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PHYSICAL OPTICS

Spectral Properties of a One-Dimensional Resonant Photonic Crystal

S. Ya. Vetrov[a], I. V. Timofeev[b], and A. Yu. Kutukova[a]

[a] *Siberian Federal University, Krasnoyarsk, 660074 Russia*
[b] *Kirensky Institute of Physics, Siberian Branch, Russian Academy of Sciences, Akademgorodok, Krasnoyarsk, 660036 Russia*
e-mail: tiv@iph.krasn.ru

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Abstract—The eigenexcitation and transmission spectra of a one-dimensional resonant photonic crystal are studied for TM and TE polarized electromagnetic waves. The crystal considered is a layered structure consisting of alternating isotropic layers and layers of a resonantly absorbing gas. The performed calculations show that the band structure of the spectra of the resonant photonic crystal significantly changes as the angle of incidence and the density of the resonant gas are varied. The structure of the spectra of additional transmission in the bandgap of the resonant photonic crystal is studied taking into account the decay of electromagnetic modes. The possibilities of controlling the spectrum of electromagnetic modes are indicated.

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INTRODUCTION

Photonic crystals, whose dielectric properties change periodically such that Bragg diffraction of light is possible, attract interest as new optical materials that possess unique properties [1–4]. Due to the occurrence of photonic bandgaps, where the density of photonic states is low, photonic crystals allow one to implement a number of interesting regimes of propagation of electromagnetic waves. For example, in three-dimensional photonic crystals, bandgaps can occur such that light of any polarization cannot enter into or emerge from them in any direction. Another important property of photonic crystals is a high degree of localization of electromagnetic waves at lattice defects. In this case, defect energy levels arise in bandgaps of photonic crystals. An atom can emit a quantum at a frequency corresponding to a defect level.

Based on photonic-crystal-materials, new types of waveguides have been created [5], methods for increasing the efficiency of nonlinear-optical processes have been proposed [6–8], and ideas on the development of hardware components of optoelectronic industry and information technologies are discussed [2].

The spectral properties of photonic crystals can also be changed significantly by placing resonant media (atoms or molecules) inside a periodic structure. We will further refer to this type of structure as a "resonant photonic crystal." The simplest resonant photonic crystals are layered structures that consist of alternating layers of two materials, one of which may be a resonantly absorbing gas. In the case of normal incidence of light, the spectral properties of such crystals have been studied in [9, 10]. Combining the resonant dispersion of a gas with the dispersion of a photonic-bandgap structure leads to qualitative changes in the spectra of photonic crystals. Thus, transmission bands appear in the photonic bandgap and additional bandgaps at the bandgap edge of the photonic-bandgap structure.

In this work, in contrast to [9, 10], we study particular features of the dispersion properties of an infinite photonic-bandgap structure, which arise as a result of a resonant dispersion of a gas in the case of an oblique incidence of radiation. We also study the effect of the resonant light absorption on the transmission spectrum upon variation of the angle of incidence, as well as of the parameters of the photonic-bandgap structure and resonant gas.

BAND STRUCTURE OF SPECTRUM OF EIGENEXCITATIONS OF AN INFINITE PHOTONIC CRYSTAL WITH RESONANT GAS

Consider a photonic crystal whose structure is an infinite layered medium one of the alternating layers of which is a resonant gas. The structure under consideration (Fig. 1) is characterized by the dielectric permittivities ε_1 and $\varepsilon_2(\omega)$ of the isotropic medium and resonant gas, respectively. The thicknesses of the layers are d_1 and d_2 and the period of the structure is $L = d_1 + d_2$.

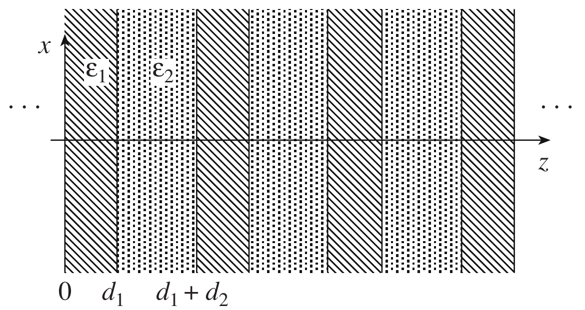

Fig. 1. Schematic of a periodic layered structure.

According to the Lorentz model, the complex dielectric permittivity of a medium is given by [11]

$$\varepsilon_2 = 1 + \frac{\omega_p^2(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2} - i\frac{\omega_p^2\gamma\omega}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}. \quad (1)$$

Here, $\omega_p^2 = 4\pi N f e^2/m$, where e is the electron charge, m is the electron mass, N is the density of resonant atoms, and f is the oscillator strength; γ is the linewidth; ω_0 is the central frequency of the resonance; and ω is the radiation frequency. In the case of TM waves (p polarization), the dispersion relation for this structure is given by the well-known expression [3, 12], the structure of which remains the same if the dispersion of the medium is taken into account,

$$\begin{aligned} \cos(k_z L) &= \cos(q_1 d_1)\cos(q_2 d_2) \\ &- (1/2)(\varepsilon_1 q_1 \varepsilon_2 q_2)^{-1}[(\varepsilon_1 q_2)^2 + (\varepsilon_2 q_1)^2] \\ &\times \sin(q_1 d_1)\sin(q_2 d_2). \end{aligned} \quad (2)$$

Here, $q_i^2 = c^{-2}\omega^2\varepsilon_i - k_x^2$, where $i = 1, 2$, c speed of light, and k_x and k_z are the components of the wave vector $\mathbf{k}$.

In this section, eigenexcitations of an infinite photonic crystal were studied taking into account mainly only the real part of dielectric permittivity (1). This simplification is a good approximation for bands whose spectral width exceeds the resonance linewidth γ. Dispersion equation (2) was solved numerically. For these numerical calculations, the parameters of the photonic crystal were chosen to be close to those used in [9]; namely, $\varepsilon_1 = 3.24$, $d_1\sqrt{\varepsilon_1} = d_2$, and $L = d_1 + d_2 = 100$ nm. Hg was chosen as a resonant gas, for which $\gamma/\omega_G = 5 \times 10^{-7}$, $\omega_p^2/\omega_G^2 = 7 \times 10^{-8}$, where $\omega_G = \pi c/L_0$ is the characteristic frequency of the bandgap, and $L_0 = d_1\sqrt{\varepsilon_1} + d_2$ is the optical thickness. The width of the resonance line at $\lambda_0 \approx 250$ nm is $\gamma \approx 10^9$ Hz.

Figure 2 shows a typical seed band structure of an infinite photonic crystal in the plane ωk_x for TM waves. The frequency ω is normalized to ω_G and the wave vector k_x is normalized to $k_G = \pi/L_0$.

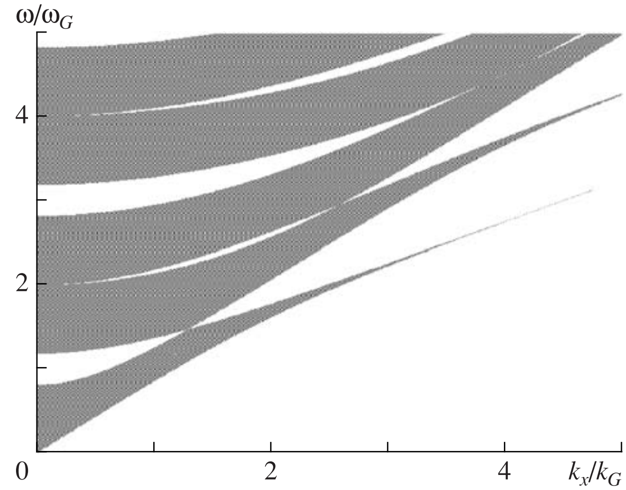

Fig. 2. Band structure of a photonic crystal in the ωk_x plane for TM waves. Bright areas correspond to bandgaps; ω is measured in units of $\omega_G = c\pi/L_0$, and k_x is expressed in units of $k_G = \pi/L_0$; $\varepsilon_1 = 3.24$, $\varepsilon_2 = 1$, $L = d_1 + d_2 = 100$ nm, $d_1\sqrt{\varepsilon_1} = d_2$, and $L_0 = d_1\sqrt{\varepsilon_1} + d_2$.

Taking into account the frequency dispersion of the real part of dielectric permittivity (1) leads to qualitative changes in the band structure of the seed spectrum. The combination of the dispersion of the photonic-bandgap structure with the resonant dispersion of the gas can lead to the appearance of an additional bandgap in the spectrum of the resonant photonic crystal. This effect is illustrated in Fig. 3. It can be seen that, in the presented fragment of the spectrum of the photonic crystal, an additional bandgap appears if the resonance frequency of the gas $\omega_0 = 1.19\omega_G$ is at the edge of the first bandgap.

A different situation occurs if the resonance frequency lies in the bandgap of a photonic crystal. Figure 4 shows a fragment of the spectrum for the case where the resonance frequency $\omega_0 = 1.18\omega_G$ is within the bandgap. In this case, an additional narrow transmission band appears in the bandgap, with the width of this band exceeding the resonance linewidth by an order of magnitude. By varying the parameters of the photonic-crystal structure and resonant gas, one can control the widths of the additional bandgap and transmission band. For example, an increase in the density of the resonant gas, all the other parameters of the resonant photonic crystal being unchanged, leads to an increase in the width of the additional transmission band.

A comparison of Figs. 5 and 4 at the wave vector $k_x = 0$ shows that a threefold increase in the gas density leads to a threefold increase in the width of the transmission band. For nonzero projections of k_x, transmission bandwidths also differ markedly.

Taking into account not only the real, but also the imaginary part of dielectric permittivity (1), results in

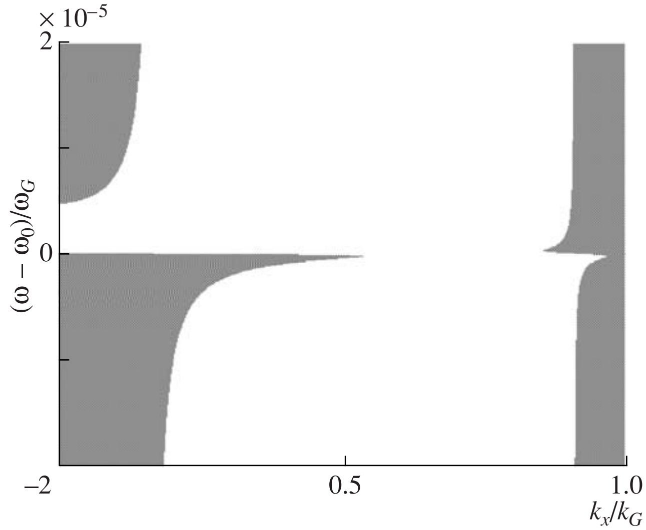

Fig. 3. Fragment of the band structure shown in Fig. 2. The resonance frequency is at the band edge, $\omega_0 = 1.19\omega_G$, $\gamma = 5 \times 10^{-7}\omega_G$, $\omega_p^2 = 7 \times 10^{-8}\omega_G^2$, and the atomic density of the resonant gas is $N = 4 \times 10^{14}$ cm^{-3}.

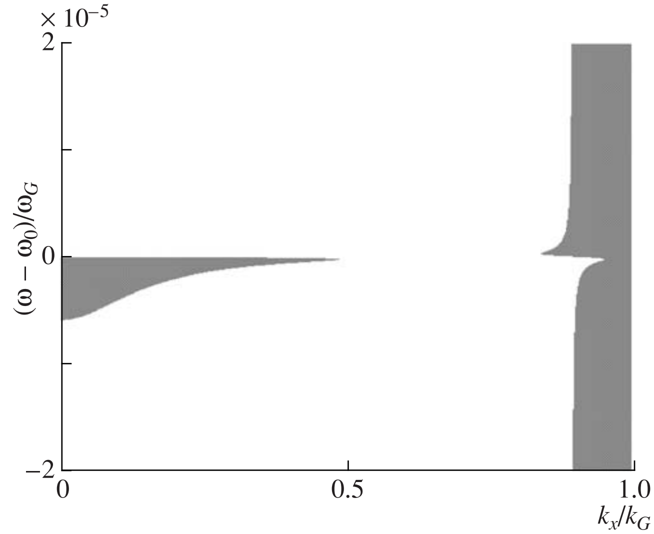

Fig. 4. Fragment of the band structure. The resonance frequency in the bandgap is $\omega_0 = 1.18\omega_G$, and the remaining parameters are the same as in Fig. 3.

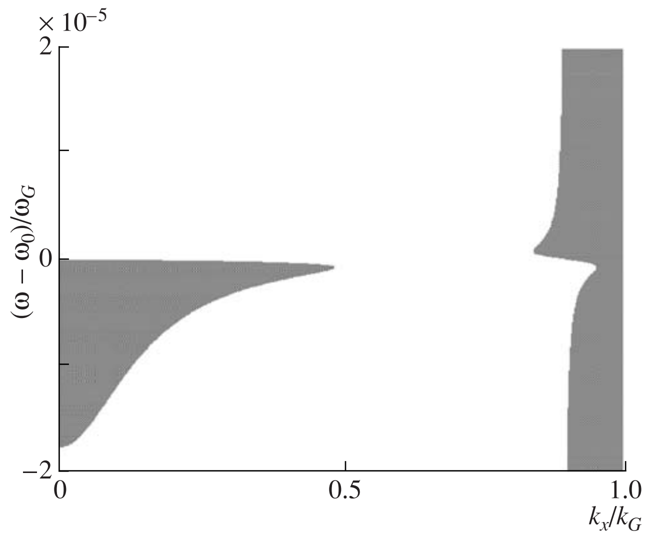

Fig. 5. Fragment of the band structure. The density of resonant atoms is $N = 1.2 \times 10^{15}$ cm^{-3}, and the remaining parameters are the same as in Fig. 4.

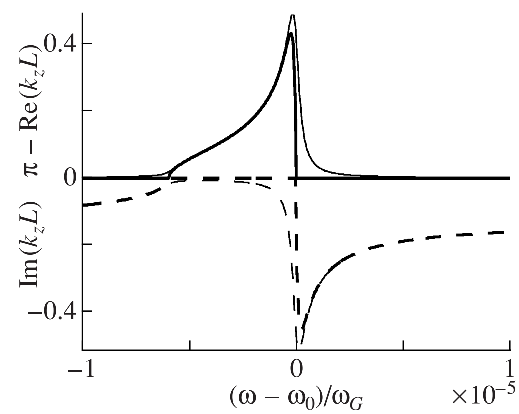

Fig. 6. Dependences of the real and imaginary parts of the wave-vector projection k_z on the frequency detuning ω from the resonance frequency of the gas for the case of the normal incidence of the radiation. The thick solid and dashed curves are the dependences for a nonabsorbing resonant photonic crystal ($\varepsilon_2(\omega) = \mathrm{Re}\varepsilon_2(\omega)$). The thin solid and dashed curves are the dependences for an absorbing resonant photonic crystal ($\varepsilon_2(\omega) = \mathrm{Re}\varepsilon_2(\omega) + i\mathrm{Im}\varepsilon_2(\omega)$). The parameters of the system are the same as in Fig. 4.

the wavenumber k_z of an absorbing resonant photonic crystal becoming complex, not only for frequencies lying in bandgaps, but also for frequencies in the allowed bands. Figure 6 presents characteristic dependences of the real and imaginary parts of k_z on the frequency detuning ω from the resonance frequency of the gas ω_0 when this frequency lies in the first bandgap and the radiation is normally incident on the crystal. For comparison, these dependences are presented for nonabsorbing and absorbing resonant photonic crystals with $\mathrm{Im}\varepsilon_2 = 0$ and $\mathrm{Im}\varepsilon_2 \neq 0$, respectively. It can be seen from the figure that taking into account the imaginary part of dielectric permittivity (1) leads to a narrowing of the additional transmission band in the bandgap of the resonant photonic crystal as a result of a significant absorption near the resonance frequency ω_0. Upon shifting from the resonance frequency ($\omega < \omega_0$), the imaginary part of the wavenumber k_z becomes much smaller its real part and, therefore, modes of the transmission band will be weakly decay. Finally, we note

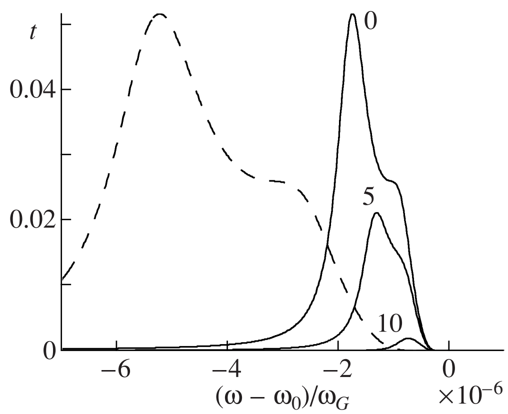

Fig. 7. Frequency dependences of the transmission coefficient $t(\omega)$ of an absorbing resonant photonic crystal. The solid curves are presented for the angle of incidence $\theta = 0°$, $5°$, and $10°$. The density of resonant atoms is $N = 4 \times 10^{14}$ cm^{-3}. The dashed curve was calculated for the normal incidence of the radiation. The density of resonant atoms, $N = 1.2 \times 10^{15}$ cm^{-3}, and the resonance linewidth, $\gamma = 1.5 \times 10^{-6}\omega_G$, are increased threefold. The resonant photonic crystal under study contains 30 periods and is 3 μm thick. The remaining parameters are the same as in Fig. 4.

that the spectral features of a resonant photonic crystal described for TM waves also are preserved in the case of TE waves.

TRANSMISSION SPECTRUM

The transmission spectrum of a finite resonant photonic crystal (Fig. 1), whose structure is characterized by the dielectric permittivity ε_1 and the complex dielectric permittivity $\varepsilon_2(\omega)$ given by (1) will be studied by the transfer-matrix method [13, 14]. For the structure under study, the distribution of the electric field in each layer has the form

$$E_x(n, t) = [A_n e^{i\alpha_n(z - z_n)} + B_n e^{-i\alpha_n(z - z_n)}]e^{-i\omega t}, \qquad (3)$$

where A_n and B_n are, respectively, the amplitudes of the incident and reflected waves in the nth layer $\alpha_n = \omega/c\sqrt{\varepsilon_n - \sin^2\theta}$, and θ is the angle of incidence of the radiation. From the continuity of the electric E_x and magnetic H_y fields at the interface $z = z_{n-1}$ between the layers, we obtain the system of equations, which can be represented as a matrix equation

$$\begin{pmatrix} A_{n-1} \\ B_{n-1} \end{pmatrix} = T_{n-1,n}\begin{pmatrix} A_n \\ B_n \end{pmatrix}, \qquad (4)$$

where

$$T_{n-1,n} = \frac{1}{2}\begin{pmatrix} (1+h)e^{-i\alpha_n\gamma_n} & (1-h)e^{i\alpha_n\gamma_n} \\ (1-h)e^{-i\alpha_n\gamma_n} & (1+h)e^{i\alpha_n\gamma_n} \end{pmatrix}. \qquad (5)$$

is the transfer-matrix. Here, $h = \sqrt{\varepsilon_n - \sin^2\theta}/\sqrt{\varepsilon_{n-1} - \sin^2\theta}$, and $\gamma_n = z_n - z_{n-1}$, where $n = 1, 2, \ldots, N$, is the thickness of the nth layer. Equation (4) yields the following relation of the amplitudes A_0 and B_0 of the waves incident on the resonant photonic crystal and reflected from it with the amplitude A_s of the wave emerged from the crystal provided that the reflection of waves from the right-hand side of the resonant photonic crystal does not occur ($B_s = 0$):

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \hat{M}\begin{pmatrix} A_s \\ 0 \end{pmatrix}. \qquad (6)$$

Here,

$$\hat{M} = \hat{T}_{01}\hat{T}_{12}\ldots\hat{T}_{N-1,N}\hat{T}_{N,S}, \qquad (7)$$

where $S = N + 1$, and $\gamma_{N+1} = 0$. Finally, using (7), we obtain the transmission coefficient

$$t(\omega) = 1/|\hat{M}_{11}|^2, \qquad (8)$$

where $\hat{M}_{11}$ is the element of the matrix $\hat{M}$.

Let us now study the particular features of the transmission spectrum of a finite resonant photonic crystal by numerically solving Eq. (8) for the transmission coefficient. Figure 7 shows the frequency dependences of the transmission coefficient of a finite absorbing resonant photonic crystal. The solid transmission curves for various angles of incidence were obtained under the same other parameters of the system as in Fig. 6. It can be seen from a comparison of these figures that, under normal incidence, the width of the frequency range of the additional transmission in Fig. 7 is consistent with the additional transmission band of the infinite resonant photonic crystal. We also note that the transmission coefficient is highly sensitive to the angle of incidence θ. Thus, as θ was increased to 10°, the transmission coefficient at the maximum of the transmission curve decreased by an order of magnitude.

In the case of the impact broadening mechanism [15], as the density of the resonant gas is increased threefold, the decay γ also increases threefold. The spectrum of the additional transmission band (dashed curve in Fig. 6) shifts away from the resonance, and the bandwidth increases threefold. The transmission coefficient at the maximum does not change.

As the resonance frequency ω_0 approaches the edge of the bandgap, with the other parameters of the system being unchanged, the transmission increases significantly; for example, at $\omega_0 = 1.185\omega_G$, it reaches 50%.

CONCLUSIONS

We found that the combination of the dispersion of a photonic-bandgap structure with the dispersion of a resonant gas leads to qualitative changes in the spectral properties of the resonant photonic crystal. We showed that there are real possibilities for changing the additional transmission curves in the resonant photonic crystal bandgap by varying the density of the resonant gas and the position of the resonance frequency with respect to the bandgap edge. The transmission can also be efficiently controlled by varying the angle of incidence of the laser radiation. In practical applications, these resonant photonic crystals can be promising in the creation of narrowband filters with tunable characteristics and in the construction of new types of optical devices.

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REFERENCES

1. J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals* (Princeton Univ. Press, Princeton, 1995).
2. K. Busch, S. Lölkes, R. B. Wehrspohn, and H. Foll, *Photonic Crystals: Advances in Design, Fabrication and Characterization* (Wiley-VCH, Weinheim, 2004).
3. A. Yariv and P. Yeh, *Optical Waves in Crystals: Propagation and Control of Laser Radiation* (Wiley, New York, 1984; Mir, Moscow, 1987).
4. V. F. Shabanov, S. Ya. Vetrov, and A. V. Shabanov, *Optics of Real Crystals: Liquid-Crystalline Defects, and Inhomogeneities* (Sib. Otd. Ross. Akad. Nauk, Novosibirsk, 2005) [in Russian].
5. A. M. Zheltikov, Usp. Fiz. Nauk **170** (11), 1203 (2000).
6. O. Painter, R. Zec, A. Yariv, et al., Science **264**, 1819 (1999).
7. M. G. Martem'yanov, T. V. Dolgova, and A. A. Fedyanin, Zh. Éksp. Teor. Fiz. **125**, 527 (2004).
8. F. Wong, S. N. Zhu, K. F. Li, et al., Appl. Phys. Lett. **88**, 071102 (2006).
9. A. M. Zheltikov, A. N. Naumov, P. Barker, and R. B. Maĭls, Opt. Spektrosk. **89** (2), 309 (2000).
10. M. Artoni, G. Rossa, and V. Bassani, Phys. Rev. E **72**, 046604 (2005).
11. S. A. Akhmanov and S. Yu. Nikitin, *Physical Optics* (Mosk. Gos. Univ., Moscow, 1998) [in Russian].
12. Shi Hang and Tsai Chie-hua, Sol. Stat. Commun. **52** (12), 953 (1984).
13. P. Yeh, J. Opt. Soc. Am. **69** (5), 742 (1979).
14. S. Ya. Vetrov, I. V. Timofeev, and A. V. Shabanov, Phys. Stat. Sol. (RRL) **1** (3), 92 (2007).
15. *Physical Encyclopedia* (Bol'sh. Ross. Éntsiklopedia, Moscow, 1998) [in Russian].

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