I_- + e^{i(\omega t+\varphi)} I_+)$$

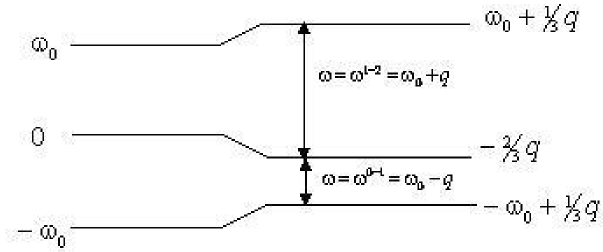


Figure 1: energy level diagram of nuclear spin states with (right) and without (left) quadrupolar interaction.

Here $I_\pm = I_x \pm iI_y$ are the increasing and decreasing operators, respectively, and $\Omega = \gamma B_1$ is the field amplitude.

The time evolution of the state in the reference frame rotating with frequency [13] is described by the evolution operator

$$U(t) = e^{-iHt} \tag{9}$$

with the time-independent effective Hamiltonian

$$H = H_0 + \omega I_z - \frac{1}{2}\Omega(e^{-i\varphi} I_- + e^{i\varphi} I_+) \tag{10}$$

The RF field phase φ determines the field direction in the rotating coordinate system. The field for $\varphi = \pi (\varphi = 0)$ and $\varphi = \frac{\pi}{2} (\varphi = -\frac{\pi}{2})$ is directed along the X $(-X)$ and Y $(-Y)$ axes, respectively.

4 Selective rotation by means of selective pulses

As shown higher, action of RF pulse is determined by evolution operator (9) with Hamiltonian (10). For obtaining of selective rotation operator (4), it is necessary to select parameters of an external field so that

$$U(t) = e^{-iHt_{\gamma}} = R_{\alpha}^{m-n}(\theta). \tag{11}$$

Rotation between $m-n$ states implements by turning RF field frequency with resonance frequency of transition between corresponding energy levels:

$$\omega^{0-1} = \omega_0 - q; \qquad \omega^{1-2} = \omega_0 + q. \tag{12}$$

In this case, other transitions should not be excited, and it is necessary for this purpose, that the field amplitude was much less than the difference between resonance frequencies of two transitions (12). Thus, the requirement $\Omega \gg q$ should be satisfied for the selective pulse. The pulse duration determine an angle of rotation that

$$t_r = \frac{\theta}{\sqrt{2}\Omega}. \tag{13}$$

Gaussian, rather than rectangular, selective pulses are used in experiments [4]

$$\Omega(t) = |Omega exp(-g^2(\frac{2t}{t_g} - 1)^2), \tag{14}$$

where t_g is pulse duration, g is the limiting exponent. For example, for $g = 1/5, exp(-g^2 = 0.105)$ and

$$t_g = 1.752\frac{\theta}{\sqrt{2}\Omega}. \tag{15}$$

A Gaussian pulse is obtained by composing a package of narrow rectangular pulses whose amplitude varies according to Eq. (14).

Nonresonant transitions between the level with energy $2q$ and two other levels with close energies $\pm\frac{Omega}{\sqrt{2}}$(we omit the term $\frac{-2q}{3}$ in Eq. (7), leading to an unobservable common phase factor) occur simultaneously with resonant transitions. These transitions lead to an error in obtained rotation operator. An additional contribution to the error comes from the phase factor $exp(-it2q)$ of the nonresonant level, because it corresponds to one in the operator $R_{\alpha}^{m-n}(\theta)$. Partially this phase error can be eliminating by selecting pulse duration that:

$$t_{r,g} = \frac{2\pi}{\omega_q}\left(\frac{\omega_q\theta}{2\sqrt{2}\pi\Omega}\right)_{round}; \omega_q = 2q. \tag{16}$$

Here expression in brackets is rounded off up to the proximal integer.

The error of operator obtained as the pulses action is estimated both with (Δ_1) and without (amplitude error Δ_2) the inclusion of the phase of matrix elements from the formulas

$$\Delta_1 = \frac{1}{3}\sqrt{\sum_{i,j}\left|U_{ij} - U_{ij}^{teor}\right|^2}; \Delta_2 = \frac{1}{3}\sqrt{\sum_{i,j}\left|\left|U_{ij}\right| - \left|U_{ij}^{teor}\right|\right|^2}, \tag{17}$$

where U_{ij}^{teor} are the matrix elements of the ideal operator $R_{\alpha}^{m-n}(\theta)$ given by Eq. (4), and U_{ij} are the matrix elements of the evolution operator given by Eq. (9) that are calculated with the use of the MATLAB package. Calculation results are shown in Fig. 2 for $R_y^{1-0}(\frac{\pi}{2})$

Oscillations with high frequency $\omega_q \approx 2q$ are observed against the monotonic increase in the error. The error is minimal if nonresonant levels are rotated by an angle multiple to 2π in time $t_r(t_g)$. The error of the absolute values of the matrix elements at the minima is smaller than the total error; therefore, a small phase difference between these elements holds. It is seen that the error of the Gaussian pulse is smaller than the error of the rectangular pulse, but the Gaussian pulse cannot be made shorter than $\sim \frac{6}{q}$, because the error begins to abruptly increase for shorter pulses.

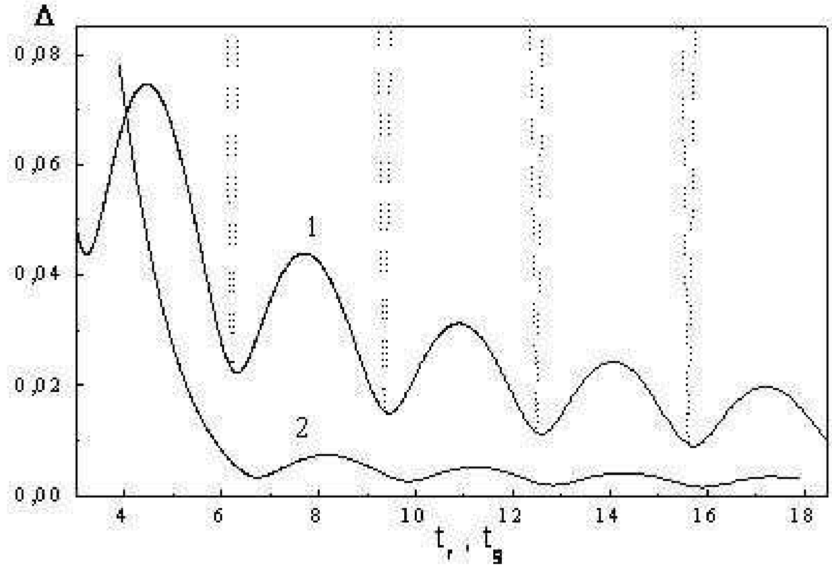


Figure 2: Selective rotation error for $R_y^{1-2}(\frac{\pi}{2})$ as a function of RF pulse duration (in the units of $\frac{1}{q}$). The solid line is the error Δ_2 for rectangular pulse (1) and Gaussian pulse (2). The dashed line is the total error Δ_1 for the Gaussian pulse.

5 Selective rotation by means of nonselective pulses

The nonselective rotation operator acts simultaneously on three state of the system

$$P_\alpha(\theta) = e^{-i\theta I_\alpha}. \tag{18}$$

For example, the rotation matrix about the Y axis has the form

$$P_y(\theta) = e^{-i\theta I_y} = \frac{1}{2}\begin{pmatrix} 1+\cos\theta & -\sqrt{2}\sin\theta & 1-\cos\theta \\ \sqrt{2}\sin\theta & 2\cos\theta & -\sqrt{2}\sin\theta \\ 1-\cos\theta & \sqrt{2}\sin\theta & 1+\cos\theta \end{pmatrix};$$

To implement the operator given by Eq. (18), the amplitude of the RF field in Eq. (10) must be much larger than the difference between the resonance frequencies of various transitions (12), i.e., $\Omega \gg q$. At $\omega = \omega_0$ for RF pulse corresponding Eq. (18), it follows from Eq. (9) that

$$\{\theta\}_\alpha = e^{-it_p(H_p - \Omega I_\alpha)}, \tag{19}$$

where $t_p = \frac{\theta}{\Omega}$ is the pulse duration.

In order to obtain the selective transition on qutrit states, we must construct an effective Hamiltonian in Eq. (9) with the form of matrices $B^{m-n}_{\alpha}[\theta](5)$. This Hamiltonian must include operators describing the allowable interactions on the system. Let us introduce the notation

$$A = e^{-i\varphi I_x}(H_q\tau_1)e^{i\varphi I_x}; B = e^{-i\psi I_y}(H_q\tau_2)e^{i\psi I_y}; C = \xi I_x + \eta I_y, \tag{20}$$

where the first two operators can be obtained from the free-evolution operators and nonselective rotation operators with the use of the property of exponential operators

$$e^{-i\varphi I_\alpha}e^{iHt}e^{i\varphi I_\alpha} = exp(e^{-i\varphi I_\alpha} \cdot iHt \cdot e^{i\varphi I_\alpha}), \tag{21}$$

and the third operator in Eq. (20) can be obtained by means of the RF field. Equating the sum of the matrices of operators (20) to Eq. (5), we arrive at the system of nine equations

$$B^{m-n}_{\alpha} = H_{eff}\tau = A + B + C = \left[\begin{array}{ccc} \frac{1}{6}((3\cos^2\varphi - 1)q\tau_1 + (3\cos^2\psi - 1)q\tau_2) & \frac{\sqrt{2}}{2}i(-q\tau_1\sin\varphi\cos\varphi + \eta + i(q\tau_2\sin\psi\cos\psi + \xi)) & -\frac{1}{2}((1-\cos^2\varphi)q\tau_1 + (\cos^2\psi - 1)q\tau_2) \\ \frac{\sqrt{2}}{2}i(q\tau_1\sin\varphi\cos\varphi - \eta + i(q\tau_2\sin\psi\cos\psi + \xi)) & -\frac{1}{3}((3\cos^2\varphi - 1)q\tau_1 + (3\cos^2\psi - 1)q\tau_2) & -\frac{\sqrt{2}}{2}i(-q\tau_1\sin\varphi\cos\varphi - \eta + i(q\tau_2\sin\psi\cos\psi - \xi)) \\ -\frac{1}{2}((1-\cos^2\varphi)q\tau_1 + (\cos^2\psi - 1)q\tau_2) & -\frac{\sqrt{2}}{2}i(q\tau_1\sin\varphi\cos\varphi + \eta + i(q\tau_2\sin\psi\cos\psi - \xi)) & \frac{1}{6}((3\cos^2\varphi - 1)q\tau_1 + (3\cos^2\psi - 1)q\tau_2) \end{array}\right]. \tag{22}$$

The solution of this system provides desired parameter values presented in the table. Note that the parameters and determining the free-evolution times must be always positive; hence, the selective rotations at positive and negative angles are implemented with different values of the RF field parameters.

	φ	ψ	$q\tau_1$	$q\tau_2$	ξ	η
B^{0-1}_{x}	$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	0
B^{0-1}_{-x}	$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	0
B^{1-2}_{x}	$\frac{\pi}{2}$	$-\frac{\pi}{4}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	0
B^{1-2}_{-x}	$\frac{\pi}{2}$	$-\frac{\pi}{4}$	$\frac{1}{2\sqrt{4}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	0
B^{0-1}_{y}	$-\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	0	$\frac{1}{2\sqrt{2}}$
B^{0-1}_{-y}	$-\frac{\pi}{4}$	$-\frac{\pi}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	0	$-\frac{1}{2\sqrt{2}}$
B^{1-2}_{y}	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	0	$\frac{1}{2\sqrt{2}}$
B^{1-2}_{-y}	$\frac{\pi}{4}$	$-\frac{\pi}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	0	$-\frac{1}{2\sqrt{2}}$

Table 1. Parameter values for obtaining B^{m-n}_{α} matrices.

Thus, at parameters presented in the table, we obtain the matrices B^{m-n}_{α} in the form of the sum

$$\theta B^{m-n}_{\alpha} = \theta(A + B + C). \tag{23}$$

Since the operators in this expression do not commute with each other, in order to obtain the desired result, we use the Trotter-Suzuki formula [14] for the exponential operators

$$\left(e^{\frac{-i\theta A}{2n}}e^{\frac{-i\theta B}{2n}}e^{\frac{-i\theta C}{n}}e^{\frac{-i\theta B}{2n}}e^{\frac{-i\theta A}{2n}}\right)^n = e^{-i\theta(A+B+C)} + O((\theta/n)^3). \tag{24}$$

In view of Eq. (23), the presented product of the operators in the limit $n \to \infty$ converges to the ideal selective rotation given by Eq. (4).

Substituting operators (20) in Eq. (24) and using Eq. (21), we obtain a product of evolution operators corresponding action of sequence of nonselective pulses separated by free evolution intervals. In particular, the sequence for the selective rotation about the Y axis is written in the form

$$\{\varphi\}_{-x} \overset{f_1/2}{\longleftarrow} \{\varphi\}_x \cdot \{\varphi\}_{-y} \overset{f_1/2}{\longleftarrow} \{\varphi\}_y \cdot \{\delta_y\}_y \cdot \{\varphi\}_{-y} \overset{f_1/2}{\longleftarrow} \{\varphi\}_y \cdot \{\varphi\}_{-x} \overset{f_1/2}{\longleftarrow} \{\varphi\}_x =$$
$$\{\varphi\}_{-x} \overset{f_1/2}{\longleftarrow} \{\varphi\}_x \cdot \{\varphi\}_{-y} \overset{f_1/2}{\longleftarrow} \delta_y\}_y \overset{f_1/2}{\longleftarrow} \varphi\}_y \cdot \{\varphi\}_{-x} \overset{f_1/2}{\longleftarrow} \{\varphi\}_x, \tag{25}$$

where

$$t_1 = \frac{\theta\tau_1}{n}; t_2 = \frac{\theta\tau_2}{n}; \delta_x = \frac{\theta\xi}{n}; \delta_x = \frac{\theta\eta}{n}, \tag{26}$$

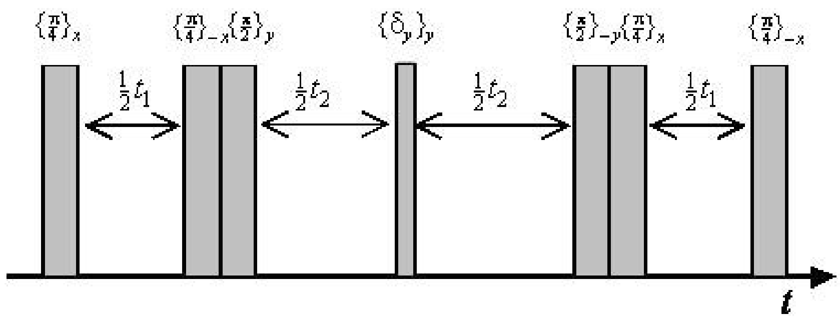


Figure 3: The pulse sequence for the selective rotation about Y between levels 1 and 2. The rectangles are the nonselective pulses, and arrows are the free-evolution intervals.

and $\overset{t}{\longleftarrow} \equiv e^{-itH_q}$ means an interval of free evolution. In transformation (25), we use the property of the rotations about one axis (Y) and remove two pulses. A similar truncated sequence can be obtained for the X rotation if operator A changes to B and vise versa. Pulse sequence (25) can be present in visual scheme in Fig. 3. Iterating this sequence n times, we obtain operator R_{α}^{m-n}

Formulas (21), (23) and (24) are strictly valid when ideal nonselective rotation operators (18) are used. In the real experiment, these operators are obtained by means of the evolution operator given by Eq. (19). The presence of the quadrupole interaction along with the RF field leads to errors disappearing only in the limit $\Omega \to \infty$. In particular, since the quadrupole interaction does not change sign when the rotation angle changes sign (the direction of the RF field changes), the condition

$$\{\theta\}_{\alpha} \cdot \{\theta\}_{-\alpha} = 1 \tag{27}$$

and, therefore, Eq. (21) are not satisfied. Numerical error (17) of operator (24) implemented by means of the sequence of RF pulses (25) is shown in Fig. 4 and Fig. 5. A feature of sequence (25) is the presence of RF pulses that are introduced

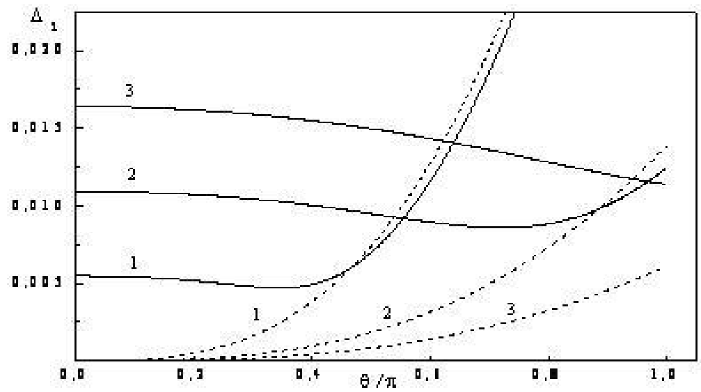


Figure 4: Total error Δ_1 for composite selective rotation operator $R_y^{1-2}(\theta)$ vs. angle θ at $\Omega = 200q$. The repetition number n in Eq. (24) is given as the digits near the lines. The dashed lines show the same error after the change of nonselective RF pulses (19) to the ideal rotation operators given by Eq. (18).

for transformation (21) of the Hamiltonian and H_q thereby do not undergoing scaling with changing n. The number of such pulses increases proportionally to n and, correspondingly, the total error caused by H_q in Eq. (19), as well as the sequence duration, increases. This is seen in Fig. 4 and Fig. 5. Therefore, in order to reduce the error associated with the violation Eq. (27), it is necessary to sufficiently increase the amplitude of the RF field. Usually $\theta \in [0, \pi]$, and so it is enough to take value $n = 1$ or $n = 2$, because in this case error associated with approximation (24) comparable with error produced not ideal action of RF pulses (Fig. 4).

Operator $R_\alpha^{m-n}(\theta)$ can be use to obtain other more complex gates, for example a quantum Fourier transform (QFT) [9-11]

$$QFT = i \cdot R_y^{1-2}\big(-\frac{\pi}{2}\big) \cdot R_y^{0-1}(-2arctg\sqrt{2}) \cdot R_z^{0-1}(\pi) \cdot R_y^{1-2}\big(\frac{\pi}{2}\big) =$$

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{-1-i\sqrt{3}}{2} & \frac{-1+i\sqrt{3}}{2} \\ 1 & \frac{-1+i\sqrt{3}}{2} & \frac{-1-i\sqrt{3}}{2} \end{pmatrix} \tag{28}$$

Rotation about the Z axis is realized in the form of composed $X - Y$ rotations

$$R_z^{0-1}(\pi) = R_y^{0-1}\big(\frac{\pi}{2}\big) \cdot R_x^{0-1}(\pi) \cdot R_y^{0-1}\big(\frac{\pi}{2}\big).$$

The result of simulation of the QFT-gate implementation both by using the rectangular selective pulses, and by using the proposed sequence of the nonselective pulses divided by the free-evolution intervals (at $n = 1$ and $n = 2$) shown in Fig. 6. Time T is the summarize duration all pulses (and delays for Fig. 6b) which necessary implements to obtain sequence of rotation in Eq. (28). It is visible, that proposed method allows to reduce the total time of experiment on the order at the comparable fidelity of QFT-gate.

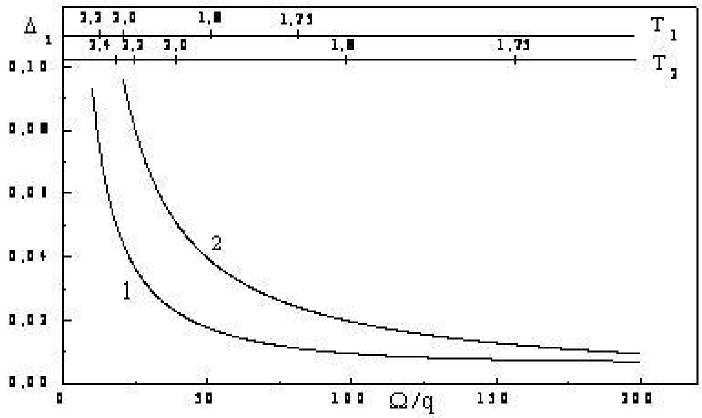
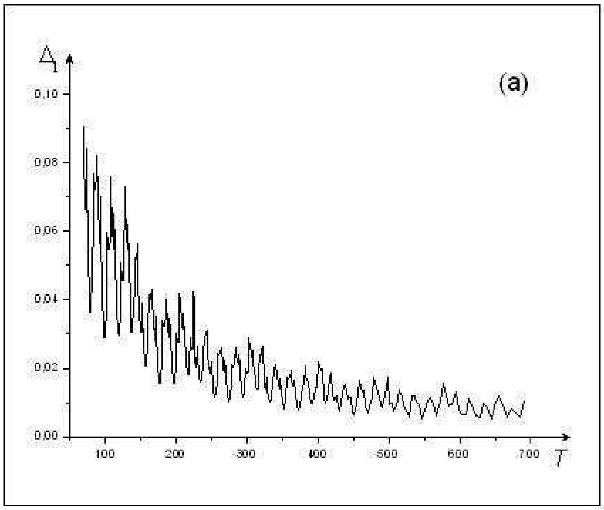


Figure 5: Total error Δ_1 for composite selective rotation operator $R_y^{1-2}(\theta)$ vs. . amplitude of the RF field at $\theta = \frac{\pi}{2}$. The repetition number n is given as the digits near the lines. The upper scale shows the total duration T_n of the operator (in the units of $\frac{1}{q}$) which is equal to the sum of the durations of RF pulses and delays.

6 Conclusion

The comparison of the results (shown in Figs. 2 and 5 as well as Fig. 6) of two methods of the implementation of selective rotations shows that, in the case of the weak quadrupole interaction, the rotation with a comparable accuracy can be implemented by means of a composite selective pulse in time $T \sim \frac{1}{q}$, which is much smaller than the duration of a simple selective pulse $t_1 \sim \frac{1}{\Omega} \gg \frac{1}{q}$.In presented scheme additional correction of external field parameters do not required rather then selective pulses which have phase errors requiring some corrections similar Eq. (16). Note that, according to the calculation, the gain in time for the $\frac{\pi}{2}$ pulse is larger than that for the $\frac{\pi}{2}$ pulse shown above.

The proposed method will allow to implement more complex quantum gates and algorithms in system with weak quadrupolar interaction (for example, in liquid crystals [3-5]). In conclusion, note that this technique of obtaining the composite selective pulse can be extended to quadrupole nuclei with many levels, as well as to the electron levels of atoms and ions in the crystal field.



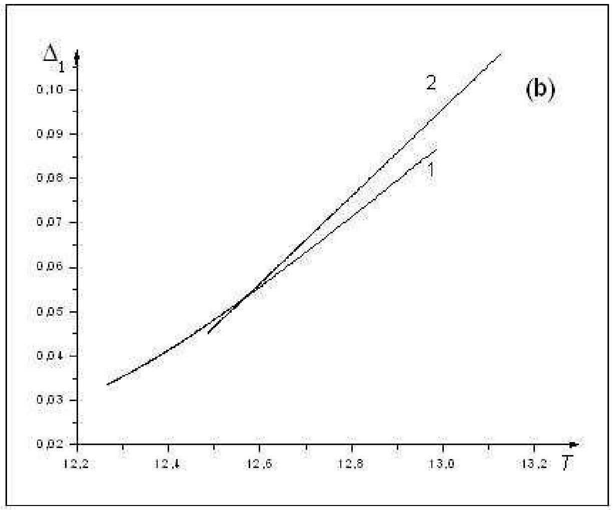


Figure 6: Total error Δ_1 for QFT-gate (28) vs. time of the experimental realization both by using selective rectangular pulses (a) and nonselective pulses divided by intervals of free evolution (b).

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