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Adder on Ternary Base Elements for a Quantum Computer

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A circuit consisting of elementary quantum logic operators has been proposed for an adder in the ternary number system. A sequence of RF magnetic field pulses has been found for its implementation by the nuclear magnetic resonance method on a chain of quadrupole nuclei with spin $I = 1$. The numerical simulation of the adder operation has been performed.

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INTRODUCTION

The ternary number system [1, 2] is a base-3 number system and exists in two variants: asymmetric (numerals 0, 1, and 2, or 0, –1, and –2) and symmetric (numerals $\bar{1} = -1$, 0, and 1). In the symmetric number system, a ternary number is represented in the form $(A_n...A_3A_2A_1)$, where $A_n = \{\bar{1}, 0, 1\}$ is the number system. A decimal number is related to a ternary number as

$$A = A_n 3^{n-1} + \ldots + A_3 3^2 + A_2 3^1 + A_1 3^0. \quad (1)$$

The symmetric number system allows the representation of negative numbers without a minus sign and, hence, provides the shortest representation of numbers. It has many other advantages [2, 3]. Table 1 presents the summation rules for two ternary numbers A and B [1].

The ternary number system has attracted interest from the very beginning of the development of digital machines in view of the arithmetic properties of the symmetric number system. The ternary number system on tristable memory devices (trits) was used in Setun' computers manufactured in a small series [2].

Quantum computational processes promising a large advance in computer science have been recently discovered [4]. Most investigations concern quantum computers on two-level quantum systems, qubits (quantum analogs of bits) ensuring computations in binary number system. Since ternary logic has a number of advantages over binary logic, quantum computers operating on three-level quantum systems, qutrits (quantum analogs of trits), can be more promising [5, 6] than those operating on qubits. However, this field has not yet been sufficiently studied. In this work, we consider an adder on qutrits; to our knowledge, the operation of such an adder has not yet been considered. An adder on a chain of qubits is well-studied theoretically [7–9]. The considered summation algorithm is not a quantum algorithm [4], but is a classical algorithm performed on a quantum computer. Although a quantum adder does not accelerate the summation process as compared to a classical adder, it is an important element of any computer.

ELEMENTARY LOGICAL OPERATORS (GATES) AND THE SCHEME OF A QUTRIT-BASED ADDER

The operation of a qubit-based adder is organized [7–9] by means of a two-qubit controlled NOT gate (CNOT) and a three-qubit Toffoli gate, which is a controlled–controlled NOT gate (CCNOT). The SUM two-qutrit gate and CSUM three-qutrit gate [10] are generalizations of the mentioned gates to the qutrit case, respectively. The operation rules for the SUM and CSUM gates are given in Tables 2 and 3, respectively (the values of the operating qutrits after the action of the corresponding operator are given in the thick-line frame); similarly, as in Table 1, A, B, and C are single-order ternary numbers.

The operation of the qutrit-based adder can be organized using these gates by the circuit shown in Fig. 1. The horizontal lines correspond to the qutrit

Table 1. Summation of two ternary numbers A and B

$B\backslash A$	–1	0	1
–1	–1 1	–1	0
0	–1	0	1
1	0	1	1 –1

states in the orders of addends (A_1, A_2, and A_3) and (B_1, B_2, and B_3), as well as qutrits used for the transfer to the next orders (C_1, C_2, and C_3). Gates are denoted by the vertical lines connecting the state lines of two (SUM) or (CSUM) three qutrits. Two three-order numbers (the order is denoted by the subscript) are taken to exemplify summation in Fig. 1. Qutrits are joined in blocks (each including three qutrits), which correspond to the orders of two addends A_n and B_n, as well as to qutrit C_n. Gates act sequentially on each order of numbers A_n and B_n. The final sum of the nth order numbers is written in the state of qutrit A_n. According to the circuit drawn in Fig. 1, the summation in the nth order is represented by the following sequence of the SUM and CSUM gates:

$$F_n = \mathrm{SUM}_{C_{n-1}A_n}\mathrm{CSUM}_{C_{n-1}A_nC_n} \times \mathrm{SUM}_{B_nA_n}\mathrm{CSUM}_{A_nB_nC_n}. \quad (2)$$

The spatial positions of qutrits are not specified in the universal logical circuit shown in Fig. 1. Let us consider an adder on the chain of qutrits, which can be quadrupole nuclei with spin $I = 1$ controlled by the NMR method (see the next section). The scheme of the arrangement of qutrits and the corresponding orders A_n and B_n of two summed ternary numbers has the form

$$A_1\ B_1\ C_1\ A_2\ B_2\ C_2\ A_3\ B_3\ C_3\ \ldots\ A_n\ B_n\ C_n.$$

Addressing necessary for a selective control of the necessary spin states by means of the RF magnetic field can be ensured, for example, by changing the static magnetic field along the chain. To perform conditional operations, we assume, as in [7, 8], the presence of spin–spin interaction between nearest neighbors.

To implement the adder on the chain of qutrits in the presence of the interaction only between the nearest neighbors, the position of the target spin in the CSUM operator should be changed. To this end, it is necessary to supplement the scheme of the permutation operators of the states of two neighboring spins (SWAP operators whose actions are specified by the rules presented in Table 4). Taking into account the new operation, expression (2) is changed to the following sequence of operators:

$$F_n = \mathrm{SUM}_{C_{n-1}A_n}\mathrm{SWAP}_{B_nC_n}\mathrm{SWAP}_{B_nA_n} \times \mathrm{CSUM}_{C_{n-1}B_nA_n}\mathrm{SWAP}_{A_nB_n}\mathrm{SWAP}_{C_nB_n} \times \mathrm{SUM}_{B_nA_n}\mathrm{SWAP}_{B_nC_n}\mathrm{CSUM}_{A_nC_nB_n}\mathrm{SWAP}_{C_nB_n}. \quad (3)$$

For summing the entire number, operator F_n should be applied to each order of the summands.

NMR IMPLEMENTATION OF LOGICAL OPERATORS AND ADDER

Let us consider a chain of quadrupole nuclei with spin $I = 1$ interacting with the axially symmetric crystal field gradient. On the one hand, such a system is sufficiently simple; on the other hand, NMR is the leading approach in the experimental simulation of quantum computations. Finally, such a system is the direct development of the previously studied model of the adder on a chain of qubits [7, 8]. The Hamiltonian of the system has the form [11]

$$H_0 = -\sum_{k=1}^{N}\omega_k I_k^z + q\sum_{k=1}^{N}\left(I_k^{z2} - \frac{2}{3}\right) - \sum_{k=1}^{N} J_k I_k^z I_{k+1}^z, \quad (4)$$

where $\omega_k = B_0\gamma + kg\gamma$ is the Larmor precession frequency in the strong static magnetic field B_0, γ is the gyromagnetic ratio, g is the gradient of the magnetic field, k is the spin number, q is the quadrupole coupling constant, J_k is the spin–spin coupling constant between nearest neighbors, and I_k^z is the operator of the projection of spin k on the Z axis. We take $\hbar = 1$; i.e., energy coincides with angular frequency.

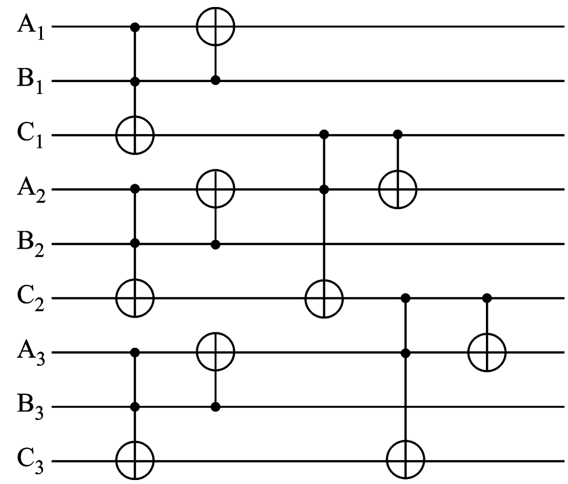

Fig. 1. Logical circuit of the operation of the qutrit-based adder.

Table 2. SUM_{AB} operator

$B\backslash A$	−1	0	1	
−1	1	−1	0	
0	−1	0	1	B
1	0	1	−1	

Table 3. Result of the application of the CSUM_{ABC} operation on $C = 0$

$B\backslash A$	−1	0	1	
−1	−1	0	0	
0	0	0	0	C
1	0	0	1	

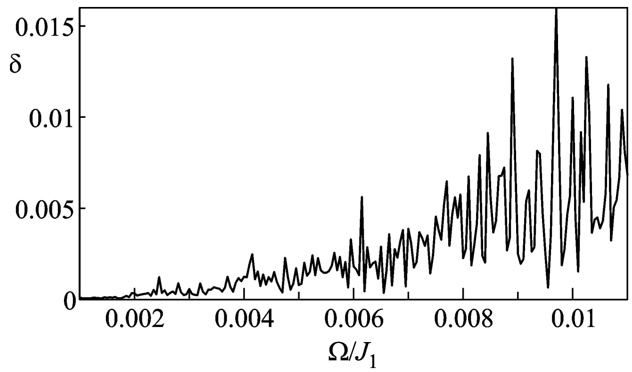

Fig. 2. Error in the summation in a single order, δ, versus the RF field amplitude Ω at $J_2 = 7.333J_1$, $J_3 = J_1$, $q = 100J_1$, and $g = 11.5J_1$.

Using the above results and a mathematical package, a numerical model of a quantum NMR adder has been developed. The summands in the nth block of an order are encoded by the initial quantum-mechanical state (column vector) formed by the direct product of state vectors of individual qutrits and the state vector of the qutrit of the order transfer in the preceding block:

$$\{|\psi_{C_{n-1}}\rangle \otimes |\psi_{A_n}\rangle \otimes |\psi_{B_n}\rangle \otimes |\psi_{C_n}\rangle\}. \quad (15)$$

The state vector of the block is represented by 1 in the corresponding place in the 3^i-component column vector, where $i = 4$. The summation is performed as a result of the action of operator (3) on each state vector of block (15) successively. According to this scheme, the summation of the order involves four nuclear spins. Correspondingly, the matrices used in calculations of the corresponding operators are 81×81 matrices. The SUM, CSUM, and SWAP gates are implemented by applying the above pulse sequences supplemented taking into account interactions with added spins. The summation of all orders of the number provides the column vectors of all orders with a changed state; decoding these column vectors, we obtain the resulting sum of two numbers in the ternary number system.

The action of each RF pulse is calculated by formulas (9) and (10). The RF field acts not only on a chosen transition of an individual spin, but also on neighboring spins, as well as on transitions close in frequency. For this reason, the multiplication of the state vector by the evolution matrix yields a superposition state. In this state, invalid answers appear with a small probability and a value different from 1 appears on the place of a valid answer. Probability P that the sum of two numbers is valid is specified by the square of the absolute value of the maximum element in the column vector; correspondingly, the summation error is $\delta = 1 - P$. The calculated error of one block of four nuclei is shown in Fig. 2. By varying the parameters, the error can be reduced to a value necessary for the operation of the adder. Since the error of an individual block is small, the summation error of the entire ternary number (all its orders) is the sum of the errors of individual blocks, i.e., increases linearly with the number of orders (the number of spins in the chain). Such a dependence for the qubit-based adder is confirmed by the numerical calculation in [8].

To conclude, we note that the above quantum schemes open the possibility of implementing the qutrit-based adder. Instead of three levels of a quadrupole nucleus, three electronic levels of ions, atoms, or molecules can be used as a working qutrit in the presence of the corresponding symmetry of the Hamiltonian. Microwave or laser pulses instead of RF pulses should be used for these systems in the universal quantum scheme.

REFERENCES

1. A. A. Papernov, *Logical Foundations of Digital Computing Technology* (Nauka, Moscow, 1968) [in Russian].
2. N. P. Brusentsov, in *Computer Techniques and Cybernetic Questions* (Mosc. Gos. Univ., Moscow, 1972), vyp. 9, p. 3 [in Russian].
3. A. Kushnerov, *Ternary Numerical Techniques. Retro and Modernity* (Univ. im. Ben-Guriona, Izrael, 2005) [in Russian].
4. K. A. Valiev and A. A. Kokin, *Quantum Computers. Hopes and Reality* (Tsentr “Regular. Khaotich. Dinamika,” Izhevsk, 2001) [in Russian].
5. A. B. Klimov, R. Guzman, J. C. Retamal, and C. Saavedra, Phys. Rev. A **67**, 062313 (2003).
6. B. Tamir, Phys. Rev. A **77**, 022326 (2008).
7. G. P. Berman, G. D. Doolen, G. V. Lopez, and V. I. Tsifrinovich, Comp. Phys. Commun. **146**, 324 (2002); quant-ph/0105133.
8. D. I. Kamenev, G. P. Berman, R. B. Kassman, and V. I. Tsifrinovich, Int. J. Quantum Inf. **2**, 323 (2004); quant-ph/0403085.
9. B.-S. Choi and R. Van Meter, quant-ph/0809.4317.
10. J. Daboul, X. Wang, and B. C. Sanders, J. Phys. A **36**, 7063 (2003).
11. Ch. Slicter, *Principle of Magnetic Resonance* (Harper and Row, New York, 1961; Mir, Moscow, 1981).
12. R. Das, A. Mitra, V. Kumar, and A. Kumar, Int. J. Quantum Inf. **1**, 387 (2003); quant-ph/0307240.
13. R. Das and A. Kumar, Phys. Rev. A **68**, 032304 (2003).
14. V. P. Shauro, D. I. Pekhterev, and V. E. Zobov, Izv. Vyssh. Uchebn. Zaved., Ser Fiz. **6**, 41 (2007).

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