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All-optical confinement of ultracold plasma with resonant ions

I.V. Krasnov

Institute of Computational Modeling, Siberian Division, Russian Academy of Sciences, 660036 Krasnoyarsk, Russia

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ABSTRACT

The solution of the problem of all-optical (nonmagnetic) confinement of ultracold electron–ion neutral plasma based on selective action on plasma ions with quantum transition $J = 1 \to J = 0$ of so-called rectified radiation forces in a strong nonmonochromatic light field is suggested. The presented scheme of the three-dimensional dissipative optical trap for plasma allows one to obtain long-lived ultracold plasma with controlled characteristics. The lifetime of the ultracold plasma in such a trap may exceed considerably (by orders of magnitude) the time of free plasma expansion and the lifetime in the (earlier proposed) optical molasses for the ultracold plasma.

1. Introduction

The methods of laser cooling and trapping of atoms and ions [1] have intensively been studied for more than three decades and are widely used in different fields of physics. A qualitatively new interesting object where these methods can certainly be applied is so-called ultracold neutral plasma (UP) [2]. UP is a classical electron–ion plasma with ultralow values (as compared to typical laboratory plasmas) of electron T_e and ion T_i temperatures: $T_e < 100$ K, $T_i \lesssim 0.1$ K. In spite of quite low density of UP, its ionic component may be strongly non-ideal ("liquid-like" or even "crystal-like"). Besides unusual physical properties of UP, the possibility of its practical application for the solution of the actual problem of generating low-temperature electron and ion beams is discussed in scientific literature [3].

The main (realized experimentally) method of creating UP is a near-threshold ionization of cooled atoms in a magneto-optical trap [2]. This method allows one to obtain only *short-lived* UP with a lifetime of ~ 100 μs, determined by the time of free plasma expansion.

The practical solution of the UP confinement problem would allow one to create a qualitatively new situation, namely, to obtain long-lived (quasi-stationary) UP with controlled characteristics [4]. And in the case of UP, containing ions with a quantum transition resonant to laser radiation it is possible to suggest the solutions of these problems based on selective action on plasma ions by the forces of resonant light pressure, which is not conventional for plasma physics [5,6]. Considered in the work [5] is the model of viscous one-dimensional (1D) UP confinement in optical molasses generated by the monochromatic standing light wave, and in work [6] – the model of UP confinement in 1D dissipative optical superlattice induced by the bichromatic standing light wave.

The main drawback of the schemes of the optical UP confinement considered in [5,6] is that they transform into complete schemes of three-dimensional (3D) UP confinement only when combined with the action of a *uniform* magnetic field suppressing plasma diffusion in the direction orthogonal to the direction of the light wave propagation. Here, the necessary value B of the magnetic field induction is rather high: $B \gtrsim 5 \times 10^3$ G [5].

The scheme of all-optical (nonmagnetic) 3D confinement of UP: the model of 3D dissipative optical trap (DOT) for UP, containing ions with quantum transition $J_b = 1 \to J_a = 0$ (where J_b and J_a are the total angular momenta in the ground and exited states, respectively) has been proposed and studied in this work. For example, the ions $^{171}Yb^+$ and $^{199}Hg^+$ have such quantum transitions. Moreover, it is these transitions that are used for laser cooling of the ions mentioned in many spectroscopic experiments (see, for example, works [7] and references in them).

In DOT considered the laser radiation has two main functions: (a) induces efficient damping of both chaotic and directed (macroscopic) motion of plasma ions as in the optical molasses for UP [5], (b) creates the deep 3D potential well for ions. As a result, the ions are cooled and confined in DOT due to the selective action of resonant light pressure forces, and the electrons are cooled and confined due to Coulomb interaction with ions. Moreover, due to

E-mail address: krasn@icm.krasn.ru.

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the slow electron–ion energy exchange the electron temperature T_e in such a system can easily be controlled and maintained, for example, by microwave heating of electrons [5,6,8].

In this DOT scheme it is assumed that the light-induced potential field force (generating 3D potential well with the depth ΔU) and velocity-linear force of viscous friction act upon plasma ions simultaneously.

The idea itself of the DOT described seems to be quite simple, but it is necessary to overcome serious obstacles to implement it. Really, the peculiarity of UP is a relatively high value of electron temperature ($T_e > 1$ K) and pressure p_e considerably exceeding ion temperature T_i ($T_i < 0.1$ K) and pressure.[1] Therefore, as in the case of 1D models of the UP confinement [5,6], high electron temperature rather than low ion temperature is taken into consideration in long-term 3D all-optical UP localization conditions:

$$\frac{\Delta U}{T_e} = \eta \gg 1, \qquad \varkappa > 0, \qquad \frac{c_s}{\varkappa} = \lambda_r \ll L, \tag{1}$$

where $\varkappa$ is a light-induced friction coefficient, T_e is in energy units, $c_s = \sqrt{T_e/m}$ – velocity of the ion sound, m – ion mass, L – characteristic spatial size of DOT. Thus, radiation forces must be quite strong.

Another additional (highly desirable) condition is the absence of vortex component of the radiation force, since vortex forces may result in instability of the ion motion [9]. The most impressive example is the impossibility of all-optical particle localization by spontaneous light pressure forces in weak resonant fields, which results from the famous optical Earnshaw theorem [10].

It will be shown that all the difficulties mentioned can successfully be overcome using so-called rectified radiation forces (RRFs), which were discovered and studied in works [9,11], and first demonstrated by experiment in [12]. Later, the idea of RRF was significantly developed in many studies (see, for example, works [13] with references).

RRF may arise in a strong nonmonochromatic light fields. This radiation force has a value of the order of magnitude of the induced light-pressure force [1] and retains its sign on macroscopic spatial scales greatly exceeding the light wavelength λ. Moreover, this RRF is not saturated with increasing radiation intensity. Therefore, such RRF allows the formation of deep potential wells for the resonant particles [14]. Another remarkable property of RRFs (useful for realizing DOT for UP) is the possibility of controlling the spatial structure of these forces [9]. In the DOT under consideration, use is made of the 3D scheme for rectifying a gradient radiation force based on employing strong partially coherent optical fields [15].

2. Action of light on plasma ions. Radiation forces

Consider electron–ion UP in the light field:

$\mathbf{E}(\mathbf{r}, t)e^{-i\omega_0 t} + \text{c.c.}$

with a carrier frequency ω_0 tuned to resonance with the $|J_b = 1, M_b = 0, \pm 1\rangle \to |J_a = 0, M_a = 0\rangle$ quantum transition of the plasma ion, where J_α is the total angular momentum and M_α denotes its projections in the ground ($\alpha = b$) and exited ($\alpha = a$) states. The field is a superposition of coherent quasi-resonant components (with three different frequencies) polarized in mutually perpendicular directions and a partially coherent (fluctuating) resonant field $\mathbf{E}'$ with a bandwidth $\sim \Gamma$:

$$E(\mathbf{r}, t) = \sum_{j=x,y,z} E_{j1}\mathbf{e}_j \exp[-i\Delta_j t] + \mathbf{E}'(\mathbf{r}, t) \tag{2}$$

where $\mathbf{e}_j$ denotes the unit basis vectors of a Cartesian coordinate system and Δ_j is detuning from the resonant frequency ω_0.

In accordance with the original conception of the effect of the radiation force rectification [9,11] (see also [15]) assume the following hierarchy of the characteristic frequencies:

$$\Delta_j,\ |\Delta_j - \Delta_l| \gg |V_{j1}|,$$

$$\Gamma \gg |U_j|,\ \frac{|V_{j1}|^2}{\Delta_j},\ \gamma,\ ks; \tag{3}$$

$$\frac{|V_{j1}|^2}{\Delta_j} \gg \frac{|U_j|^2}{\Gamma}, \tag{4}$$

$$\gamma \left|\frac{V_{j1}}{\Delta_j}\right|^2 \ll \frac{|U_j|^2}{\Gamma}, \tag{5}$$

where l and $j \neq l$ denote indices x, y, or z, $V_{j1} = dE^*_{j1}/\hbar$, $U_j = d(\mathbf{e}_j\mathbf{E}'^*)/\hbar$ are the Rabi frequencies, $d = \|d\|/\sqrt{3}$, $\|d\|$ is the reduced dipole transition matrix element, $k = \omega_0/c$ is the wave number, $s = \sqrt{T_i/m}$ is the thermal velocity of ions, T_i is in the energy units, $\gamma = \gamma'/3$, γ' – the rate of spontaneous decay of the excited state.

It should be taken into account that in the rarefied (nonrecombining) UP (which is of interest for us) having particle density $n \lesssim 10^8$ cm^{-3} and heavy ions (with a mass of $m \sim 100$ amu) the ion plasma frequency $\omega_i = \sqrt{4\pi e^2 n/m}$ is always considerably lower than the decay rate of the excited state of plasma ion: $\omega_i \ll \gamma$. Consequently, a variation in the translational state of plasma ion (conditioned by the Coulomb interaction) is adiabatic (slow) with respect to light-induced changes of its internal state and does not influence the process of forming the radiation force itself.

Due to a high frequency of the optical radiation at the assumed values of its intensity ($I < 1$ kW/cm^2), its direct (not connected with the resonant light pressure) ponderomotive action (Miller force [16]) on the charged particles is negligible.

Therefore, the light-induced force $\mathbf{F}$ acting upon the resonant ions can be defined by conventional equation [1]

$$\mathbf{F} = \hbar \sum_{j=x,y,z} \left(\rho_j \nabla \widehat{V}_j^* + \text{c.c.}\right), \tag{6}$$

where

$$\widehat{V}_j = V_{j1}(\mathbf{r}) \exp(i\Delta_j t) + U_j(\mathbf{r}, t),$$

ρ_j denotes the projections of the induced dipole moment (in d units). They are determined from the Bloch optical equations which (in the approximation of the preset motion [1]) are integrated along the unperturbed ion trajectory, $\mathbf{r} = \mathbf{v} \cdot t$. For the problem under consideration it is convenient to write these equations in the Cartesian representation (compare with [1,15,17,18]), i.e., in the representation of basic wave functions (intra-ionic motion) for the exited $|a\rangle$ and ground states $|bi\rangle$ (where $i = x, y, z$), in which the matrix elements of the dipole moment $\widehat{\mathbf{d}}$ are directed along the unit vectors of the Cartesian coordinate system:

$$\langle bi|\widehat{\mathbf{d}}|a\rangle = \mathbf{e}_i d.$$

In this case the Bloch equations take the following symmetric and compact form:

$$i\left(\frac{d}{dt} + \gamma_1\right)\rho_i = \sum_j q_{ij}\widehat{V}_j, \tag{7}$$

$$i\frac{d}{dt}q_{ij} + i\gamma\delta_{ij} \sum_{l=x,y,z} q_{ll} = i\gamma\delta_{ij} + \left(\rho_i \widehat{V}_j^* - \widehat{V}_j \rho_j^*\right) + \delta_{ij} \sum_{l=x,y,z} \left(\rho_l \widehat{V}_l^* - \text{c.c.}\right) \tag{8}$$

[1] This peculiarity of electron–ion UP results from the necessary condition for the UP stability against decay due to three-body electron–ion recombination (see, for example [5,8]).

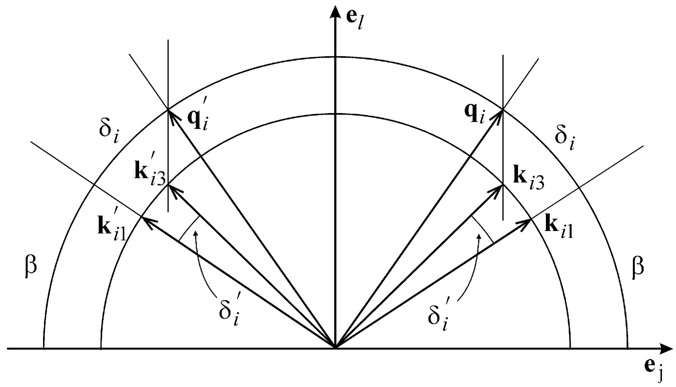

Fig. 1. Wave vectors of the light field components polarized along $\mathbf{e}_i$ and determining Rabi frequencies $U_{i1}(\mathbf{r}), U_{i3}(\mathbf{r})$ and $V_{i1}(\mathbf{r})$ in Eqs. (9), the ordered index combinations (triplets) (ijl) are: (xyz), (zxy) and (yzx), δ_i and δ_i' are small angular detunings.

where the indices $i, j = x, y, z$, q_{ii} denotes population differences between the $|bi\rangle$ and $|a\rangle$ states, q_{ij} (for $i \neq j$) characterizes the coherence between $|bi\rangle$ and $|bj\rangle$ states, $\gamma_1 = 3\gamma/2$.

Let DOT be formed by special superposition of the flat-topped light beams with rectangular profiles [19], where the Rabi frequencies U_i and V_{i1} are the following:

$$U_i(\mathbf{r}, t) = \sum_{\alpha=1}^{4} U_{i\alpha}(\mathbf{r}) \exp\bigl(i\bigl[\varphi_{i\alpha}(t)\bigr]\bigr),$$
$$V_{i1}(\mathbf{r}) = \frac{V_i}{2}\bigl(\exp\bigl(i[\mathbf{q}_i\mathbf{r} + \eta_1]\bigr) + \exp\bigl(i\bigl[\mathbf{q}_i'\mathbf{r}\bigr]\bigr)\bigr),$$
$$U_{i1}(\mathbf{r}) = \frac{U}{2}\bigl(\exp\bigl(i[\mathbf{k}_{i1}\mathbf{r}]\bigr) + a\exp\bigl(i\bigl[\mathbf{k}_{i1}'\mathbf{r}\bigr]\bigr)\bigr),$$
$$U_{i2}(\mathbf{r}) = U_{i1}^*(\mathbf{r}),$$
$$U_{i3} = \frac{\sqrt{a'}U}{2}\bigl(\exp\bigl(i[\mathbf{k}_{i3}\mathbf{r} + \zeta]\bigr) + \exp\bigl(i\bigl[\mathbf{k}_{i3}'\mathbf{r}\bigr]\bigr)\bigr),$$
$$U_{i4} = U_{i3}^*, \tag{9}$$

where the index $i = x, y, z$, η_1 and ζ are the fixed phase shifts, the parameters $a, a' < 1$, U and V_i are real amplitudes, which for the considered case of intersecting flat-topped beams do not depend on $\mathbf{r}$ in the DOT area, $\varphi_{i\alpha}(t)$ are independently fluctuating phases (with delta-correlated zero-mean derivatives), which determine the correlators of $\mathbf{E}'$ components by the relations

$$\bigl\langle \exp i\bigl[\varphi_{j\alpha}(t) - \varphi_{l\beta}(t+\tau)\bigr]\bigr\rangle = \delta_{jl}\delta_{\alpha\beta}\exp\bigl(-\Gamma|\tau|\bigr)$$

in a model of radiation with phase diffusion [20,21], angular brackets $\langle\ldots\rangle$ denote averaging over the noise. Thus, the partially coherent field $\mathbf{E}'$ has a Lorentzian spectral profile with the bandwidth Γ. It is worth noting that representation (9) for $U_i(\mathbf{r}, t)$ is valid only if the coherence length $l_c = c\tau_c = c/\Gamma$ is much greater than the characteristic trap size L: $l_c \gg L$ (see [21]).

One specifies the configuration of the light field determining wave vectors in Eq. (9) as follows (see Fig. 1):

$$\mathbf{k}_{i1} = k(\mathbf{e}_j\cos\beta + \mathbf{e}_l\sin\beta), \qquad \mathbf{k}_{i1}' = k(-\mathbf{e}_j\cos\beta + \mathbf{e}_l\sin\beta),$$
$$\mathbf{q}_i = q_i(\mathbf{e}_j\cos\beta_i + \mathbf{e}_l\sin\beta_i), \qquad \mathbf{q}_i' = q_i(-\mathbf{e}_j\cos\beta_i + \mathbf{e}_l\sin\beta_i),$$
$$\mathbf{k}_{i3} = k\bigl(\mathbf{e}_j\cos\beta_i' + \mathbf{e}_l\sin\beta_i'\bigr),$$
$$\mathbf{k}_{i3}' = k\bigl(-\mathbf{e}_j\cos\beta_i' + \mathbf{e}_l\sin\beta_i'\bigr), \tag{10}$$

where β is an angle determining main (dominant) directions of the propagation of the light beams polarized along $\mathbf{e}_i$, $\beta_i = \beta + \delta_i$, $\beta_i' = \beta + \delta_i'$, δ_i and δ_i' are small angular detunings ($|\delta_i|, |\delta_i'| \ll 1$), the ordered index combinations (ijl) are: (xyz), (zxy), (yzx). Angular detunings δ_i and δ_i' are determined so that the following relations hold:

$$\Delta\mathbf{q}_i = \mathbf{q}_i - \mathbf{q}_i' = \mathbf{k}_{i3} - \mathbf{k}_{i3}',$$
$$\Delta\mathbf{q}_i - \bigl(\mathbf{k}_{i1} - \mathbf{k}_{i1}'\bigr) = 2k\alpha_i\xi_i\cos\beta\cdot\mathbf{e}_j = \frac{2\pi}{L}\operatorname{sgn}(\xi_i\alpha_i)\cos\beta\cdot\mathbf{e}_j, \tag{11}$$

where $\alpha_i = \Delta_i/\omega_0$, $\xi_i = (1 - \delta_i^2/2\alpha_i - \delta_i\,\mathrm{tg}\,\beta/\alpha_i)$, $|\xi_i| = \pi c/|\Delta_i|L$, $L \gg \lambda = 2\pi/k$ is a macroscopic spatial scale of the problem (in fact, as one will see – the DOT size). Given the values Δ_i and L and not very small values of $\beta(\mathrm{tg}\,\beta \gg \sqrt{|\alpha_i\xi_i|})$, δ_i and δ_i' are described by simple formulae:

$$\delta_i \simeq \frac{(1-\xi_i)\alpha_i}{\mathrm{tg}\,\beta}, \qquad \delta_i' \simeq -\frac{\alpha_i\xi_i}{\mathrm{tg}\,\beta}.$$

It is necessary to pay attention that each component $E_i'\mathbf{e}_i$ of the field $\mathbf{E}'$ is a sum of two pairs of counter-propagating laser fields (vectors $\mathbf{k}_{i2} = -\mathbf{k}_{i1}$, $\mathbf{k}_{i2}' = -\mathbf{k}_{i1}'$, $\mathbf{k}_{i4} = -\mathbf{k}_{i3}$, $\mathbf{k}_{i4}' = -\mathbf{k}_{i3}'$, determining $U_{i2}(\mathbf{r})$ and $U_{i4}(\mathbf{r})$, are not given in Fig. 1).

To obtain explicit expressions for the radiation forces, we average successively (taking into account inequalities (3)–(5)) optical Bloch equations (7), (8) – at first, over the "high-frequency" oscillations with the frequencies Δ_j (as is usually done in the RRFs theory (see [9,11])), then over fluctuations of the field $\mathbf{E}'$.[2] As a result, Eqs. (7), (8) are reduced to

$$\left[\frac{d}{dt} + 2R_i(\mathbf{r}) + \gamma\right]q_i + \sum_{j\neq i}\bigl[R_j(\mathbf{r}) + \gamma\bigr]q_j = \gamma,$$
$$\mathbf{F} = -\sum_{i=x,y,z} q_i\frac{\hbar\nabla|V_{i1}(\mathbf{r})|^2}{\Delta_i}, \tag{12}$$

where q_i are averaged population differences of the quantum ion states ($i = x, y, z$), and for the averaged force $\mathbf{F}$ the previous notation is used, $R_i(\mathbf{r})$ denotes the rates of the transitions between low-lying and exited ion states induced by the field $\mathbf{E}'$:

$$R_i(\mathbf{r}) = R\bigl(1 + a_1P_i(\mathbf{r})\bigr), \quad R = \frac{U^2}{\Gamma}\bigl(1 + a^2 + a'\bigr),$$
$$P_i(\mathbf{r}) = \frac{1}{1+b}\bigl[\cos(\Delta\mathbf{k}_i\mathbf{r}) + b\cos(\Delta\mathbf{q}_i\mathbf{r} + \zeta)\bigr], \quad b = \frac{a'}{2a},$$

$a_1 = (2a + a')/(1 + a^2 + a')$, $\Delta\mathbf{k}_i = \mathbf{k}_{i1} - \mathbf{k}_{i1}'$. When writing Eqs. (12) it was taken into account that in the considered configuration of the fields the correlators

$$\int_{-\infty}^{0} d\tau\,\bigl[\bigl\langle\nabla U_i(\mathbf{r}, t)U_i^*(\mathbf{r}, t+\tau)\bigr\rangle - \text{c.c.}\bigr] \tag{13}$$

are equal to zero which provides accurate mutual compensation (in the considered approximation) of the light pressure forces $\mathbf{F}_s$, conditioned by the fluctuating field only (compare with [15]).

It should be noted that neglecting the force $\mathbf{F}_s$ may be justified at other field configurations $\mathbf{E}'$, if condition (4) is well satisfied, since in this case $|\mathbf{F}_s| \ll |\mathbf{F}|$. One can see from Eqs. (12) that radiation force (in the conditions under consideration) is a gradient force which is proportional to the sum of the population differences multiplied by intensity gradients of the coherent field $\mathbf{E}_1$ components. The 3D effect of this gradient force rectification occurs if the partially coherent field $\mathbf{E}'$ includes components capable of interfering with each other. In other words, when the values of

[2] Averaging procedure is analogous to the case of V-type atoms, described in [15]. It is based on the fact that Eqs. (7) are a system of multiplicative stochastic equations [22] and employs the smallness of the parameter ε, which is proportional to the autocorrelation time $\tau_c \sim \Gamma^{-1}$: $|U_j|\cdot\tau_c$, $|V_j|^2\tau_c/\Delta_j$, $ks\tau_c$, $\gamma\tau_c \leqslant \varepsilon \ll 1$ due to the inequalities (3).

the parameters a and a' in Eqs. (9) are not equal to zero. Then, the field $\mathbf{E}'$ induces spatial modulation of the population differences (due to the modulation of the transition rates $R_i = R_i(\mathbf{r})$), and the coherent field $\mathbf{E}_1$ induces the effective potentials (conditioned by light-induced Stark shifts of the energy levels), which determine ion motion depending on its internal state: $|bj\rangle$ or $|a\rangle$. As a result, $\mathbf{F}$ is a sum of two components:

$$\mathbf{F} = \mathbf{F}_R + \mathbf{F}_g, \quad \mathbf{F}_R = \langle \mathbf{F} \rangle_s, \tag{14}$$

where $\langle \ldots \rangle_s$ denotes averaging over the microscopic spatial oscillations with the period of the order of the light wavelength λ, $\mathbf{F}_R$ is the RRF (which varies on macroscopic spatial scales $\sim L \gg \lambda$), the force $\mathbf{F}_g$ oscillates in space with the period of the order λ, $\langle \mathbf{F}_g \rangle_s \equiv 0$.

To obtain an explicit expression for radiation forces, assume that the parameter a_1^2 is small ($a_1^2 \ll 1$), and the ions are slow:

$$\frac{ks}{\gamma}, \ \frac{ks}{R} \leqslant \varepsilon_1 \ll 1. \tag{15}$$

In the linear approximation[3] over the ion velocity $\mathbf{v}$ and the parameter a_1, the following expression is obtained from Eqs. (12) for q_i at $t \gg \gamma^{-1}, R^{-1}$:

$$\begin{aligned} q_i \simeq{}& \frac{1}{(4\chi+3)} - \frac{a_1}{(4\chi+3)^2}\big[2(2\chi+1)P_i(\mathbf{r}) - P_l(\mathbf{r}) - P_j(\mathbf{r})\big] \\ &+ \frac{a_1}{(4\chi+3)^3 R}\big[2\big(6\chi^2+8\chi+3\big)\dot{P}_i(\mathbf{r}) \\ &- \big(4\chi^2+8\chi+3\big)\big(\dot{P}_l(\mathbf{r}) + \dot{P}_j(\mathbf{r})\big)\big], \end{aligned} \tag{16}$$

where $l \neq j \neq i$, $\dot{P}_i(\mathbf{r}) = \mathbf{v}\nabla P_i(\mathbf{r})$, $\chi = R/\gamma$. Substitution of Eq. (16) into Eqs. (12), (14) allows one to obtain a common expression for $\mathbf{F}_R$ and $\mathbf{F}_g$. For clearness and simplification of the final expressions, additional assumptions should be made. Let $\zeta = \eta_1 = \pi$, the detunings $\Delta_i > 0$, $\xi_i < 0$ $(i = x, y, z)$ and Stark shifts of the energy levels induced by the waves with mutually orthogonal polarization directions are equal:

$$\frac{|V_i|^2}{\Delta_i} = \gamma\sqrt{\frac{I}{I_s}}g, \quad g = \left(\frac{V_x}{\Delta_x}\right)^2, \ \frac{I}{I_s} = \frac{V_x^2}{\gamma^2}, \tag{17}$$

where $I = I_x$ is the intensity of light beams forming $E_{x1} = (\mathbf{e}_x\mathbf{E}_1)$ a strong coherent field component, $I_s = \hbar\omega_0 k^2\gamma/6\pi$ is the intensity of radiation saturating the quantum transition.

As a result we have:

$$\begin{aligned} &\mathbf{F}_g \approx -\nabla U_g, \quad U_g = \frac{1}{4\chi+3}\sum_{i=x,y,z}\frac{\hbar|V_{i1}(\mathbf{r})|^2}{\Delta_i}, \\ &\mathbf{F}_R = \mathbf{F}_{0R} + \mathbf{F}_{1R}, \quad \mathbf{F}_{0R} = -\nabla U_R, \ \mathbf{F}_{1R} = -m\sum_{i=x,y,z} v_i \varkappa(r_i)\mathbf{e}_i, \\ &U_R = -U_0\left[\sum_{i=x,y,z}\cos\left(\frac{2\pi}{L}r_i\cos\beta\right)\right], \\ &U_0 = \hbar\gamma\frac{kL}{2\pi}\sqrt{\frac{I}{I_s}}g\frac{2\chi+1}{(4\chi+3)^2}\frac{a_1}{[1+b]}, \\ &\varkappa(r_i) = \frac{2a_1(6\chi^2+8\chi+3)\gamma}{(4\chi+3)^3 R(1+b)}\cos^2\beta\cdot\omega_R\sqrt{\frac{I}{I_s}g} \\ &\qquad \times\left[b - \cos\left(\frac{2\pi r_i}{L}\cos\beta\right)\right], \end{aligned} \tag{18}$$

where $r_i = (\mathbf{r}\mathbf{e}_i)$, $v_i = (\mathbf{v}\mathbf{e}_i)$, $\mathbf{F}_{1R}$ is the light-induced viscous friction force, $\mathbf{F}_{0R}$ is the rectified gradient force [9,11] (potential field force), $\omega_R = \hbar k^2/m$.

Thus, one can see from Eqs. (18), that the main conditions of DOT formation are satisfied (see the introduction): the light field creates a deep macroscopic 3D potential well for the ions (described by the potential $U_R(\mathbf{r})$) with the depth proportional to the larger parameter $kL \gg 1$ ($U_R \sim kLU_g \gg U_g$); besides, the light-induced friction force $\mathbf{F}_1$ influences the ions. Moreover, given $b = a'/2a > 1$, which is always to be considered further as the condition entry, friction coefficients $\varkappa(r_i) > 0$ (see condition (1)). One should pay attention to the fact that the depth ΔU of the 3D potential well, as well as the friction coefficient $\varkappa$ can be increased up to the necessary values by means of increasing intensity ($\propto I$) of the strong field components in order to satisfy main conditions (1). It is worth noting that under the condition $T_i \gg U_g$, which is always satisfied here at $a_1^2 \ll 1$, the light-induced kinetics of plasma ions is determined exactly by RRF because of the small depth of the macroscopic potential wells.

In order to describe the state of the UP ion component correctly, one should take into account another important process: ion heating due to quantum fluctuations of the radiation force (compare with [5]). The mean ion heating rate $\varLambda$ for the considered symmetric configuration of optical fields and given condition (4) ($|V_i|^2/\Delta_i \gg R$) is $\varLambda = 3mD_R$, where D_R is a so-called induced diffusion coefficient in the velocity space [1] (averaged over microscopic spatial oscillations). The order of the value of D_R is well known [1] and is determined by the multiplying gradient force square ($\mathbf{F}_g^2/m^2 \propto \hbar^2k^2\cos^2\beta\cdot V_i^4/\Delta_i^2m^2 = \hbar^2k^2\cos^2\beta\cdot\gamma^2 Ig/I_s m^2$) by the correlation time τ_g of gradient force fluctuations. In the considered case of $\varLambda$-type ions, τ_g is determined by the rate R of incoherent mixing of the ionic states: $\tau_g \sim 1/R$. Thus

$$\varLambda = C\frac{\hbar^2k^2\cos^2\beta}{m}\gamma^2 g\frac{I}{I_s R}, \tag{19}$$

where C is a proportionality factor, depending on the saturation parameter $\chi = R/\gamma$. Standard (but rather complicated) calculations within the framework of a well-developed Wigner density-matrix formalism [1] lead to the following result:

$$C = \frac{3(8\chi^3+16\chi^2+11\chi+3)}{(4\chi+3)^3}.$$

3. UP model in dissipative optical trap

From Eqs. (18) it follows that the cubic cell with the edges (the lengths being $2L_1 = L/\cos\beta \sim L$ at $\beta \sim 1$) parallel to the axes of Cartesian coordinate system and with the center in the point $\mathbf{r} = 0$ there is DOT for UP, if the main conditions are satisfied (1). The facets of this cell coincide with the boundaries of the 3D potential cubic well (described by the potential $U_R(\mathbf{r})$): $\partial U_R/\partial r_i = 0$, $\partial^2 U_R/\partial r_i^2 < 0$ at $r_i = \pm L_1$, $i = x, y, z$. The cell center corresponds to the potential well bottom: $U_R(0) = -3U_0 = \min U_R(\mathbf{r})$. Regarding the optical field configuration peculiarities (see Eqs. (9), (10) and Fig. 1) it is easy to find out that in order to implement such a DOT, intersecting (in the region $|r_i| < L_1$, $i = x, y, z$) flat-topped light beams with the rectangular profile of the characteristic size $D_1 \times D_2$ (where $D_1 > 2L_1(\cos\beta + \sin\beta)$, $D_2 > 2L_1$) are necessary.

Further analysis will be based on the same assumptions as those used in [5] when creating the optical molasses model for UP.

Consider that UP has non-ideal (liquid-like) ionic component and a weak non-ideal electron component (i.e. apart from condition (15)) the following conditions are satisfied

$$p_e \gg p_i, \qquad \varGamma_i > 1 \gg \varGamma_e, \tag{20}$$

[3] In this approximation we neglect small additives to RRF, having the order of magnitude a_1^3 and $\varepsilon_1^2 a_1$.

where p_e, p_i are the pressures and $\Gamma_\alpha = e^2/r_0 T_\alpha$ are Coulomb non-ideality parameters of the electron ($\alpha = e$) and ion ($\alpha = i$) components UP, $r_0 = (3/4\pi n_0)^{1/3}$ is the Wigner–Seitz radius, n_0 is a characteristic charged particle density $n(\mathbf{r}, t)$ value. Besides, the following hierarchy of the characteristic spatial scales is assumed:

$$\frac{2\pi}{k} = \lambda \ll r_0 \ll \lambda_e \ll L, \tag{21}$$

where $\lambda_e = \sqrt{T_e/4\pi e^2 n_0}$ is the Debye radius. The inequality on the right of (21) (the condition of the Debye radius smallness in comparison with DOT dimensions) is equivalent to the quasi-neutrality condition [23,24] of UP and, actually, is a necessary condition for the confinement of electron–ion UP by means of optical forces, selectively acting upon ions only.

To describe macroscopic motion and UP state in DOT (in the conditions under consideration) the two-fluid hydrodynamic approach will be used, representing UP as a quasi-neutral mixture of two (electron and ion) charged fluids. The electron and ion UP components interact by means of self-consistent ambipolar field $\mathbf{E}_A$ (providing quasi-neutrality [23]) and collisions with the frequency

$$\nu_{ei} \simeq (4\sqrt{2\pi}/3) e^4 n \ln\Lambda / m_e^{1/2} T_e^{3/2},$$

where $\ln\Lambda$ is the Coulomb logarithm, m_e is the electron mass. Of considerable significance is the process of collisional energy transfer from electrons to ions with the rate Q_{ei}, determined at $T_e \gg T_i$ by expression [23] (see also [25])

$$Q_{ei} \simeq T_e/\tau_\varepsilon, \quad \tau_\varepsilon^{-1} = 3\nu_{ei} m_e/m.$$

This process leads to electron cooling and ion heating. To prevent excessive electron cooling (which may result in violating conditions (20) and in developing the process of three-body electron–ion recombination) we assume that the electron temperature is maintained by heating them with the help of microwave electromagnetic radiation with the frequency $\omega_h \gg \nu_{ei}$ and the intensity I_h. The rate Λ_e [26] of the electron heating is given by $\Lambda_e \simeq (4\pi e^2/m_e c\omega_h^2) I_h \nu_{ei}$.

We are interested in the slowest (quasistationary) stage of the UP evolution in the trap, developing during the characteristic times

$$t \sim \tau > \varkappa^{-1}(L/\lambda_r)^2 \gg \varkappa^{-1}, \ \nu_{ei}^{-1}, \ \tau_\varepsilon, \tag{22}$$

where τ denotes the UP decay time due to its diffusion through the DOT boundaries. If inequalities (21), and also condition (20) are well satisfied, then the equations of the two-fluid hydrodynamic model (being macroscopic equations of momentum and energy conservation for each of the UP components) are considerably simplified.[4] They yield a rather obvious force balance equation

$$-\nabla p_e = \sum_{i=x,y,z} m\varkappa(r_i) n u_i \mathbf{e}_i + n\nabla U_R, \quad u_i = (\mathbf{e}_i \mathbf{u}), \tag{23}$$

quasi-stationary equation of the electron energy balance, averaged over the DOT volume Ω

$$3\frac{m_e}{m} T_e \langle \nu_{ei} n \rangle_\Omega \simeq \frac{4\pi e^2}{m_e c \omega_h^2} \langle \nu_{ei} n I_h \rangle_\Omega,$$
$$\langle \ldots \rangle_\Omega = \frac{1}{(2L_1)^3} \int_\Omega (\ldots)\, d\Omega, \tag{24}$$

and the equation of the local balance for the rates of ion heating and cooling processes.

$$Q_{ei} + \Lambda = \widehat{\varkappa}(\mathbf{r}) T_i, \quad T_i = \frac{\Lambda}{\widehat{\varkappa}(\mathbf{r})} + \frac{3m_e}{m}\frac{\nu_{ei}}{\widehat{\varkappa}(\mathbf{r})} T_e,$$
$$\widehat{\varkappa}(\mathbf{r}) = \sum_i \varkappa(r_i), \tag{25}$$

where $\mathbf{u}$ is the macroscopic (directed) rate of ambipolar[5] (joint) motion [23,24] of the electron and ion fluids; $\widehat{\varkappa}(r_i)$, $U_R(\mathbf{r})$ and Λ are determined from Eqs. (18), (19) and it is assumed that electrons are isothermal, $\partial T_e/\partial r_i \simeq 0$, due to high electron heat transfer rate (the isothermality condition is given in [5] and is well satisfied in a wide range of plasma parameters owing to the smallness of electron and ion mass ratio $(m_e/m) \ll 1$). Note that if the intensity I_h is uniform and time-independent ($I_h = \text{const}$), then, as it follows from Eq. (24), T_e also does not depend on time and is determined by the choice of the microwave field parameters: $T_e \simeq 4\pi e^2 m I_h / 3m_e^2 c\omega_h^2$. Further, exactly this case will be implied.

Considering the electron temperature uniformity, the equation of the state of the electron UP component: $p_e \simeq nT_e$, and combining Eq. (23) with the continuity equation

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{u}) = 0, \tag{26}$$

we obtain the following 3D Smoluchowski equation (SE) [27], describing ambipolar UP diffusion in the trap:

$$\frac{\partial n}{\partial t} = \sum_{i=x,y,z} \frac{\partial}{\partial r_i}\left(D_A(r_i)\frac{\partial n}{\partial r_i} + \frac{\partial U_R}{m\varkappa(r_i)\partial r_i} n\right), \tag{27}$$

where $D_A(r_i) = T_e/m\varkappa(r_i)$ denotes ambipolar diffusion coefficients. Eqs. (24), (25), (27) fully determine the main macroscopic UP parameters in DOT T_e, $T_i(\mathbf{r})$, and, also, the density $n = n(\mathbf{r}, t)$ and the lifetime τ of the UP in DOT, if the proper boundary conditions for SE (27) are given.

4. Diffusion decay of the UP in DOT

If DOT boundary Σ coincides with physical surfaces (the walls of a transparent cubic dielectric container into which UP is confined), then, the process of electron and ion neutralization on these surfaces is considered with the help of the effective boundary condition for Eq. (27)

$$n_\Sigma = 0, \tag{28}$$

which is usually used in plasma physics when considering problems of plasma decay due to diffusion on the container walls [23, 24] (see also Appendix B in [28]). This approach seems to be rather good even in the absence of the container, confining DOT, since the particles reaching DOT boundaries will irreversibly escape from the interior region of trap with a very large probability. Really, in the region adjacent to the boundary Σ from outside, the plasma ions are influenced by RRF $\mathbf{F}_{0R}$, pushing the ions away (from the boundary Σ) into the region in which the optical fields are absent due to the finite transverse sizes of real laser beams. Long-lived (quasi-stationary) UP states correspond to the lowest (the slowest) diffusion mode, i.e. to the solution of SE (27) of the following type (compare with the problem on the UP decay in the limited 1D optical superlattice [6]):

$$n(\mathbf{r}, t) = n_0 \prod_{i=x,y,z} \exp\left(-\frac{t}{\tau_i}\right)\Psi(r_i)\exp\left[-\frac{U(r_i)}{T_e}\right], \tag{29}$$

where $\Psi(r)$ is the eigenfunction, corresponding to the lowest eigenvalue $\lambda_1 = 1/\tau_1$ of Sturm–Liouville boundary problem (SLP)

[4] The reduction stages of these equations do not considerably differ from 1D case for two-fluid UP model and are well described in [5,6]. The order of accuracy of the reduced equations is determined by small parameters of the problem: $(\lambda_r/L)^2$, τ_ε/τ, $|p_i|/p_e \sim \Gamma_e \ll 1$.

[5] It is assumed that external electric current is absent in UP.

$$\left(\widehat{H}+\frac{\Phi}{\tau_1}\right)\Psi(r)=0, \quad \Psi(\pm L_1)=0,$$
$$\widehat{H}=\frac{\partial}{\partial r}\left(D_A\Phi\frac{\partial}{\partial r}\right), \quad \Phi=\exp\left[-\frac{U(r)}{T_e}\right], \tag{30}$$

$\tau_i=\tau_1=3\tau$ ($\forall i=x,y,z$), τ denotes the UP decay time, $U(r)=-U_0\cos(2\pi r\cos\beta/L)$, and n_0 corresponds to the UP particle density in the centre of DOT, if the eigenfunction Ψ is normalized by the condition $\Psi(0)=1$. Note, that quasi-stationary density distribution (QSD) represented in (29) is formed during the time $t<\tau$, moreover, only the density value n_0 in the trap centre depends on the initial conditions of the UP preparation.

Solution of the SLP (30) can be presented as the following Neumann series [6]:

$$\Psi=1+\sum_{n=1}^{\infty}(-1)^n\frac{1}{\tau_1^n}\widehat{E}^n\Psi_0,$$
$$\widehat{E}\varphi(r)=\int_0^r\Phi^{-1}(r_2)\frac{dr_2}{D_A(r_2)}\int_0^{r_2}\Phi(r_1)\varphi(r_1)\,dr_1, \tag{31}$$

where $\Psi_0\equiv 1$, $\Psi(-r)=\Psi(+r)$ and τ_1 is determined from the boundary condition $\Psi(L_1)=0$. In expansion (31), it suffices to take into account several first terms. When condition (1) of deep UP localization is satisfied, $\eta\gg 1$, the error in the solution for QSD at such truncation of the series (31) is exponentially small (as in a similar problem considered in [6]): $\sim\exp[-C'\eta]$, where the constant $C'\sim 1$. As a result, the following expression for the lifetime of τ UP in DOT is obtained:

$$\tau\approx\frac{\varkappa(L_1)mL_1^2}{3\pi\Delta U}\exp\left[\frac{\Delta U}{T_e}\right], \tag{32}$$

where $\Delta U=2U_0$. One can see that the lifetime of quasi-stationary UP is exponentially large with respect to the large parameter $\eta=\Delta U/T_e$: $\propto\exp\eta$. Besides, at $\eta\gg 1$ QSD is Boltzmann-like: $n\approx n_0\exp[-t/\tau]\exp[-U_R(\mathbf{r})/T_e]$, almost everywhere except for narrow regions (with a width of $\sim L_1/\sqrt{\eta}$) near the boundaries Σ of the trap.

From (32) it also follows that the lifetime of UP in DOT is determined by the dimension of the trap L, a single plasma parameter (electron temperature T_e), and also, by light field and ion quantum transition parameters. Dependence of τ on the ion mass is absent (since $\varkappa\propto m^{-1}$). The UP lifetime in DOT τ can be increased by increasing the dimensions L of the trap and the intensity of the strong field I. Fig. 2 illustrates a specific example of τ determination for DOT with the dimension $L\simeq 5$ cm at the UP electron temperature of $T_e=1.8$ K. It is visible that τ may exceed the time τ_f of free expansion of UP ($\tau_f\sim L/c_s\sim 5\times 10^{-3}$ s at $m\approx 200$ amu) more than by three orders of magnitude (at a relative intensity of the strong field $I/I_s\sim 100$)!

5. Conclusion

Thus, we have theoretically demonstrated the possibility of long 3D all-optical confinement of UP containing resonant ions with the quantum transition $J=1\to J=0$. We have found the optical field configuration and parameters at which all the necessary conditions of long-term UP confinement are satisfied, namely, the requirements to the spatial structure of the radiation forces combined with satisfying conditions (1). The suggested scheme of DOT for UP combines the features of the optical molasses and trap based on the influence of potential field force and allows not only UP confinement but also the temperature control of its ion and electron component by means of choosing the electromagnetic field parameters. Electron and ion temperatures in such a DOT can be rather low in comparison with the usual laboratory plasmas. Extending the example given in Section 4 one obtains (on the basis of Eqs. (24), (25)) the following estimations of T_i, Γ_i and other UP and DOT characteristics at $n_0\approx 10^4$ cm^{-3}, $I/I_s\approx 100$, $g=0.05$, $m\approx 200$ amu: $T_i\sim 5\times 10^{-3}$ K, $\Gamma_i\sim 15$, $\Gamma_e\sim 0.03$, $\lambda_r\lesssim 0.02$ cm, $\lambda_e\approx 5\times 10^{-2}$ cm, $\omega_i\approx 10^4$ s^{-1}, the characteristic time of three-body electron–ion recombination $\tau_r\approx 50$ s, a microwave power (necessary to maintain the electron temperature ($T_e=1.8$ K)) $P_h\simeq L^2I_h\simeq 16$ μW (at $\omega_h\simeq 3\times 10^{10}$ s^{-1}), relative intensity of the partially coherent optical field $I'/I_s\sim 5$ at $\Gamma\approx 12\gamma$. Unfortunately, the non-ideality parameter values Γ_i are far from the maximum values which, in principle, could be obtained by the laser cooling method [8]. This is an inevitable retribution for the increase of the confinement time τ in the DOT scheme considered since the increase of the optical field intensity I results in the increase of ion heating due to quantum fluctuations of the radiation force.

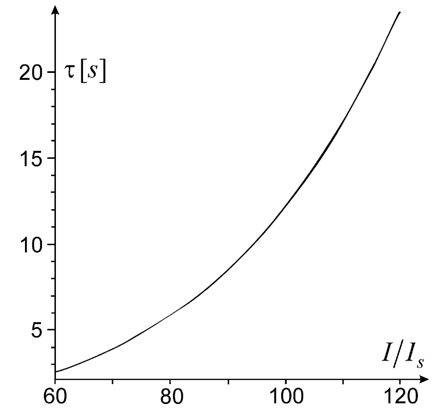

Fig. 2. Dependence of diffusion decay time τ (in seconds) of quasi-stationary UP states in the optical trap on the relative intensity of the strong coherent field I/I_s at a fixed value of $g=0.05$ and $T_e=1.8$ K, $k\approx 3.2\times 10^5$ cm^{-1}, $\gamma\approx 10^8$ s^{-1}, $a_1^2=0.2$, $b=1.5$, $L\simeq 5$ cm, $\chi=0.5$, $\beta\approx 0.15$, $\eta\gtrsim 5$.

Finally, it should be emphasized that our main purpose has been to demonstrate principally new possibilities of all-optical confinement and the UP state control with the help of the rectified radiation forces acting upon plasma ions in non-monochromatic light fields. Thus, we restricted ourselves to the choice of such conditions (in particular, assuming $a_1^2\ll 1$), which allowed obtaining approximate analytical solution of the problem. The parameter optimization of the suggested scheme of the dissipative optical trap, aimed at achieving the result desired, for example, at lower field intensities is a special challenging problem.

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