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PHYSICAL ACOUSTICS

Reflection and Refraction of Bulk Acoustic Waves in Piezoelectrics Under Uniaxial Stress

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Abstract—Basic equations that describe the conditions of reflection and refraction of bulk acoustic waves at the boundary between acentric crystalline media under uniaxial stress are presented. The effect of uniaxial stress on the anisotropy of reflection and refraction of bulk acoustic waves is numerically analyzed for boundaries of the crystal–vacuum, piezoelectric crystal–piezoelectric crystal, and piezoelectric crystal–elastic isotropic medium types.

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INTRODUCTION

The theory of reflection and refraction of bulk acoustic waves (BAWs) at the boundary between two crystalline media was described in detail in [1, 2]. This theory has been used to develop a number of acousto-electronic devices, including delay lines. The influence of homogeneous external electric field on the characteristics and conditions of reflection and refraction of BAWs at the boundary between piezoelectric crystals was considered in [3, 4]. The consideration was based on the theory of BAW propagation in piezoelectric crystals under the effects of external electric field and mechanical stress [5]. The influence of external electric field **E** and uniaxial stress **P** on the propagation of surface acoustic waves (SAWs) in piezoelectric crystals was studied in [6].

THEORY OF REFLECTION AND REFRACTION OF BULK ELASTIC WAVES AT THE BOUNDARY BETWEEN TWO PIEZOELECTRIC CRYSTALS UNDER UNIAXIAL STRESS

Using the theory developed in [5], we derive the basic equations describing the effect of **P** on the conditions of reflection and refraction of BAWs at the boundary between two piezoelectric media. In the initial coordinate system, the wave equations for small-amplitude waves in homogeneously strained acentric media and the electrostatic equation have the form [4]

$$\rho_0 \ddot{\tilde{U}}_A = \tilde{\tau}_{AB,B} + \tilde{U}_{A,FB}\bar{\tau}_{FB},\qquad \tilde{D}_{M,M} = 0. \tag{1}$$

Here, ρ_0 is the density of the crystal in the strain-free (initial) state, $\tilde{U}_A$ is the dynamic elastic displacement vector, τ_{AB} is the thermodynamic stress tensor, D_M is the electric induction vector, $\bar{\tau}_{FB} = -\bar{\tau} P_F P_B$ is the static uniaxial stress tensor (the minus sign corresponds to compression), and P_B is the unit vector in the direction of the pressure force. The tilde indicates quantities depending on time. A comma after an index denotes a spatial derivative, and the Latin coordinate indices take on the values from 1 to 3. Here and below, summation over a doubly repeated index is implied.

When the effect of **P** is taken into account, the equations of state for the dynamic components of thermodynamic stresses and the electric induction have the form

$$\tilde{\tau}_{AB} = C^*_{ABCD}\tilde{\eta}_{CD} - e^*_{NAB}\tilde{E}_N,\qquad \tilde{D}_N = e^*_{NAB}\tilde{\eta}_{AB} + \varepsilon^*_{NM}\tilde{E}_M, \tag{2}$$

where η_{AB} is the strain tensor and the elastic, piezoelectric, and dielectric effective constants are determined by the formulas

$$C^*_{ABKL} = C^E_{ABKL} - C^E_{ABKLQR}S^E_{QRMN}P_M P_N\bar{\tau},$$
$$e^*_{NAB} = e_{NAB} - e_{NABKL}S^E_{KLMD}P_M P_D\bar{\tau}, \tag{3}$$
$$\varepsilon^*_{NM} = \varepsilon^\eta_{NM} - H_{NMAB}S^E_{ABKL}P_K P_L\bar{\tau}.$$

Here, $\bar{\tau}$ is the static mechanical stress; S^E_{ABKL} is the elastic compliance tensor; and C^E_{ABKLQR}, e_{NABKL}, and H_{NMAB} are the elastic, piezoelectric, and electrostrictive nonlinear materials tensors.

To consider the problem of reflection and refraction of BAWs at the boundary between two acentric media, it is expedient to use an orthogonal coordinate system with the X_3' and X_1' axes directed along the normal to the boundary and along the boundary, respectively. We assume that an elastic wave is incident on the boundary from the side of the crystal that occupies the space $X_3' < 0$. We seek the solution to the wave equation in the form of plane waves. To study the conditions of reflection and refraction of waves, it is convenient to use the formulas for a plane elastic harmonic wave and an electric potential wave expressed in terms of the refraction vectors $\mathbf{m} = \mathbf{N}/v$ (where $\mathbf{N}$ is the unit vector of the wave normal and v is the phase velocity of BAWs):

$$\begin{aligned} U_{\mathrm{C}} &= \alpha_{\mathrm{C}} \exp[i\omega(t - m_j x_j)], \\ \varphi &= \alpha_4 \exp[i\omega(t - m_j x_j)]. \end{aligned} \tag{4}$$

Here, α_{C} and α_4 are the elastic displacement and electric potential amplitudes of the bulk wave.

Substituting Eqs. (4) into Eqs. (1) and retaining only the terms linear in $\mathbf{P}$, we obtain a system of four homogeneous equations

$$[\Gamma_{\mathrm{BC}} - \delta_{\mathrm{BC}}\rho_0]\tilde{U}_{\mathrm{B}} = 0, \tag{5}$$

where δ_{BC} is the Kronecker delta and the components of the modified Green–Christoffel tensor Γ_{BC} have the form

$$\begin{aligned} \Gamma_{\mathrm{BC}} &= (C^*_{\mathrm{ABCD}} + (2C^{\mathrm{E}}_{\mathrm{MBFN}} S^{\mathrm{E}}_{\mathrm{ADCF}} \\ &\quad + \delta_{\mathrm{BC}}\delta_{\mathrm{AM}}\delta_{\mathrm{DN}}) P_{\mathrm{M}} P_{\mathrm{N}} \bar{\tau}) m_{\mathrm{A}} m_{\mathrm{D}}, \\ \Gamma_{\mathrm{B4}} &= e^*_{\mathrm{IAB}} m_{\mathrm{I}} m_{\mathrm{A}}, \\ \Gamma_{\mathrm{4B}} &= \Gamma_{\mathrm{B4}} + 2e_{\mathrm{AFD}} S^{\mathrm{E}}_{\mathrm{MNCF}} P_{\mathrm{M}} P_{\mathrm{N}} m_{\mathrm{A}} m_{\mathrm{D}} \bar{\tau}, \\ \Gamma_{44} &= -\varepsilon^*_{\mathrm{MI}} m_{\mathrm{M}} m_{\mathrm{I}}. \end{aligned} \tag{6}$$

The determinant of the system of four homogeneous equations (5) has the form of a fourth-degree polynomial in the refraction vector component m_3 for each of the contacting media and for both reflected and refracted bulk acoustic waves at a fixed direction of the incident wave. In the general case, the m_3 components of reflected and refracted waves can be complex-valued due to the total internal reflection [2]. In this case, for the crystal occupying the lower half-space $X_3' < 0$ (reflected waves), they should have a negative imaginary part, whereas for the crystal occupying the upper half-space $X_3' > 0$ (refracted waves), they should have a positive imaginary part. This ensures the attenuation of the reflected or refracted waves deep in the corresponding media.

Determination of the refraction vectors $\mathbf{m}$ allows us to obtain the values of the angles of reflection and refraction of BAWs and the corresponding phase velocities. Important parameters are the amplitude coefficients of the reflected and refracted waves, which characterize the distribution of the incident wave energy between the reflected and refracted BAWs. To determine these coefficients, it is necessary to formulate the boundary conditions for the reflection and refraction of BAWs. In the case of a rigid acoustic contact of two crystals, the boundary conditions for the thermodynamic stress tensor are reduced to continuity of the normal components of stress tensors for the reflected or refracted BAWs and to continuity of the displacement vectors of the waves [2]. In view of the piezoelectric properties of the crystals, it is also necessary to formulate the boundary conditions for the potential wave. The continuity of the components of electric field $\mathbf{E}$ that are tangential to the boundary is ensured by the continuity condition for the electric potential φ and the continuity condition for the normal components of the induction vector $\mathbf{D}$. Thus, in the quasistatic limit, the elastic and electric boundary conditions have the form

$$\begin{aligned} \tau^{[\mathrm{I}]}_{ik} n_k &= \tau^{[\mathrm{II}]}_{ik} n_k, \\ \mathbf{U}^{[\mathrm{I}]} &= \mathbf{U}^{[\mathrm{II}]}, \\ \varphi^{[\mathrm{I}]} &= \varphi^{[\mathrm{II}]}, \\ (\mathbf{D}^{[\mathrm{I}]}, \mathbf{n}) &= (\mathbf{D}^{[\mathrm{II}]}, \mathbf{n}), \end{aligned} \tag{7}$$

where n_k is the unit vector in the direction normal to the boundary. In Eqs. (7) and in what follows, the superscript I refers to the crystal occupying the half-space $X_3' > 0$ and the superscript II to the crystal occupying the half-space $X_3' < 0$. Substituting solutions (3) into Eqs. (7) and retaining only the terms that are linear in $\mathbf{P}$, we arrive at a system of linear equations in eight amplitude coefficients of reflected and refracted waves:

$$\begin{aligned} \sum_{n=1}^{4} (b_n G^{(n)[\mathrm{I}]}_{\mathrm{B}} - a_n G^{(n)[\mathrm{II}]}_{\mathrm{B}}) &= \hat{G}^{(n)[\mathrm{II}]}_{\mathrm{B}}, \\ \sum_{n=1}^{4} (U^{(n)[\mathrm{I}]}_{\mathrm{B}} b_n - U^{(n)[\mathrm{II}]}_{\mathrm{B}} a_n) &= \hat{U}^{(n)[\mathrm{II}]}_{\mathrm{B}}, \\ \sum_{n=1}^{4} (b_n D^{(n)[\mathrm{I}]}_3 - a_n D^{(n)[\mathrm{II}]}_3) &= \hat{D}^{(n)[\mathrm{II}]}_3, \end{aligned} \tag{8}$$

where a_n are the amplitude coefficients of reflection, b_n are the amplitude coefficients of refraction, the sign "^" indicates quantities corresponding to the incident wave, and the following notation is used:

$$\begin{aligned} G^{(n)[\mathrm{I,\,II}]}_{\mathrm{B}} &= (C^{*[\mathrm{I,\,II}]}_{\mathrm{B3KL}} \delta_{\mathrm{KP}} - 2S^{[\mathrm{I,\,II}]}_{\mathrm{KPMN}} C^{[\mathrm{I,\,II}]}_{\mathrm{3BKL}} P_{\mathrm{M}} P_{\mathrm{N}} \bar{\tau}) \\ &\quad \times m^{(n)}_{\mathrm{L}} \alpha^{(n)}_{\mathrm{P}} - e^{*[\mathrm{I,\,II}]}_{\mathrm{P3B}} m^{(n)}_{\mathrm{P}} \alpha^{(n)}_4 + m^{(n)}_{\mathrm{P}} \alpha^{(n)}_{\mathrm{B}} P_3 P_{\mathrm{P}} \bar{\tau}, \end{aligned}$$

$$D_3^{(n)[\mathrm{I,II}]} = (e^{*[\mathrm{I,II}]}_{3\mathrm{AB}} + 2S^{[\mathrm{I,II}]}_{\mathrm{ABKP}} e^{[\mathrm{I,II}]}_{3\mathrm{AB}} P_\mathrm{F} P_\mathrm{P} \bar{\tau}) \times m_\mathrm{B}^{(n)} \alpha_\mathrm{A}^{(n)} - \varepsilon^{*[\mathrm{I,II}]}_{3\mathrm{K}} m_\mathrm{K}^{(n)} \alpha_4^{(n)},$$

$$\hat{G}_\mathrm{B}^{[\mathrm{II}]} = (C^{*[\mathrm{II}]}_{\mathrm{B3KL}} \delta_\mathrm{KP} - 2S^{[\mathrm{II}]}_{\mathrm{KPMN}} C^{[\mathrm{II}]}_{3\mathrm{BKL}} P_\mathrm{M} P_\mathrm{N} \bar{\tau}) \hat{m}_\mathrm{L} \hat{\alpha}_\mathrm{P} - e^{*[\mathrm{II}]}_{\mathrm{P3B}} \hat{m}_\mathrm{P} \hat{\alpha}_4 + \hat{m}_\mathrm{P} \hat{\alpha}_\mathrm{B} P_3 P_\mathrm{P} \bar{\tau}, \tag{9}$$

$$\hat{D}_3^{[\mathrm{II}]} = (e^{*[\mathrm{II}]}_{3\mathrm{AB}} + 2S^{[\mathrm{II}]}_{\mathrm{ABKP}} e^{[\mathrm{II}]}_{3\mathrm{AB}} P_\mathrm{F} P_\mathrm{P} \bar{\tau}) \times \hat{m}_\mathrm{B} \hat{\alpha}_\mathrm{A} - \varepsilon^{*[\mathrm{II}]}_{3\mathrm{K}} \hat{m}_\mathrm{K} \hat{\alpha}_4.$$

In Eqs. (8) and (9), the indices $n = 1, \ldots, 4$ denote the wave types (longitudinal $n = 1$ and transverse $n = 2, 3$) for the reflected and refracted waves; $n = 4$ corresponds to the electric potential wave.

If we consider the reflection of a wave from a crystal–vacuum boundary, we should change the boundary conditions. In this case, for mechanical quantities, stresses on the crystal surface must be absent: $\sum \tau_{3\mathrm{J}} = 0|_{X_3 = 0}$. However, if stress is applied in the direction orthogonal to the free surface, the elastic properties of the loading medium should be taken into account. Nevertheless, if we hypothetically assume that, in the given geometry, uniaxial stress occurs without any rigid elastic contact with the free surface, the mechanical boundary conditions for this case can be written in the form $\tilde{\tau}_{3\mathrm{J}} + \tilde{U}_{\mathrm{J,K}} \tilde{\tau}_{3\mathrm{K}} = 0|_{X_3 = 0}$. For electric quantities, the boundary conditions include continuity of the normal component of electric induction at the crystal–vacuum boundary and the satisfaction of the Laplace equation for the potential wave in vacuum. For example, the system of linear equations for determining four amplitude coefficients of reflection has the form

$$\sum_{n=1}^{3} -a_n \{ (C^{*[2]}_{\mathrm{B3KL}} \delta_\mathrm{KP} - 2S^{[2]}_{\mathrm{KPMN}} C^{[2]}_{3\mathrm{BKL}} P_\mathrm{M} P_\mathrm{N} \bar{\tau}) m_\mathrm{L}^{(n)} \alpha_\mathrm{P}^{(n)} + m_\mathrm{P}^{(n)} \alpha_\mathrm{B}^{(n)} P_3 P_\mathrm{P} \bar{\tau} \} + a_4 e^{*[2]}_{\mathrm{P3B}} m_\mathrm{P}^{(4)} \alpha_4^{(4)} = (C^{*[2]}_{\mathrm{B3KL}} \delta_\mathrm{KP} - 2S^{[2]}_{\mathrm{KPMN}} C^{[2]}_{3\mathrm{BKL}} P_\mathrm{M} P_\mathrm{N} \bar{\tau}) \tilde{m}_\mathrm{L} \tilde{\alpha}_\mathrm{P} + \tilde{m}_\mathrm{P} \tilde{\alpha}_\mathrm{B} P_3 P_\mathrm{P} \bar{\tau} - a_4 e^{*[2]}_{\mathrm{P3B}} \tilde{m}_\mathrm{P} \tilde{\alpha}_4, \tag{10}$$

$$\sum_{n=1}^{3} a_n \{ (e^{*[2]}_{3\mathrm{AB}} + 2S^{[2]}_{\mathrm{ABKP}} e^{[2]}_{3\mathrm{AB}} P_\mathrm{F} P_\mathrm{P} \bar{\tau}) m_\mathrm{B}^{(n)} \alpha_\mathrm{A}^{(n)} \} - a_4 (\varepsilon^{*[2]}_{3\mathrm{K}} m_\mathrm{K}^{(n)} - i\varepsilon_0) \alpha_4^{(n)} = (e^{*[2]}_{3\mathrm{AB}} + 2S^{[2]}_{\mathrm{ABKP}} e^{[2]}_{3\mathrm{AB}} P_\mathrm{F} P_\mathrm{P} \bar{\tau}) \tilde{m}_\mathrm{B} \tilde{\alpha}_\mathrm{A} - \varepsilon^{*[2]}_{3\mathrm{K}} \tilde{m}_\mathrm{K} \tilde{\alpha}_4, \tag{11}$$

where ε_0 is the dielectric constant. Note that the expressions obtained above for the boundary conditions were derived for the case of a crystal undergoing a homogeneous uniaxial external mechanical stress without considering edge effects. The derived equations take into account all the changes in the configuration of the anisotropic continuous medium that are caused by its static strain and, in particular, changes in the crystal shape, such as stretching and rotation of element lines parallel to the ribs of the sample [5, 7].

CALCULATION OF THE EFFECT OF STRESS ON REFLECTION AND REFRACTION OF ACOUSTIC WAVES AT THE FREE BOUNDARY OF A PIEZOELECTRIC CRYSTAL

To compare the external effects on the BAW reflection properties, we consider as an example the effect of external uniaxial mechanical pressure on the reflection of waves from a free boundary of a cubic piezoelectric crystal with the 23 symmetry. We apply an approach analogous to that used for studying the effect of uniaxial electric field [3]. Let the incident wave propagate in the (010) plane (the sagittal plane) and the normal to the boundary be parallel to the [001] direction. For the case of a longitudinal (L) or fast shear (FS) incident wave, the dispersion equation for the reflected waves (in the absence of pressure) has the form

$$(C_{11}^\mathrm{E} m_1^2 + C_{44}^\mathrm{E} m_3^2 - \rho_0)(C_{44}^\mathrm{E} m_1^2 + C_{11}^\mathrm{E} m_1^2 - \rho_0) - (C_{12}^\mathrm{E} + C_{44}^\mathrm{E})^2 m_1^2 m_3^2 = 0. \tag{12}$$

For a slow shear (SS) incident wave, which, in the given sagittal plane is piezoactive with polarization along the [010] direction, i.e., polarized orthogonally to the plane of incidence, the dispersion equation has the form

$$(C_{44}^\mathrm{E}(m_1^2 + m_3^2) - \rho_0))\varepsilon_{11}^\eta (m_1^2 + m_3^2) - 4e_{14}^2 m_1^2 m_3^2 = 0. \tag{13}$$

Hence, when a longitudinal wave or a fast shear wave (polarized in the plane of incidence) is incident on the boundary, only longitudinal (quasi-longitudinal, QL) and fast shear (quasi-FS, QFS) waves will be reflected. In the case of a slow shear (QSS) incident wave, the reflection only occurs for the QSS wave, whose amplitude coefficient is approximately unity. However, because this wave is piezoactive, a potential wave is present in addition to the elastic QSS wave. Therefore, although the refraction vector of the reflected QSS wave is real, its amplitude coefficient will be complex, with its imaginary part characterizing the phase shift between the incident and reflected waves.

According to the Curie principle, uniaxial pressure applied to the crystal along the [100] direction reduces the crystal symmetry to the rhombic one of the 222 class, in contrast to the case of the application of an electric field, which reduces the symmetry of the crystal to the monoclinic one of class 2. As a result, we only obtain modification of the existing materials constants, specifically

$$\begin{aligned}
&C^*_{11} = C^E_{11} + [C_{111}S_{11} + S_{12}(C_{112} + C_{113})]\bar{\tau}; && e^*_{14} = e_{14} + [e_{114}S_{11} + S_{12}(e_{124} + e_{134})]\bar{\tau};\\
&C^*_{33} = C^E_{11} + [C_{112}S_{11} + S_{12}(C_{113} + C_{111})]\bar{\tau}; && e^*_{25} = e_{14} + [e_{134}S_{11} + S_{12}(e_{114} + e_{124})]\bar{\tau};\\
&C^*_{22} = C^E_{11} + [C_{113}S_{11} + S_{12}(C_{111} + C_{112})]\bar{\tau}; && e^*_{36} = e_{14} + [e_{124}S_{11} + S_{12}(e_{134} + e_{114})]\bar{\tau};\\
&C^*_{12} = C^E_{12} + [C_{112}S_{11} + S_{12}(C_{113} + C_{123})]\bar{\tau}; && \varepsilon^*_{11} = \varepsilon^\eta_{11} + [H_{11}S_{11} + S_{12}(H_{12} + H_{13})]\bar{\tau};\\
&C^*_{13} = C^E_{12} + [C_{113}S_{11} + S_{12}(C_{123} + C_{112})]\bar{\tau}; && \varepsilon^*_{22} = \varepsilon^\eta_{11} + [H_{11}S_{11} + S_{12}(H_{21} + H_{23})]\bar{\tau};\\
&C^*_{23} = C^E_{12} + [C_{123}S_{11} + S_{12}(C_{112} + C_{113})]\bar{\tau}; && \varepsilon^*_{33} = \varepsilon^\eta_{11} + [H_{11}S_{11} + S_{12}(H_{31} + H_{32})]\bar{\tau};\\
&C^*_{44} = C^E_{44} + [C_{144}S_{11} + S_{12}(C_{166} + C_{155})]\bar{\tau}; &&\\
&C^*_{55} = C^E_{44} + [C_{155}S_{11} + S_{12}(C_{144} + C_{166})]\bar{\tau}; &&\\
&C^*_{66} = C^E_{44} + [C_{166}S_{11} + S_{12}(C_{155} + C_{144})]\bar{\tau}; &&
\end{aligned} \tag{14}$$

In this case, changes only occur in the existing components of the Green–Christoffel tensor (6) for the (010) sagittal plane. The dispersion equation (12) for the reflected waves (under uniaxial pressure applied to the crystal along the [100] direction), which corresponds to the case of a longitudinal (QL) incident wave or a shear (QFS) wave polarized in the sagittal plane, can be represented as

$$\begin{aligned}
&[C^*_{11}m_1^2 + C^*_{44}m_3^2 + 2S_{11}(C^E_{11}m_1^2 + C^E_{44}m_3^2)\bar{\tau} + m_1^2\bar{\tau} - \rho_0]\\
&\times [(C^*_{44}m_1^2 + C^*_{33}m_1^2 + (2S_{12}(C^E_{44}m_1^2 + C^E_{11}m_1^2)\bar{\tau} - \rho_0)]\\
&\quad - [C^*_{13} + C^*_{55} + (2C^E_{12}S_{12} + 2C^E_{12}S_{12} + 1)\bar{\tau}]\\
&\times [C^*_{13} + C^*_{35} + (2C^E_{12}S_{11} + 2C^E_{44}S_{12})\bar{\tau}]m_1^2m_3^2 = 0.
\end{aligned} \tag{15}$$

Similarly, the dispersion equation (13), corresponding to the case of a shear (QSS) incident wave polarized orthogonally to the plane of incidence, i.e., along the [010] direction, takes the form

$$\begin{aligned}
&[C^*_{66}m_1^2 + C^*_{44}m_3^2 + 4C^E_{66}S_{12}(m_1^2 + m_3^2)\bar{\tau} - \rho_0]\\
&\quad \times (\varepsilon^*_{11}m_1^2 + \varepsilon^*_{33}m_3^2)\\
&- (e^*_{14} + e^*_{36})[e^*_{14} + e^*_{36} + 2e_{14}(S_{11} + S_{12})\bar{\tau}]m_1^2m_3^2 = 0.
\end{aligned} \tag{16}$$

Note that in Eqs. (15), (16), the terms that are related to the effect of external uniaxial pressure characterize all the changes in the configuration of the anisotropic continuous medium due to the static strain of the medium. Effects related to the change in the geometry of the crystal, which are taken into account in Eqs. (2)–(6), lead to the violation of the symmetry of the Green–Christoffel tensor.

Application of uniaxial pressure to the crystal along the [100] direction eliminates the degeneration of the tangential type with a Poincare index of ±1 along the [001]-type crystallographic axes [8, 9]. The acoustic axis splits into two conic axes with a Poincare index of ±1/2, but, in contrast to the effect of external electric field, which leads to splitting of the acoustic axis in the (110) plane, the pressure under consideration causes splitting in the (001) plane. Because of the splitting of the acoustic axis, the incidence of a longitudinal wave gives rise to a slow shear (QSS) reflected wave in the interval between the induced acoustic axes; i.e., an "exchange" of shear elastic waves occurs within the angle of the cone formed by the induced acoustic axes. This situation is most pronounced in the (110) plane when pressure is applied to the crystal along the [110] direction, i.e., along the boundary. In the (110) plane, two acoustic axes are present: the tangential one (along [001]), which is split in two, and a conic one (along [111]), which is displaced by the pressure **P**. The splitting evidently depends on the strength of the external action [10], but it should be noted that the cone angle of the induced acoustic axes is ±8.5° whereas the exchange of shear elastic waves occurs within an interval of ±20° of angles between the wave vector of the incident elastic wave and the normal to the boundary between the media. The pressure magnitude used in our calculations was taken to be 10^8 Pa. Thus, in our case, the effect of uniaxial stress is reduced to quantitative changes in the amplitude coefficients of the reflected waves.

According to Eqs. (14), for other variants of application of pressure **P** to the crystal and for other types of incident elastic waves, we obtain only quantitative changes in the amplitude characteristics of the reflected elastic waves.

CALCULATION OF THE EFFECT OF PRESSURE ON REFLECTION AND REFRACTION OF ACOUSTIC WAVES AT THE BOUNDARY BETWEEN AN ISOTROPIC MEDIUM AND A PIEZOELECTRIC

Figure 1 shows the real parts of the amplitude coefficients of reflection and refraction for the case of an acoustic contact between two media represented by

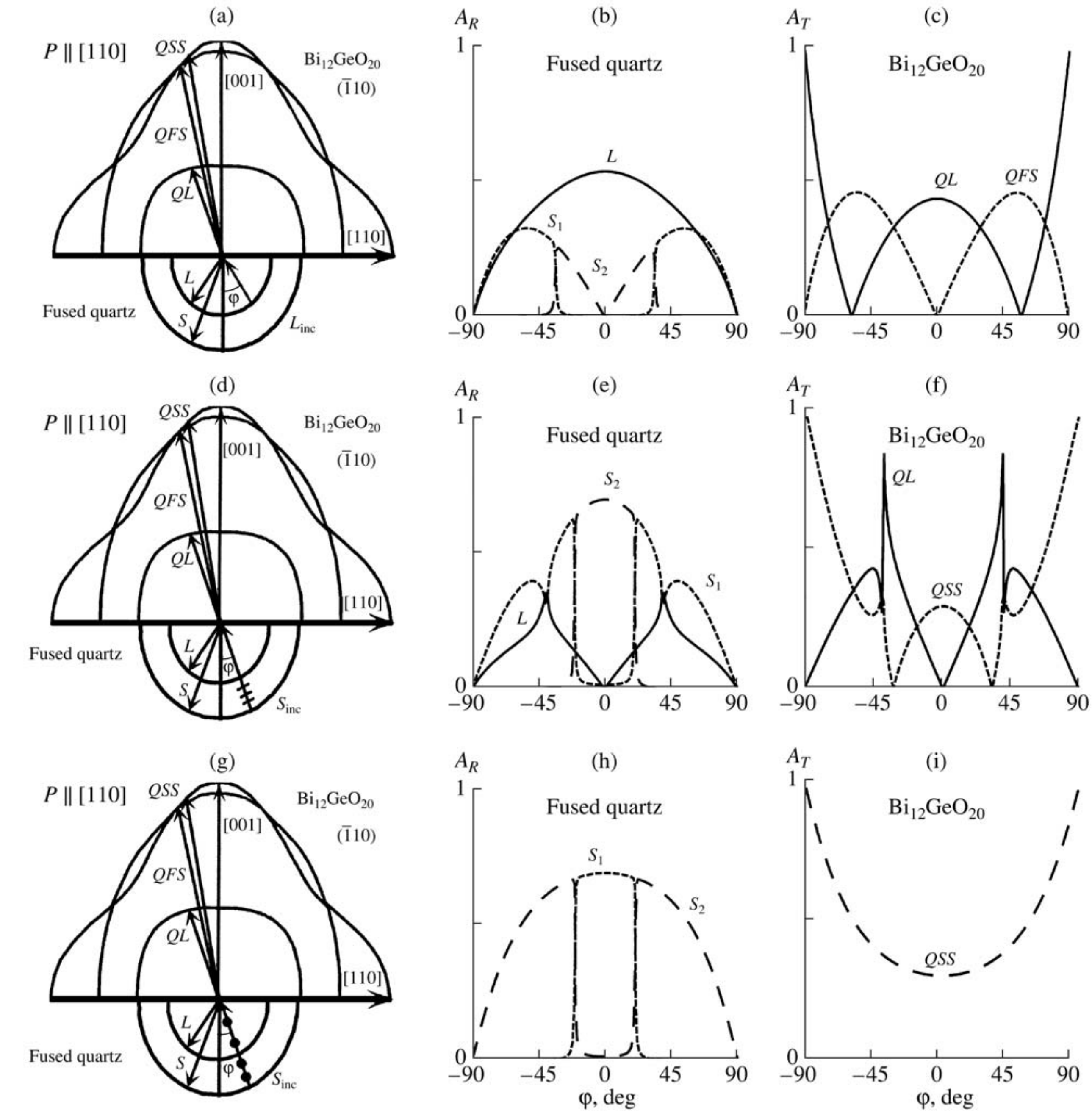

Fig. 1. (a) Patterns of refraction vectors and the amplitude coefficients of (b) reflection and (c) refraction of waves at the boundary between fused quartz and $Bi_{12}GeO_{20}$ in the (110) plane for the case of waves incident from the side of fused quartz in the presence of uniaxial pressure applied along the direction of propagation of the incident wave.

fused quartz and $Bi_{12}GeO_{20}$ (the $(\bar{1}10)$ plane) in the presence of external pressure **P** applied along the [110] direction lying in the boundary plane. In the absence of pressure, when a longitudinal wave or a shear wave polarized in the sagittal plane is incident on the boundary, only reflected and refracted longitudinal and fast shear (polarized in the plane of incidence) waves are present. When a shear wave polarized orthogonally to the sagittal plane is incident, a wave of the same type is reflected while only a slow shear wave is refracted (this situation was mentioned in [3]).

Application of uniaxial stress **P** || [110] to a $Bi_{12}GeO_{20}$ crystal reduces the crystal symmetry to a monoclinic one. As a consequence, the incidence of a longitudinal wave or a shear wave (polarized in the sagittal plane) leads to a quantitative change in the values of the amplitude coefficients. However, in the case of a shear incident wave polarized orthogonally to the

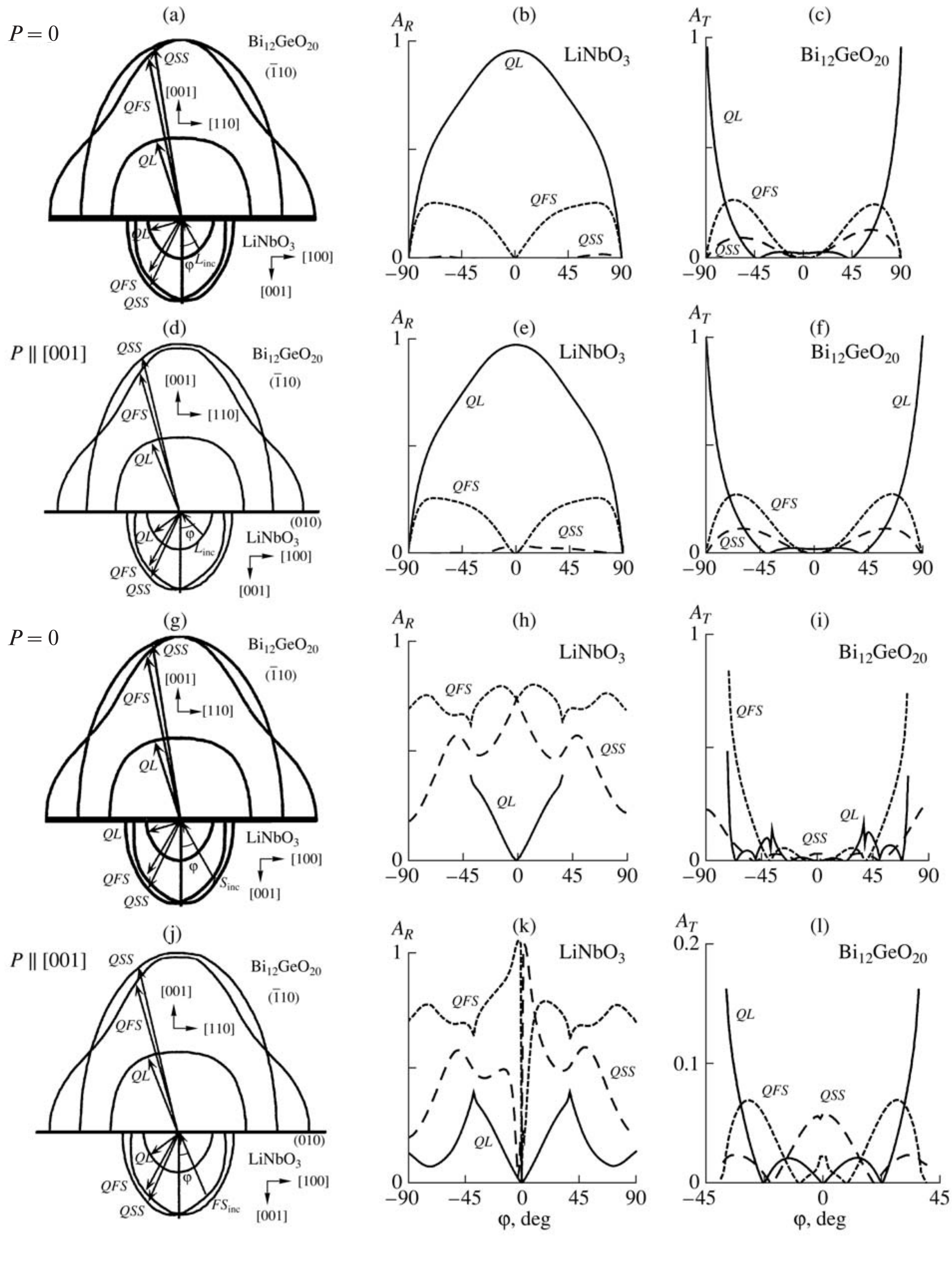

Fig. 2. Patterns of refraction vectors and the amplitude coefficients of reflection and refraction of waves at the boundary between $LiNbO_3$ and $Bi_{12}GeO_{20}$ in the (110) plane for the case of waves incident from $LiNbO_3$ in the presence of uniaxial pressure applied along the [001] direction.

plane of incidence, two reflected shear waves arise in fused quartz: one with polarization in the plane of incidence and the other with polarization orthogonal to this plane. Naturally, in the isotropic medium, only one shear wave is possible, but the change in the boundary conditions due to the uniaxial stress applied to the $Bi_{12}GeO_{20}$ crystal leads to a change in the direction of the polarization vector of the QSS wave, so that, in the case under consideration, this vector is directed at a certain angle to the plane of incidence of the wave. Note that, as was mentioned above, for the given direction of **P** application, we obtain splitting of the acoustic axis of tangential type along the [001] axis; in this case, "exchange" of shear wave solutions occurs within the interval of angles of incidence ±35° with respect to the normal to the boundary between the media for a longitudinal incident wave and within ±20° for a shear incident wave. This phenomenon manifests itself most clearly in the calculation of the hypothetical case of uniaxial stress applied along the direction of the incident wave.

When a shear wave is incident at an angle of ±40° to the normal to the boundary, we obtain a total internal reflection for the reflected longitudinal wave. When a longitudinal wave is incident on the boundary at an angle of ±57°, we obtain a transformation of the refracted elastic waves and only a fast shear refracted wave is present. The inclusion of uniaxial mechanical pressure in the calculations leads to a shift of the elastic wave transformation angle by one degree; i.e., we obtain ±58°. A similar situation arises for a shear incident wave polarized in the plane of incidence. Transformation of elastic waves occurs when the angle of incidence of the shear wave is ±32° to the normal to the boundary between the two media.

In the given geometry, the application of pressure normally to the sagittal plane or along the normal to the boundary causes rather weak and only quantitative changes in the reflection and refraction patterns.

CALCULATION OF THE EFFECT OF PRESSURE ON REFLECTION AND REFRACTION OF ACOUSTIC WAVES AT THE BOUNDARY BETWEEN TWO PIEZOELECTRICS

Figure 2 presents the real parts of the amplitude coefficients for reflection and refraction of waves at an acoustic contact between $LiNbO_3$ (the (010) plane) and $Bi_{12}GeO_{20}$ (the $(\bar{1}10)$ plane) under uniaxial pressure applied along the [001] direction, i.e., orthogonally to the boundary between the media. As was mentioned above, in the $Bi_{12}GeO_{20}$ crystal, the pressure **P** ∥ [001] eliminates the degeneration of shear elastic waves. By contrast, in the $LiNbO_3$ crystal, the application of pressure in the given direction, i.e., along the threefold axis, does not change the initial symmetry of the crystal, and the degeneration of shear waves is retained in this case.

When an elastic wave of the QL type is incident on the boundary between the two media, all three types of reflected and refracted BAWs are present, but the amplitude coefficient of the quasi-longitudinal reflected wave is dominant. This is explained by the large difference between the phase velocities in the crystals under study. The effect of the imposed uniaxial stress quantitatively changes the values of the amplitude coefficients of reflected and refracted waves, the greatest change being observed for the slow QSS reflected wave (Figs. 2d–2f). We also note the transformation of the refracted elastic waves. In the case of the normal incidence of a longitudinal wave, refraction evidently occurs for the longitudinal elastic wave, but at an angle of incidence of ±40° to the normal to the boundary between the media, and the refracted waves are only represented by quasi-shear elastic waves.

However, in the case of shear incident waves, the elimination of degeneration in the $Bi_{12}GeO_{20}$ crystal leads to considerable changes in the amplitude coefficients of reflected waves, whereas the amplitude coefficients of refracted elastic waves vary only slightly. When an elastic QFS wave is incident and the pressure **P** ∥ [001] is applied, transformation of the reflected shear waves with respect to the normal to the boundary takes place, whereas in the absence of external action, the dominant quantities are the coefficients of the reflected shear elastic waves (Figs. 2j–2l). At an angle of incidence of ±38° to the normal to the boundary between the media, the effect of total internal reflection occurs for the longitudinal wave.

Thus, using the data reported in the present paper and in [3], it is possible to analyze in detail the anisotropic character of the reflection and refraction of acoustic waves at the boundary between piezoelectric media of different kinds under a homogeneous finite external action, provided the constants characterizing the linear and nonlinear electromechanical properties of the media are known. The results obtained in this work may be useful for finding effects that are important from the practical point of view.

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