ISSN 0021-3640, JETP Letters, 2008, Vol. 87, No. 6, pp. 334–339. © Pleiades Publishing, Ltd., 2008.
Original Russian Text © V.E. Zobov, V.P. Shauro, A.S. Ermilov, 2008, published in Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, 2008, Vol. 87, No. 6, pp. 385–390.

Implementation of the Quantum Order-Finding Algorithm on Two Qudits

V. E. Zobov[a], V. P. Shauro[b], and A. S. Ermilov[a]

[a] *Kirensky Institute of Physics, Siberian Branch, Russian Academy of Sciences, Akademgorodok, Krasnoyarsk, 660036 Russia*

[b] *Siberian Federal University, Krasnoyarsk, 660041 Russia*

Received January 24, 2008; in final form, February 12, 2008

A quantum circuit has been proposed for the algorithm for finding the permutation order on two qudits with the number of levels d_1 and d_2. The sequence of the RF pulses for implementing the algorithm on two quadrupole nuclei $I_1 = 7/2$ ($d_1 = 8$) and $I_2 = 3/2$ ($d_2 = 4$) has been calculated and the algorithm has been numerically simulated. A method for preparing pseudopure states has been presented. A comparison with the implementation of the algorithm by NMR methods on five qubits has been performed.

PACS numbers: 03.67.Lx

DOI: 10.1134/S0021364008060143

INTRODUCTION

Quantum computations on d-level basis elements (qudits) with $d > 2$ can be preferable. For example, the number of qudits required for the computational basis of a given size is smaller than the number of qubits [1–4]; therefore, qudits are easily controlled. Other advantages are also expected. However, quantum computations on qudits are insufficiently studied although it is proved [1, 5] that any algorithm can be implemented using a universal set of one- and two-qudit elementary logic operators (gates) [3, 6, 7]. However, only a few particular quantum circuits were developed to elementary operations that can be implemented by experimental tools. In particular, on four levels of the quadrupole Na nucleus, the search algorithm was experimentally implemented in [8] and the Dutch algorithm was implemented in [9]. An adder was organized on eight levels of the Cs nucleus in [10]. In the mentioned works, the notion of virtual qubit, rather than qudit, was used [2].

In this work, the algorithm of finding the permutation order, which was implemented on five qubits by Vandersypen et al. [11] using NMR methods, is studied on two qudits. Such an algorithm underlies the known Shor's quantum factorization algorithm, which was demonstrated by Vandersypen et al. [12] on seven qubits.

QUANTUM ALGORITHM OF FINDING THE PERMUTATION ORDER

Let permutation s be applied on a set Y consisting of d elements. Each permutation is the product of its independent cycles [13]:

$$\begin{gathered} s = s_1 s_2 \ldots s_k, \\ \text{where } s_i = (y, s_i(y), \ldots, s_i^{l_i-1}(y)). \end{gathered} \quad (1)$$

Here, l_i is the length of the ith cycle, $s_i^{l_i}(y) = y$, and y is an element of the set Y belonging to the subset on which the permutation s_i is applied. The order r of the permutation s ($s^r = e$ is the identity permutation) is equal to the least common multiple of the lengths of the cycles l_1, l_2, …, l_k appearing in decomposition (1) of s.

To implement the quantum algorithm under consideration [11], it is necessary to write the results of multiple applications of the permutation on the states of two registers, two qudits in our case. As will be shown below, this can be made for each cycle s_i by means of the *SUM* gate [6, 7]:

$$SUM_{1i}|x\rangle_1 \otimes |y\rangle_i = |x\rangle_1 \otimes |x + y(\mathrm{mod}\, l_i)\rangle_i, \quad (2)$$

where $|x\rangle_1$ is the state of the control qudit and $|y\rangle_i$ is the target virtual qudit formed of l_i successive levels of the initial qudit (the use of the neighboring levels makes it possible to avoid forbidden transitions, but this is not important for implementing the algorithm).

The *SUM* gate for two qudits with the numbers of levels d_1 and d_2 can be implemented by the scheme [7]

$$SUM_{12} = (E_1 \otimes QFT_{d2})^{-1} P_{12} (E_1 \otimes QFT_{d2}). \quad (3)$$

Here, P_{12} is the operator of the controlled phase shift with the $(d_1 d_2 \times d_1 d_2)$ diagonal matrix in the space of

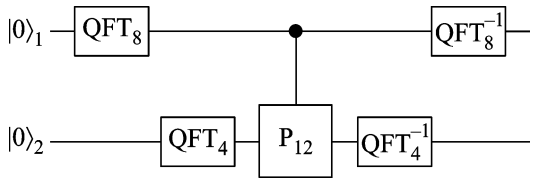

Fig. 1. Quantum circuit for the order-finding algorithm on two qudits at $d_1 = 8$ and $d_2 = 4$.

the states of two qudits whose matrix elements are defined as

$$\langle xy|P_{12}|xy\rangle = \exp\left(ixy\frac{2\pi}{d_2}\right), \quad x = 0\ldots(d_1-1), \quad y = 0\ldots(d_2-1), \tag{4}$$

E is the identity matrix, and QFT_d is the quantum Fourier transform (QFT) operator (see, e.g., [14, 15]).

For a more detailed investigation, we take qudits with eight and four levels, which correspond to the sizes of the registers in the already studied system of five qubits [11] and are convenient for comparison. The quantum algorithm for finding the permutation order on two such qudits can be implemented through the circuit that is shown in Fig. 1 and operates as follows:

(0) The first qudit (representation of the x number) and the second qudit (representation of the y number) are set in the initial state $|0\rangle_1 \otimes |0\rangle_2$.

(i) The first qudit is subjected to the quantum Fourier transform, which transforms the first register to the superposition state

$$QFT_8|0\rangle = (|0\rangle + |1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + |6\rangle + |7\rangle)/\sqrt{8}.$$

(ii) The oracle applies the *SUM* gate (with $r = 4$ or 2 at discretion), which transforms the separable state of two registers to an entangled state

$$SUM_{12}\sum_{x=0}^{7}|x\rangle_1 \otimes |0\rangle_2 = \sum_{x=0}^{7}|x\rangle_1 \otimes |s^x(0)\rangle_2.$$

(iii) The quantum Fourier transform of the states of the first qudit makes it possible to determine the period of the function $f(x) = s^x(y)$ written in these states.

(iv) The states of the first qudit are measured and the permutation order $r = 8/\Delta x$ is determined in terms of the distance Δx between the maxima (see Fig. 3).

SIMULATION OF THE ALGORITHM

To control the qubits, the rotation operators selective in spins are used [11, 12, 20], whereas to control the multilevel systems, operators selective in levels are used [2–10, 14–16] and denoted as $\{\theta\}_\alpha^{m-n}$, where θ is the angle, α is the rotation axis, and m and n are the ordinal numbers of the levels between which the transition occurs. The sequences of the selective rotation operators for the quantum Fourier transform at $d = 4$ and 8 were obtained in our work [14]. However, using the linear algebra method described in [15, 16], we obtained shorter sequences presented in Tables 1 and 2. These sequences are used in this work.

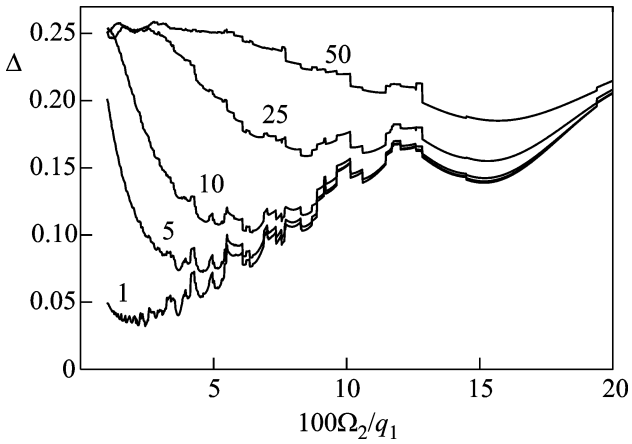

Fig. 2. RF-amplitude dependence of the error of implementing the order-finding algorithm on two qudits (see Fig. 1), $\Delta = \frac{1}{32}\sqrt{\sum_{i,j}\left|U_{ij} - U_{ij}^{\text{theor}}\right|^2}$, where U_{ij}^{theor} are the elements of the 32×32 matrix of the ideal algorithm operator as a whole and U_{ij} are the elements of the numerically calculated matrix of the algorithm operator obtained by means of the product of evolution operators (6). The magnitudes of the spin–spin interaction in units of $10^5 J/q_1$ are given near the curves. The other parameters are $\omega_1 = 3000$, $\omega_2 = 6000$, $q_1 = 100$, and $q_2 = 200$. The curves for $r = 2$ and 4 coincide.

As a physical system, we take two quadrupole nuclei with the spins $I_1 = 7/2$ and $I_2 = 3/2$ in the axially

Table 1. Parameter values in the sequence of the rotation operators $\Pi_j\{\theta_j\}_\alpha^{m-n}$ for implementing the quantum Fourier transform on the four-level quantum system

j	α	θ_j, rad	$m-n$
1	Y	$-\pi/2$	3 – 4
2	Y	$-2\arctan(\sqrt{2})$	2 – 3
3	Y	$2\pi/3$	3 – 4
4	Y	$-2\pi/3$	1 – 2
5	Z	$3\pi/4$	1 – 2
6	Z	$-\pi/2$	2 – 3
7	Z	$\pi/4$	3 – 4
8	Y	$-2\pi/3$	3 – 4
9	Y	$2\arctan(\sqrt{2})$	2 – 3
10	Y	$\pi/2$	3 – 4

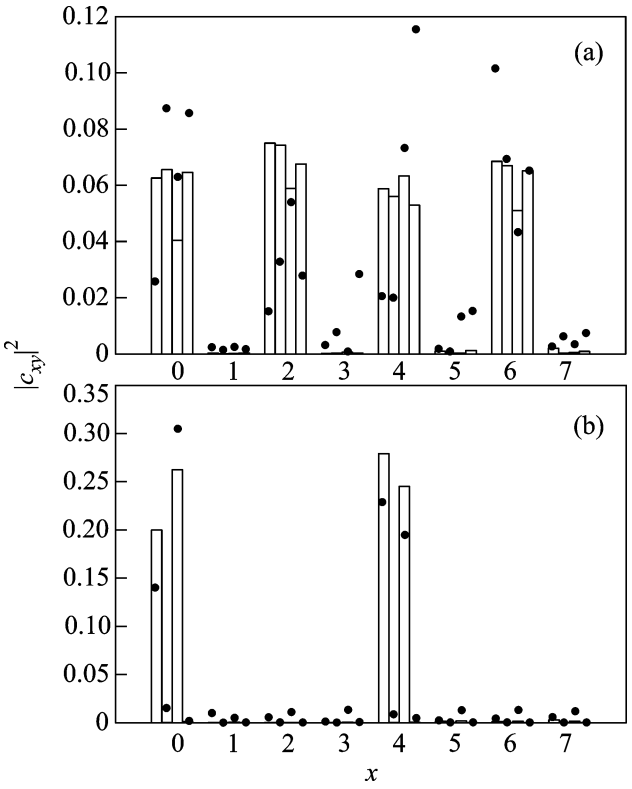

Fig. 3. Probabilities $|c_{xy}|^2$ of the $|x\rangle_1 \otimes |y\rangle_2$ states at the end of the order-finding algorithm on two qudits (see Fig. 1) for r = (a) 4 and (b) 2. Bars present the values at $J/q_1 = 10^{-5}$ and $100\Omega_2/q_1 = 2.18$ and the circles represent the values at $J/q_1 = 10^{-4}$ and $100\Omega_2/q_1 = 6.35$. The other parameters are $\omega_1 = 3000$, $\omega_2 = 6000$, $q_1 = 100$, and $q_2 = 200$. The lower digits are the ordinal numbers of the first-qudit states ($x = 0$, 1, …, 7). The states of the second qudit ($y = 0$, 1, 2, 3) are not indicated and are presented in ascending order from left to right.

symmetric crystal field and a strong static magnetic field with the Hamiltonian [17]

$$H_0 = -\omega_1 I_{1Z} - \omega_2 I_{2Z} + q_1(I_{1Z}^2 - 21/4) + q_2(I_{2Z}^2 - 5/4) - J I_{1Z} I_{2Z}. \quad (5)$$

Here, $\omega_i = B_0\gamma_i$ is the Larmor frequency of the ith-spin precession, J is the spin–spin coupling constant, q_1 and q_2 are the coupling constants of the nuclear quadrupole moments with the crystal-field gradient, and I_{1Z} is the operator of the spin projection on the direction of the static external field (the Z axis) for the corresponding nucleus. We set $\hbar = 1$; i.e., the energies are expressed in frequency units.

To control the system, we apply an RF magnetic field. An RF pulse is obtained by switching on the field with the amplitude B_1 and frequency ω during a finite time $t_p \gg 1/\omega$. In the reference frame rotating with the frequency ω [17], the time variation of the state is specified by the evolution operator

$$U(t) = e^{-iHt}. \quad (6)$$

Here, the time-independent effective Hamiltonian is given by the expression

$$H = H_0 + \omega(I_{1Z} + I_{2Z}) - \Omega_1(I_{1X}\cos\varphi + I_{1Y}\sin\varphi) - \Omega_2(I_{2X}\cos\varphi + I_{2Y}\sin\varphi), \quad (7)$$

where $\Omega_i = B_1\gamma_i$. The phase of the RF field φ determines the direction of the field in the rotating reference frame. If the frequency of the alternating field is taken to equal the frequency of the transition between the energy levels $\omega = \varepsilon_n - \varepsilon_m$, the states corresponding to given levels first change and we obtain the selective rotation $\{\theta\}_\alpha^{m-n}$ by the angle $\theta = t_p\Omega I_{mn}$, where I_{mn} is the absolute value of the matrix element of the operator I_X. The selective rotation about the Z axis is implemented following [11] by means of the phase shift of the subsequent RF pulses.

The permutation with the order $r = 4$ has one cycle $s = (0, 1, 2, 3)$ and is specified by one gate *SUM* (3), where the diagonal operator P_{12} given by Eq. (4) can be expressed in terms of the operators I_{1Z} and I_{2Z} with the use of the relation

$$\frac{\pi}{2}xy = \frac{\pi}{2}\left(\frac{7}{2} - I_{1Z}\right)\left(\frac{3}{2} - I_{2Z}\right) = \frac{\pi}{2}\left(I_{1Z}I_{2Z} - \frac{3}{2}I_{1Z} - \frac{7}{2}I_{2Z} + \frac{21}{4}\right). \quad (8)$$

The substitution of Eq. (8) into Eq. (4) transforms P_{12} to the product of three operators and a common phase factor:

$$P_{12} = \exp\left(i\frac{\pi}{2}I_{1Z}I_{2Z}\right)\exp\left(-i\frac{7\pi}{4}I_{2Z}\right) \times \exp\left(-i\frac{3\pi}{4}I_{1Z}\right)\exp\left(-i\frac{21\pi}{8}\right). \quad (9)$$

The first operator is obtained by the free evolution of the system with the spin–spin interaction Hamiltonian $H_J = -JI_{1Z}I_{2Z}$ during the time

$$t_J = \pi/2J. \quad (10)$$

To remove the phase shift caused by the quadrupole and Zeeman interactions, the time is set to be a multiple of the period $2\pi/q_1$ [9]. The second and third operators in Eq. (9) reduce to the action of additional Z pulses on the

corresponding spins. The pulses acting on spin 1 have the form

$$\left\{\frac{5}{4}\pi\right\}_{Z}^{7-8}\{\pi\}_{Z}^{6-7}\left\{\frac{3}{4}\pi\right\}_{-Z}^{5-6} \times\left\{\frac{3}{4}\pi\right\}_{-Z}^{3-4}\{\pi\}_{Z}^{2-3}\left\{\frac{5}{4}\pi\right\}_{Z}^{1-2}; \quad (11)$$

and the pulses acting on spin 2 have the form

$$\left\{\frac{5}{4}\pi\right\}_{Z}^{3-2}\{\pi\}_{-Z}^{2-3}\left\{\frac{5}{4}\pi\right\}_{Z}^{1-2}. \quad (12)$$

As a permutation with the order $r = 2$ with two cycles, we take the permutation $s = (0, 2)(1, 3)$, because it can also be specified by one *SUM* gate (3) in the operator P_{12}, where the exponent should be doubled. This means that the duration of evolution interval (10) and *Z*-rotation angles in Eqs. (11) and (12) are doubled.

Figures 2 and 3 show the computation results. For comparison, the computation results for the five-qubit algorithm that we obtained using the circuit developed in [11] with RF pulses (6) are shown in Fig. 4. In order to approach the conditions of two implementations, we choose constants that are different from the experimental values. The differences in the location of the maxima in Figs. 3 and 4 are attributed to the difference between the quantum Fourier transform operators. The fast Fourier transform leading to the inverse bit-to-bit result is implemented on qubits, whereas the complete quantum Fourier transform is implemented on qudits.

The error is associated with two basic causes. First, an RF pulse acts not only on the selective rotation on a chosen transition, but also on other nonresonant transitions. This error increases with the RF-pulse amplitude. Second, the spin–spin interaction changes the state of the system during the action of the RF pulses. This error increases with a decrease in the amplitude of the RF pulse.

Owing to these two factors, a minimum is observed on the curves plotted in Fig. 2. The error in this minimum increases with *J*. The largest contribution to the error comes from the change in the phase of the states.

PSEUDOPURE STATE

For the quantum computations using the NMR methods, it is necessary to prepare a pseudopure state from the thermal equilibrium state. The method of such preparation for one quadrupole nucleus is presented, e.g., in [8, 18]. In particular, a multifrequency (modulated) selective RF pulse, which holds the ground-state population and equates the populations of all other levels, was applied in [18] for eight levels of the ^{133}Cs nucleus with $I = 7/2$.

Table 2. Parameter values in the sequence of the rotation operators $\Pi_j\{\theta_j\}_{\alpha}^{m-n}$ for implementing the quantum Fourier transform on the eight-level quantum system

j	α	θ_j, rad	$m-n$
1	*Y*	$-\pi/2$	7 – 8
2	*Y*	$-2\arctan(\sqrt{2})$	6 – 7
3	*Y*	$-2\pi/3$	5 – 6
4	*Y*	$-2\arctan(2)$	4 – 5
5	*Y*	$-2\arctan(\sqrt{5})$	3 – 4
6	*Y*	$-2\arctan(\sqrt{6})$	2 – 3
7	*Z*	$-7\pi/4$	2 – 3
8	*Z*	–1.0409	7 – 8
9	*X*	–1.1086	7 – 8
10	*Z*	–1.5508	6 – 7
11	*X*	–1.4928	6 – 7
12	*Z*	–1.7886	5 – 6
13	*X*	–1.7680	5 – 6
14	*Z*	–1.8803	4 – 5
15	*X*	–1.9954	4 – 5
16	*Z*	0.0451	4 – 5
17	*Z*	–0.2086	7 – 8
18	*X*	–0.9564	7 – 8
19	*Z*	–0.4371	6 – 7
20	*X*	–1.3518	6 – 7
21	*Z*	0.6691	6 – 7
22	*Z*	6.9736	7 – 8
23	*X*	–0.9160	7 – 8
24	*Z*	0.3558	7 – 8
25	*Z*	6.0410	5 – 6
26	*X*	–1.6806	5 – 6
27	*Z*	–0.5993	5 – 6
28	*X*	–1.3518	6 – 7
29	*Z*	–0.4371	6 – 7
30	*X*	–0.9564	7 – 8
31	*Z*	–0.2086	7 – 8
32	*Z*	–4.6096	3 – 4
33	*X*	–2.1944	3 – 4
34	*Z*	–1.8833	3 – 4
35	*X*	–1.9954	4 – 5
36	*Z*	–1.8803	4 – 5
37	*X*	–1.7680	5 – 6
38	*Z*	–1.7886	5 – 6
39	*X*	–1.4928	6 – 7
40	*Z*	–1.5508	6 – 7
41	*X*	–1.1086	7 – 8
42	*Z*	–1.0409	7 – 8
43	*Z*	$\pi/8$	1 – 2
44	*Y*	$2\arctan(\sqrt{7})$	1 – 2
45	*Y*	$2\arctan(\sqrt{6})$	2 – 3
46	*Y*	$2\arctan(\sqrt{5})$	3 – 4
47	*Y*	$2\arctan(2)$	4 – 5
48	*Y*	$2\pi/3$	5 – 6
49	*Y*	$2\arctan(\sqrt{2})$	6 – 7
50	*Y*	$\pi/2$	7 – 8

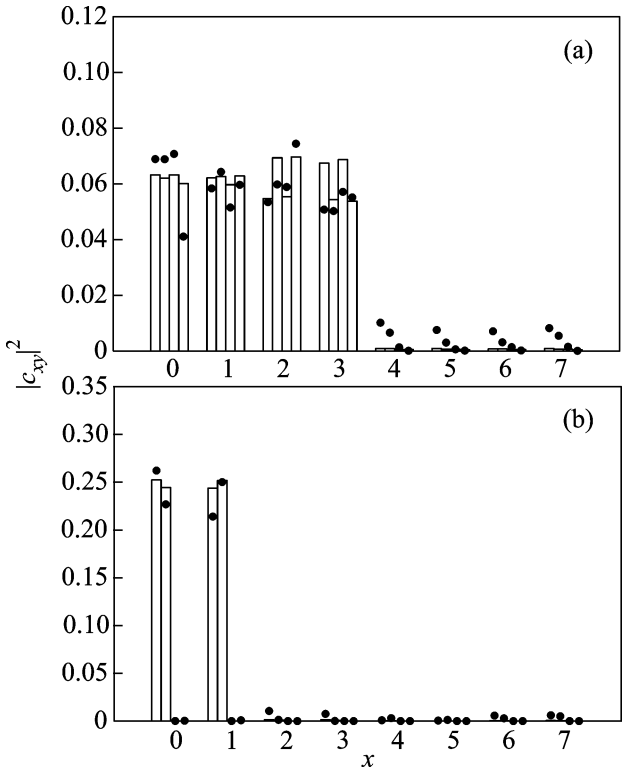

Fig. 4. Probabilities $|c_{xy}|^2$ of the $|x_1\rangle \otimes |y_2\rangle$ states at the end of the order-finding algorithm on five qubits [11] for r = (a) 4 and (b) 2 and $s = (0, 1)(2, 3)$. The bars represent the values at $J/\Delta\omega = 10^{-5}$ and the circles represent the values at $J/\Delta\omega = 10^{-4}$, where $\Delta\omega = 10000$. The other parameters are $\omega_j = j\Delta\omega$ (j = 1, 2, 3, 4, 5), $\Omega_1/\Delta\omega = \Omega_2/\Delta\omega = \Omega_3/\Delta\omega = \Omega_4/\Delta\omega = \Omega_5/\Delta\omega = 0.0125$. The lower digits are the ordinal numbers of the first-register states (x = 0, 1, …, 7). The states of the second register (y = 0, 1, 2, 3) are not indicated.

We do not know any ready procedure for creating the pseudopure state in two quadrupole nuclei. We propose to apply the filtration method by means of time-phase cycling [19]. To this end, before the above multifrequency selective pulse, spin 1 is subjected to two intense nonselective RF pulses separated by the free-evolution interval:

$$\left\{\frac{1}{2}\pi\right\}_Y \longrightarrow t_k \longrightarrow \left\{\frac{1}{2}\pi\right\}_{\varphi_k - Y}. \qquad (13)$$

The transverse component of spin 1 during time t_k acquires the phase depending on the orientation of the second spin:

$$JI_{2Z}t_k + \varphi_k = Jt_k(I_{2Z} - I_2),$$

where the phase shift of the second pulse is chosen so as to compensate for the time dependence of the ground state $I_{2Z} = I_2$. Let us perform the experiment $2I_2 + 1$ times successively with various intervals between the pulses,

$$t_k = 2\pi k/J(2I_2 + 1) \quad (k = 0, 1, \ldots, 2I_2), \qquad (14)$$

where the time is specified as a multiple of the period $2\pi/q_1$, and sum the experimental results. After that, the result corresponding to the ground state remains, whereas the contributions of the other states disappear.

In particular, four experiments should be performed for $I_2 = 3/2$. Note that the results of nine experiments were added in order to prepare the pseudopure state by means of the time averaging on five qubits in [11].

When the quantum algorithms are implemented using the NMR methods, a signal is observed from a large ensemble of quantum systems. For this reason, the quantum yield probabilities are manifested through the intensities of the lines in the NMR spectrum in the case of the spectral detection or through the elements of the density matrix in the case of the tomographic detection:

$$\rho_f = \left\{\frac{(1-\beta)}{d_1 d_2}E_1 \otimes E_2 + \beta|\Psi_f\rangle\langle\Psi_f|\right\}, \qquad (15)$$

where β is the amplitude of the pseudopure state, which is inversely proportional to the temperature, and

$$|\Psi_f\rangle = \sum_{y=0}^{3}\sum_{x=0}^{7} c_{xy}|x\rangle_1 \otimes |y\rangle_2 = U|0\rangle_1 \otimes |0\rangle_2. \qquad (16)$$

CONCLUSIONS

Thus, we derive the formulas that make it possible to implement the quantum algorithm for finding the permutation order on two quadrupole nuclei with I_1 = 7/2 and I_2 = 3/2 instead of five spins I = 1/2 [11]. The implementation requires 70 RF pulses in the first case and 80 RF pulses for r = 2 (100 pulses for r = 4) in the second case. The simulation showed that the accuracies of the two methods are close to each other at comparable parameters. To reduce the error, the case of simple pulse sequences should be changed to more complex sequences. Necessary special techniques have already been developed for qubits [11, 12, 20], whereas these technique for quadrupole nuclei have not yet been developed.

As an example, we take two quadrupole nuclei controlled by the RF field, because NMR is a leading technique for simulating quantum algorithms [20]. However, after the appropriate changes, formulas can be extended to other multilevel systems such as molecular magnetic materials, impurities in crystals, atoms and ions in traps, etc. The multilevel basis elements (ququarts [21]) were also obtained in optics, but, for example, the Shor's algorithm was experimentally demonstrated on qubits [22, 23]. The main difference of such linear optical schemes from our scheme is that the

entanglement of the photon states in them is obtained without direct interaction between these photons.

The work of A.S.E. was supported by the Krasnoyarsk Science Foundation (project no. 17G092).

REFERENCES

1. D. Gottesman, Lect. Not. Comp. Sci. **1509**, 302 (1999).
2. A. R. Kessel and V. L. Ermakov, Pis'ma Zh. Éksp. Teor. Fiz. **70**, 59 (1999) [JETP Lett. **70**, 61 (1999)]; Pis'ma Zh. Éksp. Teor. Fiz. **71**, 443 (2000) [JETP Lett. **71**, 307 (2000)].
3. A. Muthukrishnan and C. R. Stroud, Jr., Phys. Rev. A **62**, 052309 (2000).
4. J. L. Cereceda, quant-ph/0407253.
5. A. Yu. Vlasov, J. Math. Phys. **43**, 2959 (2002).
6. S. D. Bartlett, H. de Guise, and B. C. Sanders, Phys. Rev. A **65**, 052316 (2002).
7. J. Daboul, X. Wang, and B. C. Sanders, J. Phys. A **36**, 7063 (2003).
8. V. L. Ermakov and B. M. Fung, Phys. Rev. A **66**, 042310 (2002).
9. R. Das and A. Kumar, Phys. Rev. A **68**, 032304 (2003).
10. K. V. R. M. Murali, N. Sinha, T. S. Mahesh, et al., Phys. Rev. A **66**, 022313 (2002).
11. L. M. K. Vandersypen, M. Steffen, G. Breyta, et al., Phys. Rev. Lett. **85**, 5452 (2000).
12. L. M. K. Vandersypen, M. Steffen, G. Breyta, et al., Nature **414**, 883 (2001).
13. V. N. Sachkov, *Introduction to Combinatorial Methods of Discrete Mathematics* (Nauka, Moscow, 1982) [in Russian].
14. V. E. Zobov and A. S. Ermilov, Pis'ma Zh. Éksp. Teor. Fiz. **83**, 539 (2006) [JETP Lett. **83**, 467 (2006)].
15. A. S. Ermilov and V. E. Zobov, Opt. Spektrosk. **103**, 994 (2007) [Opt. Spectrosc. **103**, 969 (2007)].
16. G. K. Brennen, D. P. O'Leary, and S. S. Bullock, Phys. Rev. A **71**, 052318 (2005).
17. C. P. Slichter, *Principles of Magnetic Resonance* (Harper and Row, New York, 1963; Mir, Moscow, 1981).
18. A. K. Khitrin and B. M. Fung, Phys. Rev. A **64**, 032306 (2001).
19. J.-S. Lee and A. K. Khitrin, J. Chem. Phys. **4**, 041101 (2005).
20. L. M. K. Vandersypen and I. L. Chuang, Rev. Mod. Phys. **76**, 1037 (2004).
21. E. V. Moreva, G. A. Maslennikov, S. S. Straup, and S. P. Kulik, Phys. Rev. Lett. **97**, 023602 (2006).
22. C.-Y. Lu, D. E. Browne, T. Yang, and J. W. Pan, Phys. Rev. Lett. **99**, 250504 (2007).
23. B. P. Lanyon, T. J. Weinhold, N. K. Langford, et al., Phys. Rev. Lett. **99**, 250505 (2007).

Translated by R. Tyapaev