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MAGNETISM AND FERROELECTRICITY

Effects of Cross Correlations between Exchange and Magnetic-Anisotropy Inhomogeneities on the Spectrum and Damping of Spin Waves

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Abstract—Effects of cross correlations between inhomogeneities of the exchange and magnetic-anisotropy parameters on the spectrum and damping of spin waves in a ferromagnet are investigated. One- and three-dimensional inhomogeneities are considered. It is demonstrated that the positive cross correlations lead to an increase in the modification of the dispersion law and the damping of spin waves. The negative cross correlations result in the opposite effects: a decrease in the modification of the dispersion law and a decrease in the damping of spin waves. A comparison of the specific features revealed in this work and the results of targeted experimental investigations of modifications of the dispersion laws and damping in inhomogeneous magnets would make it possible to determine the contribution of the cross correlations to the formation of the stochastically inhomogeneous ground state in amorphous magnetic alloys.

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1. INTRODUCTION

Amorphous and nanocrystalline magnetic alloys offer a number of advantages over polycrystalline materials, such as the low coercive force, high susceptibility, narrow magnetic resonance line, etc. The technology of their production is simpler than that of single crystals. This opens up wide possibilities of using the above alloys for the design of devices intended for magnetic recording of information and high-frequency equipment.

Theoretically, amorphous and nanocrystalline magnetic alloys are characterized by the two main properties: (1) inhomogeneity of all parameters of the spin Hamiltonian (exchange, magnetic anisotropy, and other parameters) and (2) extended correlations of these inhomogeneities with the correlation length varying over a wide range (several tens and several hundreds of interatomic distances). The presence of large correlation lengths prevents the use of well-developed theoretical methods accounting for the effect of uncorrelated (δ-correlated) inhomogeneities for the calculation of a number of effects in these materials.

Effects of inhomogeneities with arbitrary correlation lengths on the spectrum and damping of spin waves in terms of the continuum model were taken into account in the first nonvanishing approximation of the perturbation theory in our earlier works [1–3]. More recently, in the same approximation, effects of correlated inhomogeneities on the spin wave spectrum were taken into account in the lattice model of ferromagnets [4, 5] and the continuum model [6].

The main result of the theory proposed in [1–3] is that, in the vicinity of the correlation wave number $k_c = r_c^{-1}$ (where r_c is the correlation length of the inhomogeneity), the laws of dispersion $\omega'(k)$ and damping $\omega''(k)$ should be modified and the corresponding modification should have different forms for inhomogeneities of different physical parameters. This theory was used for developing the experimental method of correlation spin-wave spectroscopy, which made it possible to measure the correlation lengths of inhomogeneities in many amorphous and nanocrystalline alloys [7].

In all works, except for [3], effects of inhomogeneities of each Hamiltonian parameter on the spin-wave spectrum were considered separately, because the inhomogeneities of different parameters were assumed to be stochastically independent of each other. In our previous work [3], apart from stochastically independent inhomogeneities of the exchange constant $\alpha(\mathbf{x})$ and the magnitude of the magnetization vector $M(\mathbf{x})$ (where $\mathbf{x} = (x, y, z)$), we analyzed the inhomogeneities of these parameters in the presence of their mutual correlations (cross correlations). In this case, we investigated the limiting case of strong cross correlations when the stochastic mutual correlation of random functions $\alpha(\mathbf{x})$ and $M(\mathbf{x})$ transforms into a functional relation between them. The purpose of the present work is to calculate the combine effect of inhomogeneities of the exchange constant $\alpha(\mathbf{x})$ and the magnetic anisotropy $\beta(\mathbf{x})$ on the spectrum and damping of spin waves in the presence of cross correlations with an arbitrary magnitude and sign between the random functions $\alpha(\mathbf{x})$ and $\beta(\mathbf{x})$.

2. THE MODEL AND METHOD

Let us consider the model of a ferromagnet in which the exchange parameter $\alpha(\mathbf{x})$ and the uniaxial anisotropy $\beta(\mathbf{x})$ (where $\mathbf{x} = \{x, y, z\}$) are inhomogeneous. The anisotropy direction $\mathbf{n}$ is assumed to be constant and coinciding with the direction of the external magnetic field $\mathbf{H}$. The equation of motion of the magnetization vector $\mathbf{M}$ represents the Landau–Lifshitz equation

$$\frac{\partial \mathbf{M}}{\partial t} = -g[\mathbf{M}, \mathbf{H}^e], \quad (1)$$

where g is the gyromagnetic ratio and $\mathbf{H}^e$ is the effective magnetic field defined by the expression

$$\mathbf{H}^e = -\frac{\partial W}{\partial \mathbf{M}} + \frac{\partial}{\partial \mathbf{x}} \frac{\partial W}{\partial(\partial \mathbf{M}/\partial \mathbf{x})}. \quad (2)$$

The energy density W in our model has the form

$$W = \frac{1}{2}\alpha(\mathbf{x})\left(\frac{\partial \mathbf{M}}{\partial \mathbf{x}}\right)^2 - \mathbf{MH} - \frac{1}{2}\beta(\mathbf{x})(\mathbf{Mn})^2. \quad (3)$$

The exchange parameter $\alpha(\mathbf{x})$ and the anisotropy parameter $\beta(\mathbf{x})$ can be represented in the form

$$\begin{aligned} \alpha(\mathbf{x}) &= \alpha[1 + \gamma_\alpha \rho_\alpha(\mathbf{x})], \\ \beta(\mathbf{x}) &= \beta[1 + \gamma_\beta \rho_\beta(\mathbf{x})], \end{aligned} \quad \gamma_\alpha = \frac{\Delta\alpha}{\alpha}, \quad \gamma_\beta = \frac{\Delta\beta}{\beta}, \quad (4)$$

where α, $\Delta\alpha$ and β, $\Delta\beta$ are the means and the root-mean-square fluctuations of the above parameters, and $\rho_\alpha(\mathbf{x})$ and $\rho_\beta(\mathbf{x})$ are the dimensionless centered ($\langle\rho_\alpha(\mathbf{x})\rangle = 0$, $\langle\rho_\beta(\mathbf{x})\rangle = 0$) and normalized ($\langle\rho_\alpha^2(\mathbf{x})\rangle = 1$), $\langle\rho_\beta^2(\mathbf{x})\rangle = 1$) random functions of the coordinates. Curly brackets indicate the averaging over an ensemble of random realizations of the corresponding random functions.

We represent the magnetization $\mathbf{M}(\mathbf{x}, t)$ in the form $\mathbf{M}_0 + \mathbf{m}(\mathbf{x}, t)$, and linearize Eq. (2) in a conventional manner ($M_z \approx M_0$; $m_x(\mathbf{x}, t)$, $m_y(\mathbf{x}, t) \ll M_0$). By assuming that the magnetization projections can be written in the form $m_x(\mathbf{x}, t)$ and $m_y(\mathbf{x}, t) \sim \exp(i\omega t)$, we obtain the following system of linear equations with coefficients dependent on $\mathbf{x}$ for these projections:

$$\begin{aligned} -\frac{i\omega}{g} m_x &= M_0\beta m_y + M_0\beta\gamma_\beta\rho_\beta m_y + H m_y \\ &- M_0\alpha(\nabla^2 m_y + \gamma_\alpha\rho_\alpha\nabla^2 m_y + \gamma_\alpha\nabla\rho_\alpha\nabla m_y), \\ -\frac{i\omega}{g} m_y &= M_0\beta m_x + M_0\beta\gamma_\beta\rho_\beta m_x + H m_x \\ &- M_0\alpha(\nabla^2 m_x + \gamma_\alpha\rho_\alpha\nabla^2 m_x + \gamma_\alpha\nabla\rho_\alpha\nabla m_x). \end{aligned} \quad (5)$$

We perform the Fourier transform

$$\begin{aligned} m(\mathbf{x}) &= \int m(\mathbf{k}) e^{i\mathbf{kx}} d\mathbf{k}, \\ m(\mathbf{k}) &= \left(\frac{1}{2\pi}\right)^d \int m(\mathbf{x}) e^{-i\mathbf{kx}} d\mathbf{x}, \end{aligned} \quad (6)$$

where $\mathbf{k}$ is the wave vector; d is the space dimensionality; and the integration is carried out over the entire space of wave vectors and the Cartesian coordinates, respectively.

By introducing the circular projections

$$m^\pm(\mathbf{k}) = m_x(\mathbf{k}) \pm i m_y(\mathbf{k}), \quad (7)$$

we find the integral equation for the resonance projection $\mathbf{m}^+(\mathbf{k})$ (hereafter, the index "+" will be omitted)

$$\begin{aligned} (\nu - \mathbf{k}^2) m(\mathbf{k}) &= \gamma_\alpha \int (\mathbf{k}\mathbf{k}_1)\rho_\alpha(\mathbf{k} - \mathbf{k}_1) m(\mathbf{k}_1) d\mathbf{k}_1 \\ &+ \frac{\beta\gamma_\beta}{\alpha} \int \rho_\beta(\mathbf{k} - \mathbf{k}_1) m(\mathbf{k}_1) d\mathbf{k}_1, \end{aligned} \quad (8)$$

where we introduced the designation $\nu = (\omega - \omega_0)/gM_0\alpha$. Here, ω_0 is the ferromagnetic resonance frequency. These stochastic equations will be investigated using the perturbation theory method proposed in [1–3].

The averaging of Eq. (8) over random realizations of the functions $\rho_\alpha(\mathbf{k} - \mathbf{k}_1)$ and $\rho_\beta(\mathbf{k} - \mathbf{k}_1)$ leads to the relationship

$$\begin{aligned} (\nu - \mathbf{k}^2)\langle m(\mathbf{k})\rangle &= \gamma_\alpha \int (\mathbf{k}\mathbf{k}_1)\langle\rho_\alpha(\mathbf{k} - \mathbf{k}_1) m(\mathbf{k}_1)\rangle d\mathbf{k}_1 \\ &+ \frac{\beta\gamma_\beta}{\alpha} \int \langle\rho_\beta(\mathbf{k} - \mathbf{k}_1) m(\mathbf{k}_1)\rangle d\mathbf{k}_1. \end{aligned} \quad (9)$$

By expressing the quantity $\mathbf{m}(\mathbf{k})$ from Eq. (8) and increasing the subscripts on the wave vector $\mathbf{k}$ by unity, we derive

$$\begin{aligned} m(\mathbf{k}_1) &= \gamma_\alpha \int \frac{(\mathbf{k}_1\mathbf{k}_2)\rho_\alpha(\mathbf{k}_1 - \mathbf{k}_2) m(\mathbf{k}_2)}{\nu - \mathbf{k}_1^2} d\mathbf{k}_2 \\ &+ \frac{\beta\gamma_\beta}{\alpha} \int \frac{\rho_\beta(\mathbf{k}_1 - \mathbf{k}_2) m(\mathbf{k}_2)}{\nu - \mathbf{k}_1^2} d\mathbf{k}_2. \end{aligned} \quad (10)$$

At the first step, we decouple the means of the products of the functions ρ_i and m in Eq. (9) according to the general rules into the products of the means and the correlator of the product of the centered values of these functions; that is,

$$\langle\rho_i m\rangle = \langle\rho_i\rangle\langle m\rangle + \langle\rho_i \mathring{m}\rangle, \quad (11)$$

where

$$\mathring{m}(\mathbf{k}_1) = m(\mathbf{k}_1) - \langle m(\mathbf{k}_1)\rangle. \quad (12)$$

Since the product of the means in relationship (10a) vanishes because the functions ρ_i are centered, the averaged equation should not contain terms proportional to the first powers of the quantities γ_i. For the same reason, one of the terms $\langle\rho_i \mathring{m}\rangle$, which is formed after substitution of expression (10b) into relationship (10a), vanishes in the correlator $\langle\rho_i\langle m(\mathbf{k}_1)\rangle\rangle$, and relationship (10a) transforms into a trivial identity. Therefore, the next step begins with the substitution of expression (10) into Eq. (9). As a result, we obtain

$$(\nu - \mathbf{k}^2)\langle m(\mathbf{k})\rangle$$
$$= \iint \frac{\gamma^2(\mathbf{k}_1\mathbf{k}_2)(\mathbf{k}\mathbf{k}_1)\langle\rho_\alpha(\mathbf{k}-\mathbf{k}_1)\rho_\alpha(\mathbf{k}_1-\mathbf{k}_2)m(\mathbf{k}_2)\rangle d\mathbf{k}_1 d\mathbf{k}_2}{\nu - \mathbf{k}_1^2}$$
$$+ \iint \frac{\gamma\eta_\beta(\mathbf{k}\mathbf{k}_1)\langle\rho_\alpha(\mathbf{k}-\mathbf{k}_1)\rho_\beta(\mathbf{k}_1-\mathbf{k}_2)m(\mathbf{k}_2)\rangle d\mathbf{k}_1 d\mathbf{k}_2}{\nu - \mathbf{k}_1^2} \tag{13}$$
$$+ \iint \frac{\gamma\eta_\beta(\mathbf{k}_1\mathbf{k}_2)\langle\rho_\beta(\mathbf{k}-\mathbf{k}_1)\rho_\alpha(\mathbf{k}_1-\mathbf{k}_2)m(\mathbf{k}_2)\rangle d\mathbf{k}_1 d\mathbf{k}_2}{\nu - \mathbf{k}_1^2}$$
$$+ \iint \frac{\eta_\beta^2\langle\rho_\beta(\mathbf{k}-\mathbf{k}_1)\rho_\beta(\mathbf{k}_1-\mathbf{k}_2)m(\mathbf{k}_2)\rangle d\mathbf{k}_1 d\mathbf{k}_2}{\nu - \mathbf{k}_1^2},$$

where we introduced the designations $\gamma = \gamma_\alpha$ and $\eta_\beta = \beta\gamma_\beta/\alpha$.

The means of the products of three random functions under the integral sign are decoupled in the first nonvanishing approximation of the perturbation theory (the Bourret approximation [8]); that is,

$$\langle m(\mathbf{k}_2)\rho_i(\mathbf{k}-\mathbf{k}_1)\rho_j(\mathbf{k}_1-\mathbf{k}_2)\rangle \approx \langle m(\mathbf{k}_2)\rangle\langle\rho_i(\mathbf{k}-\mathbf{k}_1)\rho_j(\mathbf{k}_1-\mathbf{k}_2)\rangle, \tag{14}$$

where each of the subscripts i and j takes on values α and β. In this relationship, the correlator $\langle\mathring{m}(\mathbf{k}_2)\mathring{P}_{ij}\rangle$ (where $\mathring{P}_{ij} = \rho_i\rho_j - \langle\rho_i\rho_j\rangle$) on the right-side is rejected. The substitution of expression (10b) into this correlator with an increase in the subscripts on the wave vector $\mathbf{k}$ by unity would lead to the next term of the expansion of the perturbation theory, etc.

Since $\rho_i(\mathbf{x})$ and $\rho_j(\mathbf{x})$ are homogeneous random functions, they satisfy the relationship

$$\langle\rho_i(\mathbf{k}')\rho_j^*(\mathbf{k}'')\rangle = S_{ij}(\mathbf{k}')\delta(\mathbf{k}'-\mathbf{k}''), \tag{15}$$

where $S_{ij}(\mathbf{k})$ is the spectral density of the correlation function $K_{ij}(\mathbf{r})$ of inhomogeneities. The correlation function is defined by the expression

$$K_{ij}(\mathbf{r}) = \langle\rho_i(\mathbf{x})\rho_j(\mathbf{x}+\mathbf{r})\rangle, \tag{16}$$

where $\mathbf{r}$ is the distance between two points. The correlation function $K_{ij}(\mathbf{r})$ and the spectral density $S_{ij}(\mathbf{k})$ are related by the Fourier transform (the Wiener–Khintchin theorem for homogeneous random functions)

$$K_{ij}(\mathbf{r}) = \int S_{ij}(\mathbf{k})e^{i\mathbf{k}\mathbf{r}}d\mathbf{k}, \quad S_{ij}(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^d \int K_{ij}(\mathbf{r})e^{-i\mathbf{k}\mathbf{r}}d\mathbf{r}. \tag{17}$$

Taking into account expressions (12) and (13), Eq. (11) is integrated over the vector $\mathbf{k}_2$. Thereafter, the quantity $\langle m(\mathbf{k})\rangle$ can be removed from the integral sign, and we obtain the dispersion relation for spin waves in the general form

$$\nu - \mathbf{k}^2 = \int \frac{\gamma^2(\mathbf{k}\mathbf{k}_1)^2 S_{\alpha\alpha}(\mathbf{k}-\mathbf{k}_1) + 2\gamma\eta_\beta(\mathbf{k}\mathbf{k}_1)S_{\alpha\beta}(\mathbf{k}-\mathbf{k}_1) + \eta_\beta^2 S_{\beta\beta}(\mathbf{k}-\mathbf{k}_1)}{\nu - \mathbf{k}_1^2}d\mathbf{k}_1. \tag{18}$$

In this relation, the terms proportional to γ^2 and η_β^2 account for effects of the exchange and anisotropy inhomogeneities, respectively. The term proportional to the product $\gamma\eta_\beta$ includes effects of correlations between the exchange and anisotropy inhomogeneities.

3. DISPERSION AND DAMPING LAWS FOR SPIN WAVES

Let us consider separately the cases of one-dimensional and three-dimensional inhomogeneities.

3.1. One-Dimensional Inhomogeneities

It is assumed that the correlations exponentially decay for the exchange autocorrelation function $K_{\alpha\alpha}(r)$, the anisotropy autocorrelation function $K_{\beta\beta}(r)$, and the cross correlation function $K_{\alpha\beta}(r)$ between the exchange and anisotropy fluctuations; that is,

$$K_{\alpha\alpha}(r) = K_{\beta\beta}(r) = \exp(-k_c r), \quad K_{\alpha\beta} = \kappa\exp(-k_c r), \tag{19}$$

where $r = |x - x'|$, k_c is the correlation wave number ($r_c = 1/k_c$ is the correlation length of inhomogeneities), and κ is the dimensionless cross correlation coefficient lying in the range $-1 < \kappa < 1$. For simplicity, we assume that the correlation length r_c is identical for all three correlation functions.

According to formula (15), at $d = 1$, to these correlation functions there correspond the spectral densities

$$S_{\alpha\alpha}(k) = S_{\beta\beta}(k) = \frac{1}{\pi}\frac{k_c}{k_c^2 + k^2},$$
$$S_{\alpha\beta}(k) = \frac{\kappa}{\pi}\frac{k_c}{k_c^2 + k^2}. \quad (20)$$

After the substitution of these expressions into Eq. (16), this equation takes the form

$$\nu - k^2 = I_{\alpha\alpha} + I_{\alpha\beta} + I_{\beta\beta}, \quad (21)$$

where

$$I_{\alpha\alpha} = \frac{\gamma^2 k^2 k_c}{\pi}\int_{-\infty}^{+\infty}\frac{k_1^2}{(\nu - k_1^2)(k_c^2 + (k - k_1)^2)}dk_1,$$
$$I_{\alpha\beta} = \frac{2\kappa\gamma\eta_\beta k k_c}{\pi}\int_{-\infty}^{+\infty}\frac{k_1}{(\nu - k_1^2)(k_c^2 + (k - k_1)^2)}dk_1, \quad (22)$$
$$I_{\beta\beta} = \frac{\eta_\beta^2 k_c}{\pi}\int_{-\infty}^{+\infty}\frac{1}{(\nu - k_1^2)(k_c^2 + (k - k_1)^2)}dk_1.$$

The calculations of these integrals with the use of the theory of residues lead to the relationships

$$I_{\alpha\alpha} = \gamma^2 k^2\frac{k^2 + ik_c\sqrt{\nu} + k_c^2}{(\sqrt{\nu} - ik_c)^2 - k^2},$$
$$I_{\alpha\beta} = \frac{2\kappa\gamma k^2\eta_\beta}{(\sqrt{\nu} - ik_c)^2 - k^2}, \quad (23)$$
$$I_{\beta\beta} = \frac{\eta_\beta^2}{(\sqrt{\nu} - ik_c)^2 - k^2}\left(1 - \frac{ik_c}{\sqrt{\nu}}\right).$$

We consider the complex dispersion law (19) in the first order of the perturbation theory by setting $\sqrt{\nu} \approx k$ on the right-hand sides of expressions (21). By representing the quantity ν in the form $\nu = \nu' + i\nu''$, we obtain the dispersion law for spin waves with allowance made for the mutual correlations between inhomogeneities of the exchange and magnetic anisotropy parameters in the form

$$\nu'/k_c^2 = u^2\left(1 - \gamma^2\frac{1 + 3u^2}{1 + 4u^2} - \frac{2\kappa\gamma\eta}{1 + 4u^2}\right) + \frac{\eta^2}{1 + 4u^2} \quad (24)$$

and the damping of spin waves in the following form:

$$\nu''/k_c^2 = u^2\left(\gamma^2\frac{u(1 + 2u^2)}{(1 + 4u^2)} + \frac{4\kappa\gamma\eta u}{(1 + 4u^2)}\right) + \frac{\eta^2(1 + 2u^2)}{u(1 + 4u^2)}, \quad (25)$$

where we introduced the dimensionless quantities $u = k/k_c$ and $\eta = \eta_\beta/k_c^2$.

3.2. Three-Dimensional Inhomogeneities

It is assumed that the decay of correlations is characterized by isotropic correlation functions dependent only on the magnitude of the radius vector $r = |\mathbf{r}|$; that is,

$$K_{ii}(r) = K_{jj}(r) = \exp(-k_c r),$$
$$K_{ij}(r) = \kappa\exp(-k_c r). \quad (26)$$

According to formula (15), to these correlation functions there correspond the spectral densities

$$S_{\alpha\alpha}(\mathbf{k}) = S_{\beta\beta}(\mathbf{k}) = \frac{1}{\pi^2}\frac{k_c}{(k_c^2 + \mathbf{k}^2)^2},$$
$$S_{\alpha\beta}(\mathbf{k}) = \frac{\kappa}{\pi^2}\frac{k_c}{(k_c^2 + \mathbf{k}^2)^2}. \quad (27)$$

We substitute relationships (25) into Eq. (16) and divide the integral in this equation into three integrals

$$\nu - \mathbf{k}^2 = I_{\alpha\alpha} + I_{\alpha\beta} + I_{\beta\beta}, \quad (28)$$

where

$$I_{\alpha\alpha} = \frac{\gamma^2 k_c}{\pi^2}\int_{\nu}\frac{(\mathbf{k}\mathbf{k}_1)^2}{(\nu - \mathbf{k}_1^2)[(k_c^2 + (\mathbf{k} - \mathbf{k}_1)^2)]^2}d\mathbf{k}_1,$$
$$I_{\alpha\beta} = \frac{2\kappa\gamma k_c\eta_\beta}{\pi^2}\int_{\nu}\frac{(\mathbf{k}\mathbf{k}_1)}{(\nu - \mathbf{k}_1^2)[k_c^2 + (\mathbf{k} - \mathbf{k}_1)^2]^2}d\mathbf{k}_1, \quad (29)$$
$$I_{\beta\beta} = \frac{\eta_\beta^2 k_c}{\pi^2}\int_{\nu}\frac{1}{(\nu - \mathbf{k}_1^2)[k_c^2 + (\mathbf{k} - \mathbf{k}_1)^2]^2}d\mathbf{k}_1.$$

By changing over to the spherical coordinate system; integrating over the azimuthal angle φ; making the replacement $x = \cos\theta$; and introducing the dimensionless quantities $u = k/k_c$, $u_1 = k_1/k_c$, and $u_\nu = \sqrt{\nu}/k_c$, we find

$$I_{\alpha\alpha} = \frac{2\gamma^2 k_c^2 u^2}{\pi}\int_{-\infty}^{+\infty}\int_0^1\frac{u_1^4 x^2 du_1 dx}{(u_\nu^2 - u_1^2)(1 + u^2 + u_1^2 - 2uu_1 x)^2},$$

$$I_{\alpha\beta} = \frac{4\kappa\gamma\eta_\beta}{\pi}\int_{-\infty}^{+\infty}\int_0^1 \frac{uu_1^3 x du_1 dx}{(u_\nu^2 - u_1^2)(1 + u^2 + u_1^2 - 2uu_1x)^2}, \quad (30)$$

$$I_{\beta\beta} = \frac{2\eta_\beta^2}{\pi k_c^2}\int_{-\infty}^{+\infty}\int_0^1 \frac{u_1^2 du_1 dx}{(u_\nu^2 - u_1^2)(1 + u^2 + u_1^2 - 2uu_1x)^2}.$$

Here, we changed the limits of integration with the use of the relationship $\int_0^{+\infty}\int_{-1}^1 \longrightarrow \int_{-\infty}^{+\infty}\int_0^1$ valid for the above integrands.

The integrals over the variable u_1 were calculated by the method of residues. This results in cumbersome expressions, which in the first order of the perturbation theory at $u_\nu \approx u$ take the form

$$I_{\alpha\alpha} = 2\gamma^2 k_c^2 u^5 i\int_0^1 \frac{x^2 dx}{A} - \gamma^2 k_c^2 u^2 \int_0^1 x^2 D^3 \frac{C(D^2 - 2u^2)dx}{B} - \gamma^2 k_c^2 u^4 \int_0^1 x^2 D^4 \frac{dx}{B},$$

$$I_{\alpha\beta} = 4\gamma\kappa\eta_\beta u^3 i\int_0^1 \frac{xu^3 dx}{A} + \gamma\kappa\eta_\beta u\int_0^1 xD^2 \frac{C[D^2 - 3u^2]dx}{B} + \gamma\kappa\eta_\beta u\int_0^1 xD^3 \frac{(D^2 + u^2)}{B}, \quad (31)$$

$$I_{\beta\beta} = -2\frac{\eta_\beta^2}{k_c^2} ui\int_0^1 \frac{dx}{A} + \frac{\eta_\beta^2}{k_c^2}\int_0^1 D\frac{(-D^3 + u^2 C)dx}{B},$$

where $A = (2u^2(x + 1) + 1)^2$, $B = (u^2 - (ux + i\sqrt{1 + u^2(1 - x^2)})^2)^2(1 + u^2(1 - x^2))^{3/2}$, $C = ux - i\sqrt{1 + u^2(1 - x^2)}$ and $D = ux + i\sqrt{1 + u^2(1 - x^2)}$.

By integrating over the variable x in relationships (29), substituting them into Eq. (26), and separating the real and imaginary parts, we obtain the dispersion law $\nu'(k)$ and the damping law $\nu''(k)$ for spin waves with due regard for the cross correlations between the three-dimensional inhomogeneities of the exchange and magnetic anisotropy parameters in the form

$$\nu'/k_c^2 = u^2\left(1 - \gamma^2\left(\frac{1 + 5u^2}{1 + 4u^2} + \frac{1}{u^2} - \frac{(1 + 2u^2)\arctan(2u)}{2u^3}\right)\right) + 2\kappa\gamma\eta\left(\frac{\arctan(2u)}{2u} - \frac{3u^2 + 1}{1 + 4u^2}\right) - \frac{\eta^2}{(1 + 4u^2)}, \quad (32)$$

$$\nu''/k_c^2 = \gamma^2 u^3\left(\frac{1}{u^2} + \frac{2u^2}{1 + 4u^2} - \frac{(1 + 2u^2)\ln(1 + 4u^2)}{4u^4}\right) + 2\kappa\gamma\eta\left(\frac{u(1 + 2u^2)}{1 + 4u^2} - \frac{\ln(1 + 4u^2)}{4u}\right) + \frac{2\eta^2 u}{(1 + 4u^2)}. \quad (33)$$

In these expressions, the terms proportional to the quantities γ^2 and η^2 were derived in our earlier works [1–3] and the terms proportional to $\gamma\eta$ describing the mutual correlations were included for the first time in this work.[1]

4. INVESTIGATION OF THE DISPERSION AND DAMPING LAWS

The dispersion and damping laws for the one-dimensional inhomogeneities (relationships (22), (23)) and three-dimensional inhomogeneities (expressions (30), (31)) were investigated using the analytical and numerical methods.

4.1. One-Dimensional Inhomogeneities

The modifications of the dispersion law due to the one-dimensional exchange and anisotropy inhomogeneities (described by formula (22)) are shown in Figs. 1a and 1b.

The dotted line in Fig. 1a represents the dispersion law $\nu' = k^2$ corresponding to a homogeneous ferromagnet ($\gamma = \eta = 0$). Thin curve *1* corresponds to the presence of only exchange inhomogeneities ($\gamma \neq 0$, $\eta = 0$), and thin curve *2* corresponds to the presence of only anisotropy inhomogeneities ($\gamma = 0$, $\eta \neq 0$). In accordance with expression (22), curve *1* has a bending in the vicinity of the point $(k/k_c)^2 = 1/4$, which is poorly seen on scales of Fig. 1a. As can be seen from curve *2*, the magnetic-anisotropy inhomogeneities lead to a shift in the ferromagnetic resonance frequency corresponding to $k = 0$ in addition to the modification of the dependence $\nu'(k)$. The thick solid curve in Fig. 1a indicates the total modification of the dispersion law due to the presence of both the exchange and anisotropy inhomogeneities in the absence of their mutual correlations ($\gamma \neq 0$, $\eta \neq 0$, $\kappa = 0$).

Figure 1b illustrates effects of cross correlations between the exchange and anisotropy inhomogeneities on the dispersion law. The thick solid curve in this figure reproduces the same curve in Fig. 1a and corresponds to the combined effect of mutually uncorrelated exchange and anisotropy inhomogeneities ($\gamma \neq 0$, $\eta \neq 0$, $\kappa = 0$). The dashed and dot-dashed curves indicate effects of the positive ($\kappa > 0$) and negative ($\kappa < 0$) cross correlations, respectively. Moreover, we studied the analytical expressions corresponding to the expansion

[1] The approximate formulas for the terms proportional to γ^2 were given in [1–3]. The complete expressions were published in the preprint [9].

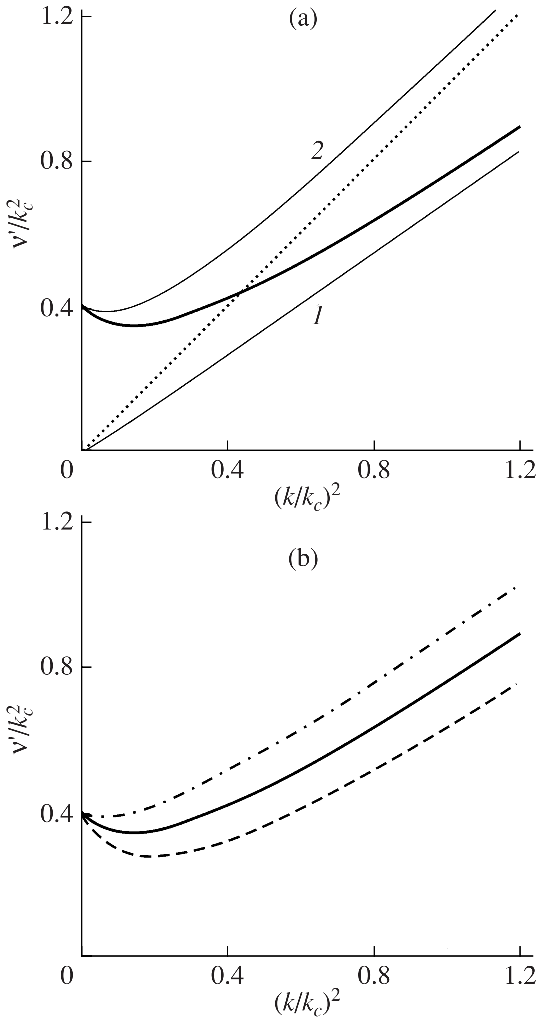

Fig. 1. Dispersion laws for spin waves in a ferromagnet with one-dimensional exchange and anisotropy inhomogeneities for κ = 0 (thick solid curves), 0.8 (dashed curve), and –0.8 (dot-dashed curve). Thin solid curves *1* and *2* correspond to the dispersion laws in the case of only exchange and only anisotropy inhomogeneities, respectively. The dotted line represents the dispersion law in a homogeneous ferromagnet.

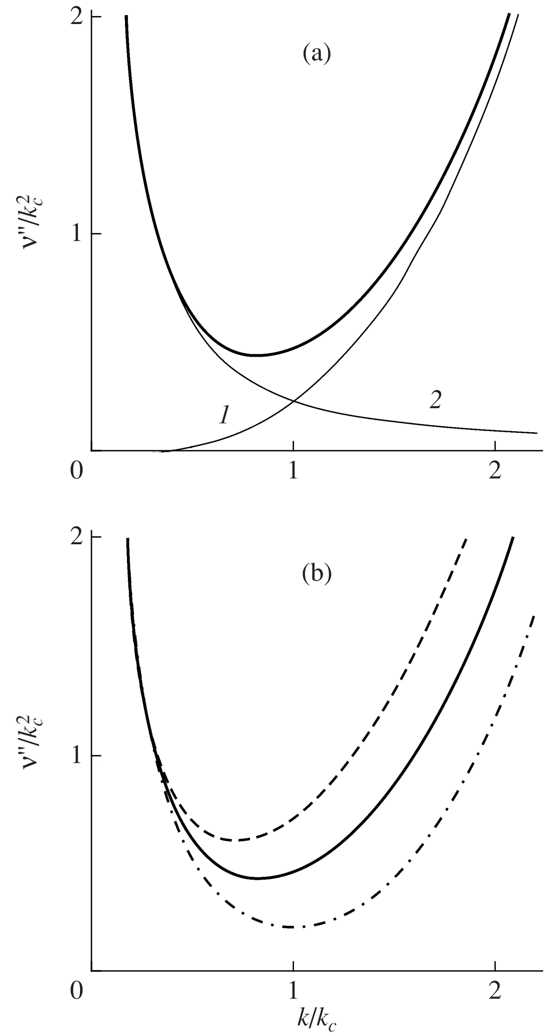

Fig. 2. Damping of spin waves in a ferromagnet with one-dimensional exchange and anisotropy inhomogeneities. Designations of the curves are the same as in Fig. 1.

of the exact relationship (22) into a series at $k \longrightarrow 0$ and the asymptote of this relationship at $k \longrightarrow \infty$; that is,

$$\nu'/k_c^2 \approx \begin{cases} u^2(1-\gamma^2-2\kappa\gamma\eta-4\eta^2)+\eta^2, & 2u \ll 1 \\ u^2\left(1-\frac{3}{4}\gamma^2\right)-\frac{\gamma^2}{16}-\frac{1}{2}\kappa\gamma\eta, & 2u \gg 1. \end{cases} \quad (34)$$

It can be seen from these expressions that, at $2u \ll 1$, the effective spin-wave stiffness (the coefficient of the quantity u^2) depends on both the exchange and anisotropy inhomogeneities, as well as on the magnitude and the sign of the correlation coefficient κ.

In the coordinate system (ν', k^2), the straight lines representing the tangents to the dashed and dot-dashed curves in Fig. 1b correspond to expressions (32). Despite a substantial difference between the exact curves $\nu'(k^2)$ corresponding to the positive and negative cross correlations, the tangents to these curves and the solid curve intersect in the vicinity of the same value $(k/k_c)^2 = 1/4$. This is associated with the same form of the denominator in all terms of relationship (22) that describe the modification of the dispersion law.

The damping $\nu''(k)$ determined by the one-dimensional exchange and anisotropy inhomogeneities and described by formula (23) is illustrated in Figs. 2a and 2b. In Fig. 2a, thin curve *1* corresponds to the damping associated with the exchange inhomogeneities ($\gamma \neq 0$, $\eta = 0$), and thin curve *2* indicates the damping due to the anisotropy inhomogeneities ($\gamma = 0$, $\eta \neq 0$). The thick solid curves in Figs. 2a and 2b represent the combined effect of both the exchange and anisotropy inhomogene-

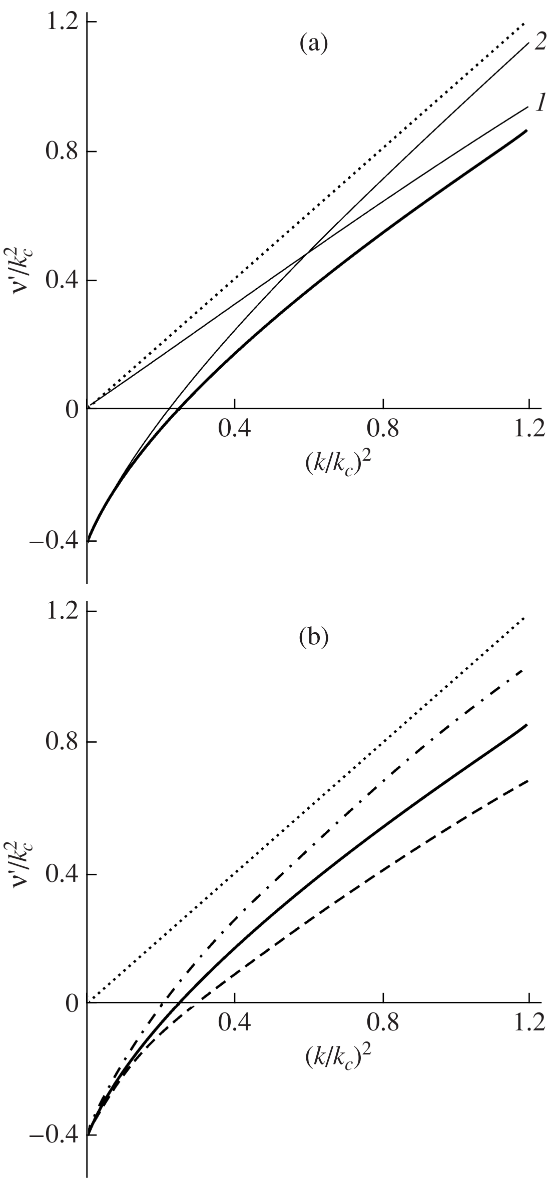

Fig. 3. Dispersion laws for spin waves in a ferromagnet with three-dimensional exchange and anisotropy inhomogeneities. Designations of the curves are the same as in Fig. 1.

ities in the absence of their mutual correlations ($\gamma \neq 0$, $\eta \neq 0$, $\kappa = 0$). The divergence of the damping at $k \longrightarrow 0$ in the case of the one-dimensional anisotropy inhomogeneities indicates that the theory is inapplicable in the range of small wave numbers k. This is natural because, for waves in the presence of one-dimensional inhomogeneities, there exists a finite localization radius l and the dispersion and damping laws can be approximately investigated only for wavelengths smaller than this localization radius l ($k \gg l^{-1}$).

Figure 2b shows effects of cross correlations between the exchange and anisotropy inhomogeneities on the damping of spin waves. The dashed and dot-dashed curves correspond to $\kappa > 0$ and $\kappa < 0$, respectively. It can be seen from Fig. 2b that the positive cross correlations lead to an increase in the damping of spin waves, whereas the negative cross correlations result in a decrease in their damping. The approximate expressions corresponding to the expansions of the exact formula (23) into series at $k \longrightarrow 0$ and $k \longrightarrow \infty$ have the form

$$\nu''/k_c^2 \approx \begin{cases} \gamma^2 u^3 + 4\kappa\gamma\eta u^3 + \eta^2 \frac{1}{u}, & 2u \ll 1, \\ \frac{1}{2}\gamma^2 u^3 + \kappa\gamma\eta u + \eta^2 \frac{1}{u}, & 2u \gg 1. \end{cases} \quad (35)$$

4.2. Three-Dimensional Inhomogeneities

The modifications of the dispersion law due to the three-dimensional exchange and anisotropy inhomogeneities (described by formula (30)) are shown in Figs. 3a and 3b.

The dotted lines in these figures represent the dispersion law $\nu' = k^2$ corresponding to a homogeneous ferromagnet ($\gamma = \eta = 0$). Thin curve *1* corresponds to the presence of only exchange inhomogeneities ($\gamma \neq 0$, $\eta = 0$), and thin curve *2* corresponds to the presence of only anisotropy inhomogeneities ($\gamma = 0$, $\eta \neq 0$). The thin solid curves in these figures indicate the total modification of the dispersion law due to the presence of both the exchange and anisotropy inhomogeneities in the absence of their mutual correlations ($\gamma \neq 0$, $\eta \neq 0$, $\kappa = 0$). It can be seen that the anisotropy inhomogeneities lead to a shift in the ferromagnetic resonance frequency corresponding to $k = 0$ in addition to the modification of the dependence $\nu'(k)$. Unlike the case of one-dimensional inhomogeneities (Fig. 1a), this shift corresponds to a decrease in the ferromagnetic resonance frequency. Effects of cross correlations between the exchange and anisotropy inhomogeneities on the dispersion law are illustrated in Fig. 3b. It can be seen from this figure that the positive cross correlations (dashed curve, $\kappa > 0$) result in a larger deviation of the dispersion law and, by contrast, the negative cross correlations (dot-dashed curve, $\kappa < 0$) decrease the modification of the dispersion law as compared to that associated with the combined effect of the uncorrelated exchange and anisotropy inhomogeneities. The approximate expressions corresponding to the expansions of the exact formula (30) into series at $k \longrightarrow 0$ and $k \longrightarrow \infty$ are represented in the form

$$\nu'/k_c^2 \approx \begin{cases} u^2\left(1 - \frac{\gamma^2}{3} - \frac{2}{3}\kappa\gamma\eta + 4\eta^2\right) - \eta^2, & 2u \ll 1 \\ u^2\left(1 - \frac{5}{4}\gamma^2\right) + \frac{\pi}{2}\gamma^2 u - \frac{23}{16}\gamma^2 - \frac{3}{2}\kappa\gamma\eta, & 2u \gg 1. \end{cases} \quad (36)$$

As for the one-dimensional inhomogeneities, the effective spin-wave stiffness for the three-dimensional inho-

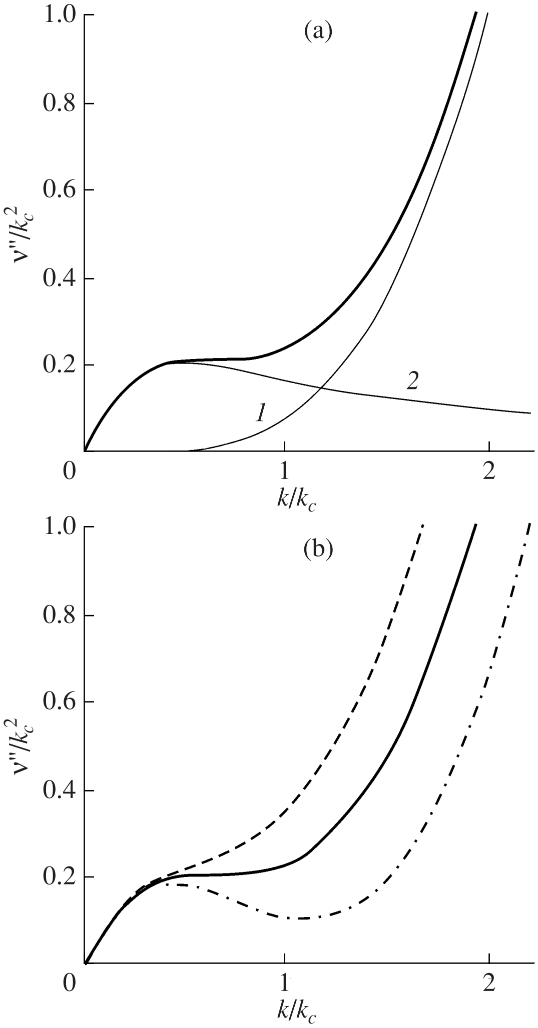

Fig. 4. Damping of spin waves in a ferromagnet with three-dimensional exchange and anisotropy inhomogeneities. Designations of the curves are the same as in Fig. 1.

mogeneities at $2u \ll 1$ depends on the quantities γ^2, η^2, and κ. The intersection point for the limiting curves described by expressions (34) is determined by the more complex relationships as compared to the straight lines represented by expressions (32). For some parameters, this point for the curves described by expressions (34) can be absent. However, it can be seen from the general form of expressions (34) that the point $(k/k_c)^2 = 1/4$ $(2u = 1)$ for both one-dimensional and three-dimensional cases is the crossover point in the dispersion curve, in the vicinity of which specific features corresponding to small wave numbers k give way to specific features characteristic of large wave numbers k.

The damping $\nu''(k)$ associated with the three-dimensional exchange and anisotropy inhomogeneities and described by formula (31) is illustrated in Figs. 4a and 4b. The thick solid curves in these figures represent the combined effect of the presence of both the exchange and anisotropy inhomogeneities in the absence of their mutual correlations ($\gamma \neq 0$, $\eta \neq 0$, $\kappa = 0$). Curve *1* corresponds to the damping determined by the exchange inhomogeneities ($\gamma \neq 0$, $\eta = 0$), and curve *2* indicates the damping due to the anisotropy inhomogeneities ($\gamma = 0$, $\eta \neq 0$). Figure 4b shows effects of cross correlations between the exchange and anisotropy inhomogeneities on the damping of spin waves. It can be seen from Fig. 4b that, as for the case of one-dimensional inhomogeneities, the positive cross correlations (dashed curve, $\kappa > 0$) lead to an increase in the damping of spin waves, whereas the negative cross correlations (dot-dashed curve, $\kappa < 0$) result in a decrease in their damping. A change in the damping due to the cross correlations is a function of the wave number k or, correspondingly, the frequency. For $\kappa < 0$, the damping should decrease most strongly in the vicinity of $k \sim k_c$.

The approximate expressions corresponding to the expansions of the exact formula (31) into series at $k \longrightarrow 0$ and $k \longrightarrow \infty$ take the form

$$\nu''/k_c^2 \approx \begin{cases} 2\eta^2 u - 8\eta^2 u^3 + 2u^5\left(\dfrac{\gamma^2}{3} + \dfrac{35}{3}\kappa\gamma\eta + 16\eta^2\right), \\ 2u \ll 1, \\ \dfrac{1}{2}\gamma^2 u^3 + u\left(\dfrac{7}{8}\gamma^2 - \gamma^2 \ln(2u) + \kappa\gamma\eta\right) \\ + \dfrac{1}{2u}\left(\eta^2 + \dfrac{1}{2}\kappa\gamma\eta - \gamma^2\ln(2u)\right), \\ 2u \gg 1. \end{cases} \quad (37)$$

5. CONCLUSIONS

Thus, in this work, we investigated effects of mutual correlations (cross correlations) between inhomogeneities of the exchange and magnetic anisotropy parameters on the modification of the dispersion law and the damping of spin waves in a ferromagnet. We considered both one- and three-dimensional inhomogeneities. The investigation was performed in the first nonvanishing approximation of the perturbation theory to which there correspond squares of the relative root-mean-square deviations of the exchange (γ^2) and magnetic anisotropy (η^2) inhomogeneities, as well as the product of the root-mean-square deviations ($\gamma\eta$) for the term describing effects of cross correlations. The last term is characterized by the dimensionless correlation coefficient lying in the range $-1 < \kappa < 1$. Specific values of the correlation coefficient κ are determined by the microscopic model of inhomogeneities (consideration of this model is beyond the scope of our work). In this respect, the general analytical expressions obtained for the dispersion law $\nu'(k)$ and the damping law $\nu''(k)$ were numerically investigated for three correlation coefficients $\kappa = -0.8$, 0, and 0.8. The correlation coefficient $\kappa = 0$ corresponds to the combined effect of the mutually uncorrelated exchange and anisotropy inhomogeneities on the spectrum and damping of spin waves. For correlation coefficients $\kappa = 0.8$ and -0.8, the graphs of the derived expressions illustrate the effects of positive and negative mutual correlations, respectively.

It was demonstrated that the positive cross correlations lead to a larger deviation of the dispersion law $\nu'(k)$ from the unperturbed dispersion law as compared to the combined effect of mutually uncorrelated inhomogeneities. Moreover, the positive cross correlations result in an increase in the damping of spin waves. The negative cross correlations lead to the opposite effects: a decrease in the modification of the dispersion law and the damping. Furthermore, the behavior of the dependence of a decrease in the damping ν'' on the wave number k also changes: there appears a specific "window" with a considerably lower damping in the vicinity of $k \sim k_c$.

The correlations between inhomogeneities of different parameters of the ferromagnet arise, in particular, in the case where there is a common origin of these inhomogeneities. For example, both the exchange and anisotropy inhomogeneities in amorphous alloys are associated, to a considerable extent, with the inhomogeneities of interatomic distances in these materials. Therefore, the existence of correlations ($\kappa \neq 0$) between the exchange and magnetic-anisotropy inhomogeneities can be expected in these alloys. Since the amorphous alloys upon annealing transform into the nanocrystalline state, the above factor responsible for the cross correlations disappears. Consequently, it can be expected that, upon annealing of amorphous alloys, the magnitude of the cross correlation coefficient should decrease or even vanish. This will lead to a change in the dispersion and damping laws for spin waves.

Therefore, a comparison of the specific features revealed in this work with the results of the targeted experimental investigations of modifications of the dispersion and damping laws in inhomogeneous magnets would make it possible to determine the contribution of cross correlations to the formation of the stochastically inhomogeneous ground state in amorphous magnetic alloys.

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