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**NONLINEAR AND QUANTUM OPTICS**

# Anisotropy of Nonlinear Optical Transmission at the Edge of the Photonic Band Gap of an Apodized Layered Medium

**S. Ya. Vetrov[a], I. V. Timofeev[b], and A. V. Shabanov[b]**

[a] *Polytechnic Institute, Siberian Federal University, Krasnoyarsk, 660074 Russia*
[b] *Kirensky Institute of Physics, Siberian Branch, Russian Academy of Sciences, Akademgorodok, Krasnoyarsk, 660036 Russia*
*e-mail: tiv@iph.krasn.ru*

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**Abstract**—The effect of Kerr nonlinearity on the transmission of laser radiation in a one-dimensional photonic crystal has been investigated by the transfer-matrix method, modified to describe nonlinear effects. The crystal under study is a thin-film multilayer structure with a spatial distribution of the refractive index, which makes it possible to eliminate side bands in the transmission spectrum at each side of the photonic band gap and significantly increase the slope of the transmission curve. The transmission spectrum of such a photonic crystal structure has been studied for two opposite directions of laser radiation propagation. The anisotropy of nonlinear transmission is most pronounced near the edge of the photonic crystal band gap, which lies in the near-IR region. The proposed structure, having strong transmission anisotropy and sufficiently low reflection in the forward direction, can operate as an optical diode (analog of an electron diode).

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## INTRODUCTION

Photonic crystals (PCs) are structures with spatial modulation of dielectric properties on the scale of about one light wavelength. Due to the presence of photonic band gaps (PBGs) and unusual dispersion properties, PCs make it possible to implement original methods for controlling many characteristics of electromagnetic radiation [1–6]. In this context, PCs can be considered as optical analogs of semiconductors, in which periodic potential field changes the properties of electrons due to the existence of a band gap. In development of new methods for controlling propagation of electromagnetic radiation in PC structures, a key concept is the use of nonlinear PCs, where change in the light reflection and transmission depends on the laser intensity. Nonlinearity of PCs is important in design of such devices as optical diodes [7–9] and switches and limiters [9–11], whose operation is based on the optical Kerr effect, i.e., dependence of the effective refractive index on the radiation intensity.

Optical diode is one of the main components for completely optical signal processing and communication systems. As electron diodes, which are widely used for such processing, optical diodes are designed for unidirectional transfer of optical signals of certain frequency and intensity. The first optical diode was proposed in [7], where the dynamic shift of the edge of the band gap of a nonlinear PC with a gradient of optical layer thickness was used. An optical diode was implemented in practice on a lithium niobate crystal [12]. It was shown that energy transfer from the main to the second-harmonic signal depends on the wave propagation direction in the waveguide. In [13], the results of theoretical and experimental investigations of an electrically controlled optical diode were reported. The diode was developed on the basis of a structure composed of an anisotropic nematic layer placed between two cholesteric liquid-crystal layers with different helical pitches. Optical diodes based on nonlinear PC defects were investigated in [14, 15]. In [16], it was shown that an asymmetrically apodized nonlinear system composed of two waveguide channels, coupled via periodically repeating round microcavities, can be used as an optical diode. Apodization only from the left side of such a system leads to the following effect: light of certain intensities incident from the left can easily penetrate the structure, whereas light incident from the right is almost completely reflected.

Note that qualitatively new features arise in the reflection and transmission spectra of one-dimensional PCs with a gradual change in the spatial distribution of permittivity (apodization). It turned out that oscillations on the reflection curve can be eliminated and its slope can be significantly increased (to almost rectangular shape) [17].

The results of studying the effect of Kerr nonlinearity on laser radiation transmission in layered media having an almost rectangular reflection curve were reported in [18]. In particular, it was shown that the proposed nonlinear asymmetric multilayer PC has strong anisotropic transmission, which is due to a great extent to the smoothed (apodized) PBG edge, which lies in the visible spectral range. The smoothed PBG edge, being a descent with an almost constant slope from total transmission to almost total reflection, leads to a stepwise decrease in nonlinear transmission by several tens times even before the onset of bistability. Currently, PCs with a PBG lying in the near-IR region ($\lambda \sim 1$–$1.5$ μm) are of great interest, for example, for telecommunication applications [2, 19, 20].

In this paper, we report the results of studying the influence of Kerr nonlinearity on the laser radiation transmission in asymmetric one-dimensional layered media, having an almost rectangular reflection curve in the near-IR region. The symmetry of the structure is violated by the substrate. In contrast to [18], the case of a nonabsorbing substrate is considered. The dependence of PC transmission near the PBG edge on the wavelength and intensity of incident radiation was investigated by the transfer-matrix method, modified to describe nonlinear effects. The spatial distribution of the field in the sample was analyzed. The proposed diode structure has a low operating intensity ($I \sim 10^7$ W/cm$^2$), high transmission (90%), and 30-fold ratio of transmittances in opposite directions.

## PC STRUCTURE AND TRANSMITTANCE

The distribution of the refractive index in a layered medium with a rectangular reflection curve will be set in the form

$$n(z) = \begin{cases} n_0, & z \le z_0, \\ n_0 + (-1)^l \Delta n \sin^2[\pi(l-1/2)/N], & \\ z_{l-1} < z \le z_l, & l = 1, 2, 3, \dots, N, \\ n_f, & z_N < z \le z_{N+1}, \\ n_0, & z_{N+1} < z, \end{cases} \quad (1)$$

where it is assumed that there is a homogeneous medium beyond the crystal whose refractive index $n_0$ coincides with its mean value for PC and $n_f$ is the refractive index of the substrate. In addition, we assume below that all layers have the same thickness $(z_l - z_{l-1})$.

The Maxwell equation, with regard to the cubic nonlinearity of a one-dimensional PC, can be written in the form [21]

$$d^2E/dz^2 + \omega^2/c^2[n^2(z) + 4\pi\chi^{(3)}|E|^2]E = 0, \quad (2)$$

where $E(z) \equiv E_y(z)$; $\chi^{(3)}$ is the cubic susceptibility; $n(z)$ is the linear refractive index, which is determined by expression (1); and $\omega$ is the wave frequency. The effective refractive index has the form

$$\tilde{n}(z) = [n^2 + 4\pi\chi^{(3)}|E|^2]^{1/2} \approx n + n^{(2)}I(z), \quad (3)$$

where the intensity $I(z) = (c/8\pi)n|E(z)|^2$.

Obviously, with an increase in the intensity of incident radiation, the refractive indices change in the layers under the field action, due to the high-frequency Kerr effect. The change in the refractive indices leads to a change in the field amplitudes; this change, in turn, causes variation in the refractive indices, etc.

We will model the structure with the use of the transfer-matrix method, adapted to description of nonlinear effects [22]. To this end, the medium will be divided into a large number of sublayers, such that the effective refraction index within an $m$th sublayer can be considered as constant,

$$\tilde{n}_m = n_m + n_m^{(2)}I(z_m). \quad (4)$$

Distribution of the electric and magnetic fields in the sublayers has the form

$$\begin{aligned} E_m(z) &= A_m e^{i\tilde{k}_m(z-z_m)} + B_m e^{-i\tilde{k}_m(z-z_m)}, \\ H_m(z) &= i\tilde{k}_m A_m e^{i\tilde{k}_m(z-z_m)} - i\tilde{k}_m B_m e^{-i\tilde{k}_m(z-z_m)}, \\ &\text{at} \quad z_{m-1} < z \le z_m, \end{aligned} \quad (5)$$

where $A_m$ and $B_m$ are the amplitudes of the incident and reflected waves in the $m$th sublayer, $\tilde{k}_m = 2\pi\tilde{n}_m/\lambda$ is the wavenumber, and $\lambda$ is the light wavelength in vacuum. From the condition of field continuity at the interface between the media, $z = z_{m-1}$, we obtain the following relation between the complex amplitudes of the incident and reflected waves in sublayers $m-1$ and $m$:

$$\begin{bmatrix} A_{m-1} \\ B_{m-1} \end{bmatrix} = \hat{T}_{m-1,m} \begin{bmatrix} A_m \\ B_m \end{bmatrix}, \quad (6)$$

where

$$\hat{T}_{m-1,m} = \frac{1}{2}\begin{bmatrix} g_m^{-1}(1 + \tilde{n}_m/\tilde{n}_{m-1}) & g_m(1 - \tilde{n}_m/\tilde{n}_{m-1}) \\ g_m^{-1}(1 - \tilde{n}_m/\tilde{n}_{m-1}) & g_m(1 + \tilde{n}_m/\tilde{n}_{m-1}) \end{bmatrix}, \quad (7)$$

$g_m = \exp(i\tilde{k}_m(z_m - z_{m-1}))$, $m = 1, 2, \dots, M, M+1$.

Thus, the field incident on the sample and the field at the output of the sample are related as follows:

$$\begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = \hat{T} \begin{bmatrix} A_{M+1} \\ 0 \end{bmatrix}, \quad (8)$$

where $\hat{T} = \hat{T}_{01}\hat{T}_{12}\dots\hat{T}_{M-1,M}\hat{T}_{M,M+1}$ and $z_{M+1} - z_M = 0$. The amplitude $B_{M+1} = 0$ because reflection of electromagnetic waves from the right side of the sample is absent. The transmittance $t(\omega)$ is determined by the expression

$$t(\omega) = 1 - |T_{21}/T_{11}|^2, \quad (9)$$

where $T_{11}$ and $T_{21}$ are the $\hat{T}$-matrix elements.

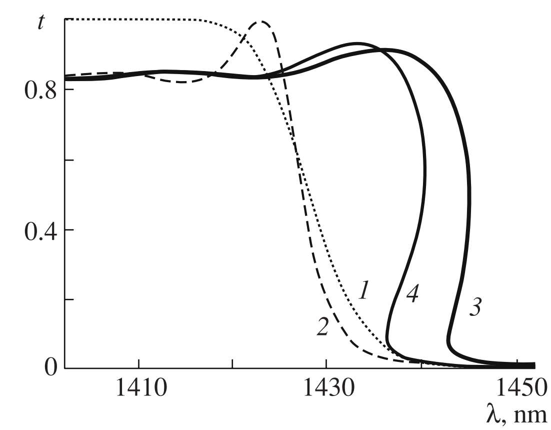


**Fig. 1.** The transmission spectra of different PCs near the left PBG edge: (*1*) linear PC without a substrate; (*2*) linear PC with violated spatial symmetry; and (*3*, *4*) PC with Kerr nonlinearity for radiation propagating from left to right and vice versa, respectively. Each PC consists of 50 pairs of layers; each layer has a thickness of 170 nm and a refractive index set by $n_0 = 2.5$ and $\Delta n = 1$. The substrate on the right side of the crystal has the thickness $d = 700$ nm and the refractive index $n_f = 1.6$. The nonlinear-interaction parameter $\beta = 0.008$. The diode effect is observed in the wavelength range from 1436.5 to 1442.9 nm.

Numerical calculation of the field distribution in the sample within the linear approximation is reduced to determination of the incident and reflected waves from formula (6) successively in each layer, beginning with the right boundary of the medium. With an increase in the intensity of incident radiation, the refractive indices of the layers change under the field action. Initially, a linear refractive index is substituted into (7), and the field amplitudes found within this approximation give a refined refractive index, according to (4), which is used as a new seed value. The efficiency of nonlinear interaction (3) is characterized by the dimensionless parameter $\beta = 4\pi\chi^{(3)}|A_0|^2$. When $\beta$ is about several percent, the refractive index in the *m*th layer is found after 3 iterations. At the output of the sample, one should choose such an amplitude $A_{M+1}$ from the interval [0, 1], which would give a unit field amplitude at the input: $A_0 = 1$.

## RESULTS AND DISCUSSION

The calculation was performed for a PC composed of 101 layers, each having thickness $d = 170$ nm and refractive index set by $n_0 = 2.5$ and $\Delta n = 1$. The symmetry of the structure is violated by the substrate with the thickness $d_f = 700$ nm and refractive index $n_f = 1.6$. The total thickness of the multilayer medium under consideration is $L = 17.87$ μm; thus, at the pulse duration $\tau \sim$ 1 ps, we can use the stationary approximation $L \ll \tau c$. The dimensionless parameter $\beta$, characterizing the efficiency of nonlinear interaction, is equal to 0.008. For example, the cubic susceptibility $\chi^{(3)}$ of silicon is about $10^{-8}$ cm$^3$/erg [23] and, therefore, the input intensity $I_0 \sim 10^7$ W/cm$^2$.

Figure 1 shows the transmission spectra of a linear PC and a PC with cubic nonlinearity for the cases where light propagates through the sample from left to right and vice versa. It can be seen that, for the linear PC, the presence of the substrate leads to a sharp increase in the transmittance at the PBG boundary (up to unity). Bleaching of the structure is explained by destructive interference of the reflections from the substrate of certain optical thickness and the layered apodized structure. In this case, the minimum in the transmission spectrum of the substrate lies at the PBG edge of the symmetric PC. It is of importance that the noted feature facilities an increase in nonlinear trans-

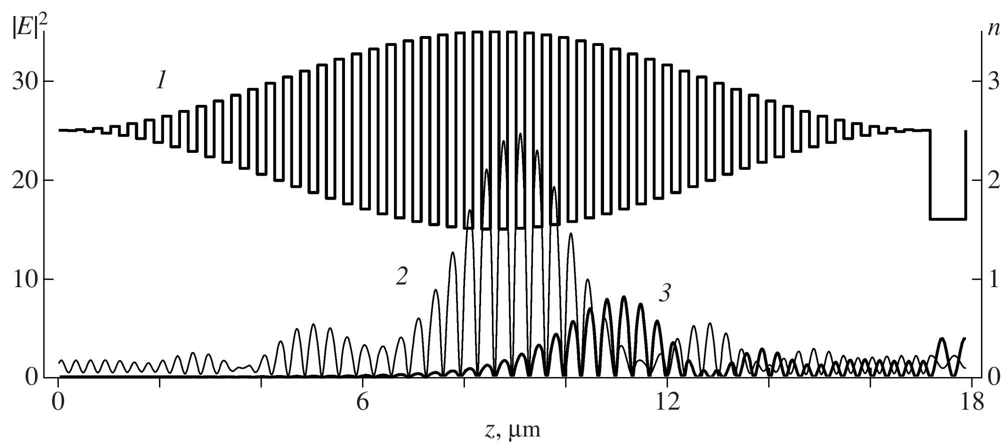


**Fig. 2.** Spatial distributions of the (*1*) refractive index (according to (1)) and (*2*, *3*) the square of the electric field magnitude in the medium for radiation propagation (*2*) from left to right and (*3*) vice versa. The field is normalized to $|A_0|^2$; the incident radiation wavelength $\lambda = 1438$ nm in vacuum; the transmittances in the forward and backward directions are 90.3 and 3.2%, respectively; and the structure parameters are the same as in Fig. 1.

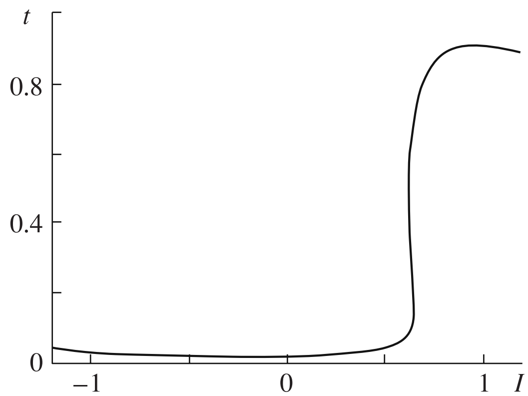


**Fig. 3.** Dependence of the transmittance on the radiation intensity for $\lambda = 1438$ nm. Positive intensities correspond to the case where radiation is incident on the left side and emerges from the right side of the sample; negative intensities correspond to the opposite case. $I = \pm 1$ corresponds to $\beta = 0.008$. Other parameters are the same as in Fig. 1.

mission in the forward direction, retaining significant anisotropy of the structure transmission. Figure 1 shows a large red shift of the transmission curves of the nonlinear crystal, which is due to the increase in the optical density of layers. It can also be seen in Fig. 1 that the diode effect—high ratio of the transmittances for waves propagating in opposite directions—is observed in a wide wavelength range: from 1436.5 to 1442.9 nm. On the short-wavelength edge of this interval, the diode is bistable for the radiation propagating from right to left. However, at a monotonic increase in the intensity, from two stable states, a state with a lower transmission manifests itself. In the entire wavelength range under consideration, the forward transmittance is high (up to 90%); this is a favorable feature of the optical diode analyzed here in comparison with the analogs considered in the Introduction.

Figure 2 shows the intensity distribution over the sample for a field with the wavelength $\lambda = 1438$ nm. For opposite light propagation directions, the intensities are significantly different and the fields are localized in both cases in layers with a low refractive index. For light incident from the left, the optical field increases by a factor of 25 in the central layers of the medium. Despite such an increase in the field, the nonlinear change in the refractive index in the center of the medium does not exceed 3%; thus, the Kerr approximation remains justified.

In the model under consideration, a thin-film layered medium, which exhibits strong anisotropic optical transmission, is an optical analog of an electron diode. The dependence of the transmittance on the input intensity (Fig. 3) can be considered as an analog of the current–voltage characteristic. The diode effect, as can be seen in Fig. 3, is observed in a wide range of input intensities.

Finally, we should note that, for specified structure parameters, distribution (1) in the near-IR range can apparently be implemented in structures based on porous silicon. It is known that, by choosing the silicon treatment parameters, one can control the optical parameters (including refractive index) in a wide range [24].

## CONCLUSIONS

The effect of Kerr nonlinearity on the transmission of laser radiation in one-dimensional layered media with a specific structure, providing an almost rectangular reflection curve, has been investigated. The proposed diode structure has attractive features, including high energy transmission and significant transmission anisotropy in a fairly wide operating range of wavelengths and intensities.

The record value of nonlinear transmittance is due to the disappearance of reflective ability at the PBG boundary of a linear PC, which is obtained by varying the optical thickness of the substrate. It is noteworthy that such transmission in the forward direction coexists with strong transmission anisotropy.

We should also note that, due to the structure apodization, the absence of dispersion at the PBG edges offers additional possibilities for controlling light pulses without distortion.

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