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Transformable broad-band transparency and amplification in negative-index films

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The possibility to produce laser-induced optical transparency of a metamaterial slab through the entire negative-index frequency domain is shown above a certain control laser field intensity threshold. © *2008 American Institute of Physics*. [DOI: 10.1063/1.3028651]

Negative-index metamaterials (NIMs) exhibit highly unusual optical properties and promise a great variety of unprecedented applications.[1] However, strong optical absorption inherent to NIMs imposes severe limitation on their applications. The possibility to overcome such obstacle was proposed[2] based on the coherent energy transfer from the control field to the signal through a three-wave mixing accompanied by the optical parametric amplification (OPA) of the signal. It was shown that the transparency exhibits an extraordinary resonance behavior as a function of intensity of the control field and the NIM slab thickness which occurs due to the backwardness of the light waves in NIMs. Basically, such resonances are narrow and the sample remains opaque anywhere beyond the resonance field and sample parameters. Herein, we show an extraordinary dependence of the transmission for the negative-index backward-wave signal on the absorption index for the coupled ordinary, positive-index idler. With the aid of a realistic numerical model, we prove that the transparency and amplification of the NIMs slab at the signal frequency can be made robust and achievable through a wide range of the control field intensity and the NIM slab thickness.

The basic idea of the proposal is as follows. We consider three coupled optical electromagnetic waves with wave vectors k_j ($j=\{1,2,3\}$) codirected along the z axis.[2] The waves propagate through a slab of thickness L that possesses optical nonlinearity $\chi^{(2)}$. Here, we assume magnetic nonlinearity. The outcomes do not change in the case of electric nonlinearity. Only two waves enter the slab, strong control field at ω_3 and weak signal at ω_1, which then generate a difference-frequency idler at $\omega_2=\omega_3-\omega_1$. The idler contributes back to the signal through the same three-wave mixing process, $\omega_1=\omega_3-\omega_2$, and thus provides OPA of the signal. The signal wave $H_1(\omega_1)$ is assumed backward due to negative refractive index $n(\omega_1)<0$. It means that the energy flow S_1 is directed antiparallel to k_1 and, therefore, opposite to the z axis. Two other waves, the idler H_2 and the control field H_3, are ordinary waves with $k_{2,3}$ and $S_{2,3}$ directed along the z axis since $n(\omega_2)>0$ and $n(\omega_3)>0$. Consequently, the control wave enters the slab at $z=0$, whereas the signal at $z=L$. Generated idler travels along the z axis. Unlike early proposals[3–5] and recent breakthrough experiment on backward-wave optical parametric oscillation in the submicrometer periodically poled nonlinear-optical (NLO) crystal,[6,7] here all wave vectors are codirected. This is crucially important because it removes severe technical problem of phase matching of the coupled waves with otherwise counterdirected wave vectors. The later is intrinsic to ordinary, positive-index materials.

The equations for the slowly varying normalized amplitudes $a_j=\sqrt[4]{\epsilon_j/\mu_j}h_j/\sqrt{\omega_j}$ for the signal and idler waves can be written in the forms[2]

$$da_1/dz=-iga_2^* \exp[i\Delta kz]+(\alpha_1/2)a_1, \tag{1}$$

$$da_2/dz=iga_1^* \exp[i\Delta kz]-(\alpha_2/2)a_2. \tag{2}$$

Here, $g=(\sqrt{\omega_1\omega_2}/\sqrt[4]{\epsilon_1\epsilon_2/\mu_1\mu_2})(8\pi/c)\chi^{(2)}h_3$ is the NLO coupling coefficient, h_3 is the amplitude of the control field, ϵ_j and μ_j are the electric permittivity and magnetic permeability of the slab's material, phase mismatch $\Delta k=k_3-k_2-k_1$, k_j are wave vectors directed along the z axis, and α_j are the absorption indices. The quantities $|a_{1,2}|^2$ are proportional to the number of photons at the corresponding frequencies. The transmission factor for the negative-index signal, $T_1(z)=|a_1(z)/a_{1L}|^2$, and energy conversion factor for the idler, $\eta_2(z)=|a_2(z)/a_{1L}|^2$, are derived from the solution to the coupled Eqs. (1) and (2), whereas the control field is assumed homogeneous through the slab. Transmission is given by[2]

$$T_1(z=0)=T_{10}=\left|\frac{\exp\{-[(\alpha_1/2)-s]L\}}{\cos RL+(s/R)\sin RL}\right|^2. \tag{3}$$

Here, $s=(\alpha_1+\alpha_2)/(4)-i(\Delta k/2)$, and $R=\sqrt{g^2-s^2}$. The fundamental difference between the spatial distribution of the waves in ordinary and NI materials is most explicitly seen in the limiting case $\alpha_j=0$, $\Delta k=0$. Then Eq. (3) reduces to

$$T_1=1/[\cos(gL)]^2, \tag{4}$$

which depicts periodical oscillations of the transmission as a function of the slab thickness L and of the parameter g that is proportional to the control field strength. Transmission $T_1\to\infty$ at $gL\to(2j+1)\pi/2$, which indicates self-oscillations. Such a set of the geometrical transmission resonances ($j=\{0,1,2,\ldots\}$), even at $\Delta k=0$, is in stark contrast with the propagation properties in ordinary NLO crystals. In the later case, for all three wave vectors codirected and $\Delta k=0$, the signal and the idler would exponentially grow across the slab without any singularities as $|h_{1,2}|^2\propto\exp(2gz)$. Such unusual transmission properties of the NIM slab occur because the signal and the idler grow towards opposite directions, and

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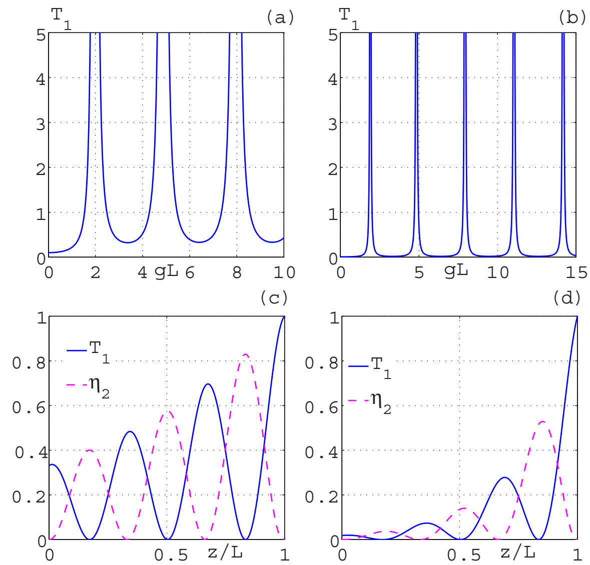

FIG. 1. (Color online) [(a) and (b)] Typical dependence of the output signal at $z=0$ on the intensity of the control field and the slab's thickness at different absorption indices for the signal and the idler and [(c) and (d)] distribution of the fields inside the slab. $\Delta k=0$.[(a) and (c)] $\alpha_2 L=0.1$, $\alpha_1 L=2.3$. [(b) and (d)] $\alpha_2 L=-3$, $\alpha_1 L=5$. (c) $gL=9.51$. (d) $gL=9.48$.

they are determined by the boundary conditions on the opposite sides of the NIM slab.

Figures 1(a) and 1(b) present numerical analysis of Eq. (3). They depict a typical resonance dependence of the transmission factor for the originally strongly absorbing slabs on the parameter gL. It is known that even weak amplification per unit length may lead to lasing provided that the corresponding frequency appears in a high-quality cavity or distributed feedback resonances. Such resonances are equivalent to a great extension of the effective length of a low-amplifying medium. Figures 1(a) and 1(b) demonstrate the same behavior. The stronger the signal absorption is, the more fine-tuning to the OPA resonance is required. A *strong resonance* dependence of the NIM slab transparency on the parameter gL indicates the necessity of *fine-tuning* for the intensity of the control field. This work is to show that such dependence can be transformed so that absolute transparency and amplification become robust and achievable through a *wide range* of the parameter gL. As seen from Eqs. (1) and (2), local NLO energy conversion rate for each of the waves is proportional to g factor and to the amplitude of another coupled wave. Hence, the facts that the waves decay towards opposite directions have a significant influence on the entire NLO propagation process and on the entire transmission properties of the slab. As an example, Figs. 1(c) and 1(d) display unusual distributions of the fields inside the slab which correspond to the third transmission minima. Oscillation amplitudes grow sharply with approaching the resonances. Unless optimized, the signal maximum inside the slab may appear much greater than its output value at $z=0$. The comparison of Figs. 1(a) and 1(b) suggests that the greater the difference between the signal and the idler absorption indices is, the more opaque the slab beyond the resonances is. A typical NIM slab absorbs about 90% of light at the frequencies which are in the NI frequency range. Such absorption corresponds to $\alpha_1 L \approx 2.3$. The slab becomes *transparent within the broad range of the slab thickness and the control field intensity* if the transmission in the minimums

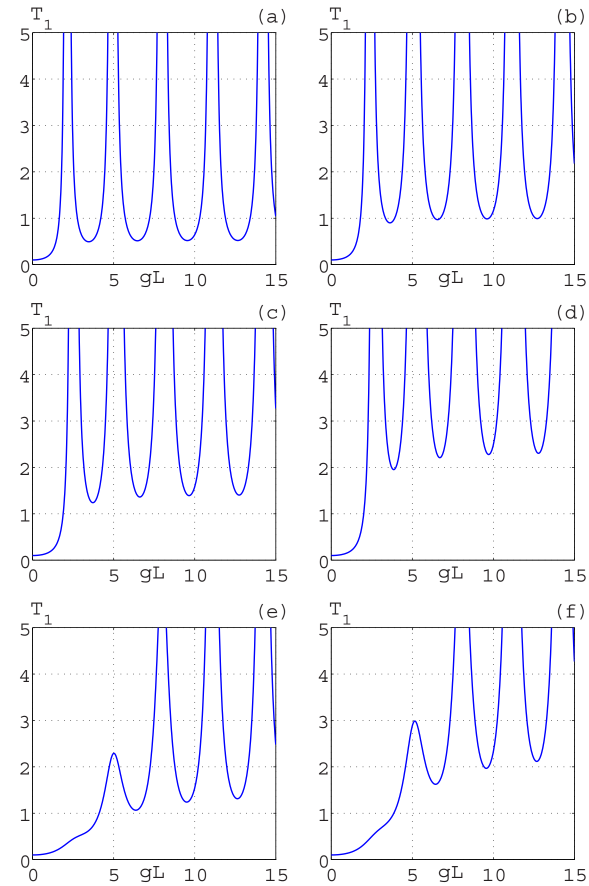

FIG. 2. (Color online) Transmission of the NIM slab at $\alpha_1 L=2.3$ and different values of $\alpha_2 L$. [(a)–(d)] $\Delta kL=0$. [(e)–(f)] $\Delta kL=\pi$. (a) $\alpha_2 L=1$. (b) $\alpha_2 L=2.3$. [(c) and (e)] $\alpha_2 L=3$. [(d) and (f)] $\alpha_2 L=4$.

is about or more than 1. The examples of numerical analysis given below are to prove the possibility to achieve such optical properties by the appropriate adjustment of the absorption indices. Figure 2 depicts transmission properties of the NIM slab at $\alpha_1 L=2.3$ and different magnitudes of the absorption index $\alpha_2 L>0$ which are less, equal, and greater than $\alpha_1 L$. It is seen that the transmission in minima becomes about or larger than 1 at $\alpha_2 \geqslant \alpha_1$. Figure 3 presents transmission at different $\alpha_2=\alpha_1$. Being compared with Fig. 2(b), it proves that the transmission in the minima depends rather on the ratio of the signal and the idler absorption indices than on their magnitude. Transmission does not drop below 1 both at low and high absorption indices provided that $\alpha_2 \geqslant \alpha_1$ and the gL factor is larger than a certain magnitude. The smaller the magnitudes of equal absorption indices are, the closer to 1 the transmission in the first minimum is. At larger $\alpha_2=\alpha_1$,

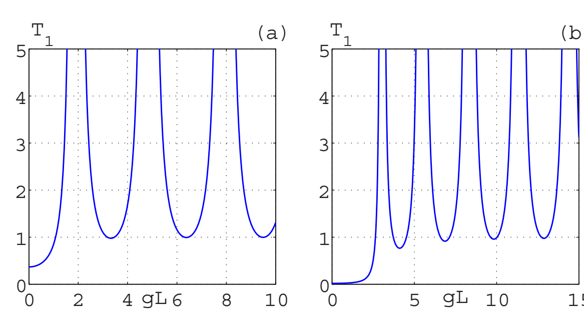

FIG. 3. (Color online) Laser-induced transparency at equal absorption indices for the signal and the idler. $\Delta k=0$. (a) $\alpha_2 L=\alpha_1 L=1$. (b) $\alpha_2 L=\alpha_1 L=4$.

TABLE I. Dependence of the transmission minima on the absorption index $\alpha_2 L$. $\alpha_1 L=2.3$.

	$\alpha_2 L=1$, $\Delta k=0$			$\alpha_2 L=2.3$, $\Delta k=0$		
gL	3.465	6.465	9.54	3.63	6.555	9.615
T_1	0.493	0.514	0.518	0.9	0.969	0.986
	$\alpha_2 L=1$, $\Delta kL=\pi$			$\alpha_2=2.3$, $\Delta kL=\pi$		
gL		6.24	9.405		6.315	9.465
T_1		0.448	0.487		0.789	0.896
	$\alpha_2 L=3$, $\Delta k=0$			$\alpha_2 L=4$, $\Delta k=0$		
gL	3.72	6.6	9.66	3.87	6.69	9.72
T_1	1.24	1.362	1.392	1.953	2.211	2.278
	$\alpha_2 L=3$, $\Delta kL=\pi$			$\alpha_2 L=4$, $\Delta kL=\pi$		
gL		6.36	9.51		6.42	9.57
T_1		1.065	1.24		1.624	1.968

it tends to 1 in the next minima with the increase in the parameter gL. The resonances become narrower with the increase in the signal absorption index and experience a shift towards the larger magnitudes of the parameter gL. Figures 2(e) and 2(f) show that phase mismatch decreases transparency in the first maxima but does not noticeably change it in the minima so that it remains about or larger than 1 if $\alpha_2 \geqslant \alpha_1$. It is seen that the effect of phase mismatch decreases with the increase in the parameter gL. Table I explicitly proves the above presented conclusions regarding the changes in the positions and corresponding magnitudes of the transmission minima.

To conclude, we have investigated the possibility to transform optical properties of the metamaterial slab in the negative-index frequency domain. Strong absorption of negative-index electromagnetic waves is inherent to such materials. It is crucially important for many applications of this revolutionary class of optical materials to ensure a robust transparency of the metamaterial slabs for the negative-index signals. The basic idea of the proposed approach is coherent NLO energy transfer from the control electromagnetic wave to the negative-index signal through a three-wave mixing and OPA. We have revealed an extraordinary dependence of transmission of the negative-index backward-wave signal coupled with the ordinary, positive-index control and idler waves on the ratio of their absorption indices. Such coupling scheme is intrinsic to NIMs. With the aid of numerical simulations, we have shown the way to ensure nearly 100% transparency or amplification of the negative-index signal within a broad range of intensities of the control fields and the slab's thicknesses. Among the counterintuitive conclusions of the work is the recommendation to adjust the absorption index for the idler by increasing it. Thus, we have shown that the minimum transparency can be transformed, increased, and even turned into amplification, so that it remains robust for any magnitudes of the slab thickness and the control field intensity above a certain threshold.

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