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A METHOD FOR COMPUTING THE MICROWAVE ABSORPTION SPECTRUM IN A DISCRETE MODEL OF A FERROMAGNETIC

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An effective method based on linearization of the Landau–Lifshitz equation has been developed to determine normal magnetization oscillation modes in a discrete model of a condensed medium. The possibility to calculate microwave absorption spectra for ferromagnetic specimens of any shape is shown.

Keywords: magnetization oscillation, Landau–Lifshitz equation, absorption spectrum.

The fundamental equation that describes the dynamics of a magnetic system under the action of external constant and variable magnetic fields is the Landau–Lifshitz nonlinear differential equation. For a discretized model of an inhomogeneous medium, it is necessary to solve a system of connected nonlinear differential equations. To solve systems of this type, numerical integration based on an algorithm similar to the Runge-Kutta algorithm is widely used [1]. The main advantage of this approach is the possibility to predict the evolution of a magnetic system, starting from the initial distribution of magnetic moments in the discrete model. The main disadvantages of this type of algorithm are substantial computational burden and the inconstancy of the magnetic moment vector in discrete elements in iterative calculations, which in fact gives no way of investigating systems that consist of a large number of elements. Therefore, undoubtedly, the search for new approaches and algorithms which would allow one to solve complicated problems of this type is of importance and urgency in modern physics. The potentialities of multilayer magnetic structures used in microelectronic devices also promote the extension of related research works.

In recent years, to study microdiscrete models of complicated magnetic structures, approaches have been developed that have been actively used in optics [2–4]. They are based on a description of the motion of magnetic moments as the sum of the natural oscillations of the normal magnetic modes for the overall system. In this paper, the idea described elsewhere [2] is developed as applied to a discrete model [5], which has shown rather high efficiency.

A ferromagnetic is represented as a discrete medium consisting of N identical (of volume V_0) dipoles $\boldsymbol{\mu}^{(i)}$ ($i = 1, 2, \ldots, N$) that, having a constant saturation magnetization M_s, fill uniformly the whole of the body. Denoting the direction of the ith dipole by $\boldsymbol{M}^{(i)}$, we write an expression for the free energy density of the system, F, taking into account the Zeeman energy, the exchange and dipole interaction energies, and the energy of uniaxial magnetic anisotropy:

$$F(\boldsymbol{M}^{(1)}, \boldsymbol{M}^{(2)}, \ldots, \boldsymbol{M}^{(N)}) = -M_s V_0 \boldsymbol{H} \sum_{i=1}^{N} \boldsymbol{M}^{(i)} + J V_0 \sum_{i=1}^{N} \sum_{j=1}^{N_i} (1 - \boldsymbol{M}^{(i)} \boldsymbol{M}^{(j)})$$
$$+ \frac{M_s^2 V_0^2}{2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \boldsymbol{M}^{(i)} \overleftrightarrow{A}_{ij}^{\mathrm{dip}} \boldsymbol{M}^{(j)} - V_0 \sum_{i=1}^{N} K_i (\boldsymbol{M}^{(i)} \boldsymbol{n}^{(i)})^2. \quad (1)$$

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Here $\boldsymbol{H}$ is the external magnetic field, J is the exchange interaction constant (here the second summation over j in the exchange energy term refers only to the nearest N_i neighbors of the ith dipole), K_i is the constant of uniaxial magnetic anisotropy, and $\boldsymbol{n}^{(i)}$ is a unit vector aligned with the easy magnetization axis. The magnetostatic energy conditioned by the dipole-dipole interaction between discrete elements is described by the tensor $\overleftrightarrow{A}_{ij}^{\mathrm{dip}}$. To calculate its components, one uses either an approximation based on the interaction of a pair of point dipoles [5] or an exact analytic expression [6], which involves time-consuming computations.

It has been shown [5] that the expression for the free energy (1) can be represented as

$$F(\boldsymbol{M}^{(1)}, \boldsymbol{M}^{(2)}, ..., \boldsymbol{M}^{(N)}) = -M_s V_0 \sum_{i=1}^{N} \left[\boldsymbol{H}\boldsymbol{M}^{(i)} + \frac{1}{2} \sum_{j=1}^{N} \boldsymbol{M}^{(i)} \overleftrightarrow{A}_{ij} \boldsymbol{M}^{(j)} \right], \tag{2}$$

where $\overleftrightarrow{A}_{ij}$ is the effective tensor of the interaction between the ith and the jth dipole, whereas the effective local magnet field acting on the kth dipole is described as

$$\boldsymbol{H}^{\mathrm{eff}(k)}(\boldsymbol{M}^{(1)}, \boldsymbol{M}^{(2)}, ..., \boldsymbol{M}^{(N)}) = -\frac{1}{M_s V_0} \frac{\delta F}{\delta \boldsymbol{M}^{(k)}} = \boldsymbol{H} + \sum_{j=1}^{N} \overleftrightarrow{A}_{kj} \boldsymbol{M}^{(j)} . \tag{3}$$

The equation of motion for the kth dipole ($k = 1, \ldots, N$) is the Landau–Lifshitz equation

$$\frac{\partial \boldsymbol{M}^{(k)}}{\partial t} = -\gamma \left[\boldsymbol{M}^{(k)} \times \boldsymbol{H}^{\mathrm{eff}(k)} \right] - \alpha\gamma \boldsymbol{M}^{(k)} \left[\boldsymbol{M}^{(k)} \times \boldsymbol{H}^{\mathrm{eff}(k)} \right]. \tag{4}$$

Here the first term of the sum describes the precession of the magnetic dipole relative to the effective local magnetic field $\boldsymbol{H}^{\mathrm{eff}(k)}$, the second one describes the damping in the system, γ is the gyromagnetic ratio, and α is the damping parameter. Using the method of successive approximations, we seek a solution in the form $\boldsymbol{M}^{(k)} = \boldsymbol{M}_0^{(k)} + \boldsymbol{m}^{(k)}(t)$, $\boldsymbol{H}^{\mathrm{eff}(k)} = \boldsymbol{H}_0^{\mathrm{eff}(k)} + \boldsymbol{h}^{\mathrm{eff}(k)}(t)$, where $|\boldsymbol{m}^{(k)}| << |\boldsymbol{M}_0^{(k)}|$ and $|\boldsymbol{h}^{\mathrm{eff}(k)}| << |\boldsymbol{H}_0^{\mathrm{eff}(k)}|$; $\boldsymbol{M}_0^{(k)}$, which sets the equilibrium direction of the kth dipole, is determined by solving the system of linear inhomogeneous equations [5]

$$\boldsymbol{H}_0^{\mathrm{eff}(k)}(\boldsymbol{M}_0^{(1)}, \boldsymbol{M}_0^{(2)}, ..., \boldsymbol{M}_0^{(N)}) - \nu_k \boldsymbol{M}_0^{(k)} = 0 , \tag{5}$$

with undetermined Lagrange multipliers ν_k; $\boldsymbol{m}^{(k)}(t)$ is the variable component of the magnetic moment the kth dipole. At the same time, the static and dynamic parts of the effective field $\boldsymbol{H}^{\mathrm{eff}(k)}$ are determined, in view of (3) and (5), as

$$\begin{gathered} \boldsymbol{H}_0^{\mathrm{eff}(k)} = \sum_{i=1}^{N} \overleftrightarrow{A}_{ki} \boldsymbol{M}_0^{(i)} + \boldsymbol{H}_0 = \nu_k \boldsymbol{M}_0^{(k)}, \\ h^{\mathrm{eff}(k)}(t) = \sum_{i=1}^{N} \overleftrightarrow{A}_{ki} \boldsymbol{m}^{(i)}(t) + \boldsymbol{h}^{rf(k)} = \boldsymbol{h}^{(k)} + \boldsymbol{h}^{rf(k)}. \end{gathered} \tag{6}$$

Considering only linear terms and taking into account that $[\boldsymbol{M}_0^{(k)} \times \boldsymbol{H}_0^{\mathrm{eff}(k)}] = 0$, we can write the equation of motion (4) in the form

$$\frac{\partial \boldsymbol{m}^{(k)}}{\partial t} = \sum_{i=1}^{N} \overleftrightarrow{B}_{ki} \boldsymbol{m}^{(i)} + \overleftrightarrow{N}_0^{(k)} \boldsymbol{h}^{rf(k)} . \tag{7}$$

The following designations have been used:

$$\vec{M}_{0k} = \begin{bmatrix} 0 & -M_{0z}^{(k)} & M_{0y}^{(k)} 0 \\ M_{0z}^{(k)} & 0 & -M_{0x}^{(k)} \\ -M_{0y}^{(k)} & M_{0x}^{(k)} & 0 \end{bmatrix}, \quad \begin{array}{l} \vec{N}_0^{(k)} = -\gamma\left(\vec{M}_{0k} - \alpha\vec{M}_{0k}^2\right), \\ \vec{B}_{ki} = \begin{cases} \vec{N}_0^{(k)}\left(\vec{A}_{kk} - \nu_k \vec{E}\right), & k = i, \\ \vec{N}_0^{(k)}\vec{A}_{ki}, & k \neq i, \end{cases} \end{array} \tag{8}$$

where $M_{0x}^{(k)}$, $M_{0y}^{(k)}$, and $M_{0z}^{(k)}$ are the $\boldsymbol{M}_0^{(k)}$ vector components and $\vec{E}$ is a 3×3 unit matrix.

To determine the normal magnetic modes, let us consider the case of free magnetization oscillations. Here, in the absence of a variable field, $\boldsymbol{h}^{rf(k)} = 0$, the expression (7) has the form

$$\frac{\partial \boldsymbol{m}^{(k)}}{\partial t} = \sum_{i=1}^{N} \vec{B}_{ki}\boldsymbol{m}^{(i)} \ \ (k = 1, 2, \dots, N). \tag{9}$$

We seek a solution to the system of linear differential equations (9) in the form $\boldsymbol{m}^{(i)}(t) = \boldsymbol{V}^{(i)}e^{\lambda t}$. Here $\lambda = -i\omega$, where ω is the precession frequency of the dipole. As a result, we obtain

$$\sum_{i=1}^{N} \vec{B}_{ki}\boldsymbol{V}^{(i)} = \lambda \boldsymbol{V}^{(k)} \ \ (k = 1, 2, ..., N). \tag{10}$$

Solving equation (10) for the eigenvectors and eigenvalues, we can write the general solution of the homogeneous system (9) as an expansion over the (linearly independent) eigenvectors of the normal magnetic oscillation modes:

$$\boldsymbol{m}^{(i)}(t) = \sum_{m=1}^{M} C_m \boldsymbol{V}_m^{(i)} e^{\lambda_m t} \ \ (i = 1, 2, \dots, N), \tag{11}$$

where M is the number of oscillation modes ($M \leq 2N$) involved in the calculation, λ_m is the eigenvalue (natural resonance frequency of the mode) corresponding to the eigenvector $\boldsymbol{V}_m=[\boldsymbol{V}_m^{(1)}, \boldsymbol{V}_m^{(2)}, \dots, \boldsymbol{V}_m^{(N)}]^{\mathrm{T}}$ (prefix "T" implies transposition), and $\boldsymbol{V}_m^{(1)}, \boldsymbol{V}_m^{(2)}, \dots, \boldsymbol{V}_m^{(N)}$ are the amplitudes of dipole oscillations at the frequency of the mth mode. For stimulated oscillations, it is necessary to solve the system of inhomogeneous equations (7); therefore, we use the method of variation of constants and seek a solution of the form

$$\boldsymbol{m}^{(i)}(t) = \sum_{m=1}^{M} C_m(t) \boldsymbol{V}_m^{(i)} e^{\lambda_m t} \ \ (i = 1, 2, \dots, N). \tag{12}$$

Substituting expression (12) in the system of equations (7), we obtain

$$\sum_{m=1}^{M} \boldsymbol{V}_m^{(k)} e^{\lambda_m t} \frac{\partial C_m(t)}{\partial t} = \vec{N}_0^{(k)} \boldsymbol{h}^{rf(k)} \ \ (k = 1, 2, \dots, N). \tag{13}$$

For the (linearly independent) vectors $\boldsymbol{V}_m$, designating $\boldsymbol{U}_m = \boldsymbol{V}_m^{\mathrm{T}}$, we bring the system of equations (13) to the form

$$\frac{\partial C_m(t)}{\partial t} = e^{-\lambda_m t} \sum_{j=1}^{N} \boldsymbol{U}_m^{(j)} \vec{N}_0^{(j)} \boldsymbol{h}^{rf(j)} \ \ (m = 1, 2, ..., M). \tag{14}$$

Integrating these differential equations, we find

$$C_m(t) = C_m^{\text{free}} + \int e^{-\lambda_m t} \sum_{j=1}^{N} \boldsymbol{U}_m^{(j)} \vec{N}_0^{(j)} \boldsymbol{h}^{rf(j)} dt \,. \tag{15}$$

The expression obtained determines the oscillation amplitude of the mth mode for the excitation of the system by high-frequency signals of arbitrary waveform. We restrict our consideration to the practically important case where the magnetic system is excited by a sinusoidal high-frequency field of frequency ω, for which we have $\boldsymbol{h}^{rf(j)}(t) = \boldsymbol{h}_0^{rf(j)} e^{-i\omega t}$. Then the corresponding expression for the amplitude will take the form

$$C_m(t) = C_m^{\text{free}} + \frac{e^{-(\lambda_m + i\omega)t}}{-(\lambda_m + i\omega)} \sum_{j=1}^{N} \boldsymbol{U}_m^{(j)} \vec{N}_0^{(j)} \boldsymbol{h}^{rf(j)} \,. \tag{16}$$

Substituting expression (16) in (12), we obtain the general solution of the equation of motion for the ith dipole

$$\boldsymbol{m}^{(i)}(t) = \sum_{m=1}^{M} C_m^{\text{free}} \boldsymbol{V}_m^{(i)} e^{\lambda_m t} + \sum_{m=1}^{M} \sum_{j=1}^{N} \frac{\boldsymbol{U}_m^{(j)} \vec{N}_0^{(j)} \boldsymbol{h}^{rf(j)}}{-(\lambda_m + i\omega)} \boldsymbol{V}_m^{(i)} e^{-i\omega t} \,. \tag{17}$$

Note that the first term of the expression describes the free oscillations of the magnetic system. This is easy to verify by setting $\boldsymbol{h}^{rf(j)}$ equal to zero. For a dumped steady-state oscillation, we can write $\boldsymbol{m}^{(i)}(t)$ as

$$\boldsymbol{m}^{(i)}(t) = \sum_{m=1}^{M} \sum_{j=1}^{N} \frac{\boldsymbol{U}_m^{(j)} \vec{N}_0^{(j)} \boldsymbol{h}^{rf(j)}}{-(\lambda_m + i\omega)} \boldsymbol{V}_m^{(i)} e^{-i\omega t} \,. \tag{18}$$

Knowing the equation of motion of a magnetic dipole, we can determine the absorption energy of the system under consideration by using the following expression [7]:

$$E = \omega V_0 \sum_{i=1}^{N} \mathrm{Im}\left[\boldsymbol{m}^{(i)} \boldsymbol{h}^{rf(i)*} \right]. \tag{19}$$

To demonstrate the efficiency of the approach described, we perform a numerical simulation of the spin-wave resonance (SWR) spectrum of a ferromagnetic film magnetized to saturation orthogonal to the film plane by an external constant field H_0. Let the film be a thin disc of diameter 500 μm and thickness $L = 0.5$ μm. Let the magnetic parameters of the film correspond to those of yttrium iron ferrite structured as $Y_3Fe_5O_{12}$ garnet (YIG), with saturation magnetization $M_s = 139$ G, exchange constant $A = J \times d^2 = 10^{-6}$ erg/cm (d is the distance between neighboring discrete dipole elements), and the magnetization precession damping coefficient $\alpha = 0.005$. Note that to simplify calculations in solving the problem under consideration, one can neglect the crystallographic magnetic anisotropy of the specimen. Let the discrete elements constituting the film be 100×100×0.01 μm in size; then their number in the whole of the film volume equals 5×5×50. As the chosen geometry of the "unit cell" is highly sensitive to shape anisotropy, we used an exact analytical expression [6] to calculate the magnetostatic interaction tensor. The external magnetic field H_0 was set equal to 3 kOe, which provided reliable magnetization of the specimen orthogonal to the film plane.

The dispersion relation for spin waves in a normally magnetized ferromagnetic film is well known [8]:

$$\omega = \gamma \left[H_0 - 4\pi M_0 + \frac{2A}{M_s} k_z^2 \right], \tag{20}$$

where k_z is the wave number (projection of the wave vector on the z axis directed orthogonal to the film plane).

Relation (20) describes the spin wave frequency as a function of the wave number, which is determined by the boundary conditions on the top and on the bottom surface of the film. It is well known that in the case of a "free" film (when magnetic moments are not fixed on its surfaces), a linearly polarized uniform microwave field applied in the

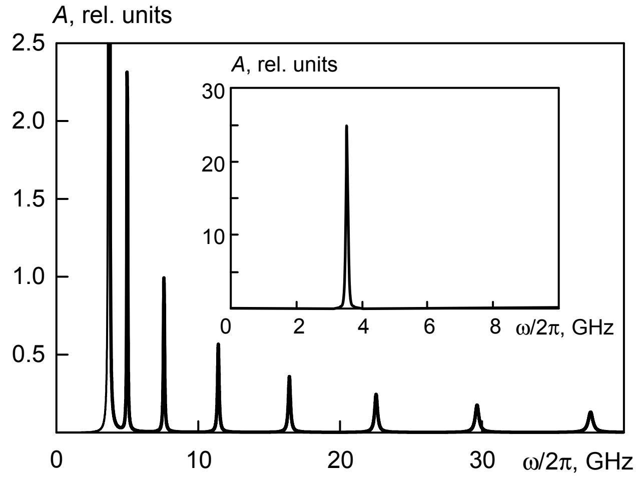

Fig. 1. The SWR spectrum calculated for a normally magnetized ferromagnetic film with magnetic moments rigidly fixed on its surface. The inset shows a uniform FMR peak for a film with nonfixed magnetic moments.

plane can excite only a homogeneous magnetization oscillation mode ($k_z = 0$): homogeneous ferromagnetic resonance (FMR). In the case that magnetic moments are rigidly fixed on both surfaces of the film, nonhomogeneous types of precession are excited, and their wave numbers are determined by the relation

$$k_z = \pi n/L \ (n = 1, 2, 3, \ldots), \tag{21}$$

where n is the number of the spin-wave resonance mode. However, in the case under consideration, as shown elsewhere [8], in the film subject to the action of a linearly polarized planar microwave field, only odd-number ($n = 1, 3, 5, \ldots$) standing spin waves could be excited. Resonances of waves of this type were observed by many researchers, but at a fixed pump frequency and a swept constant magnetic field H_0 [8].

A numerical analysis performed for the given micromagnetic model of a YIG thin film with magnetic moments fixed on the surfaces by using the method described above, has predicted the spectrum of spin-wave resonances in a wide frequency range at a constant magnetic field (Fig. 1). The first resonance obviously corresponds to the first oscillation mode ($k_z = \pi/L$), its amplitude being an order of magnitude greater than the amplitude of the second peak corresponding to the second spin-wave resonance mode ($n = 3$). The rest peaks in the spectrum are associated with the resonances of higher oscillation modes corresponding to $n = 5$, 7, 9, 11, 13, and 15. The inset in Figure 1 shows the spectrum obtained for the same specimen with nonfixed magnetic moments. As expected, the spectrum has a single peak associated with uniform FMR.

The results of the numerical analysis are given in Fig. 2*a*, where curve *1* is a plot of the spin-wave resonance frequency as a function of the squared mode number. In the same figure, curve *2* presents a plot of the dispersion relation (20) for the same variables. It can be seen that both relations are linear and almost coincident. A slight difference for the highest oscillation modes seems to be related to that the dispersion relation (20) has been derived without regard for spin wave damping. In the numerical analysis, however, the damping was taken into consideration and resulted in a certain decrement in resonance frequencies which increased with n.

Of great interest is the behavior of the spin-wave resonance peak amplitude as a function of the mode number. It has been shown [8] that the SWR mode intensity is a linear function of $1/n^2$. The results of the numerical simulation of a micromagnetic model of the YIG film under consideration presented in Fig. 2*b* correlate well with this conclusion.

Thus, the study of SWR spectra performed proves the legitimacy and high effectiveness of the proposed method of numerical analysis of the microwave properties of magnetic films. It should be noted that the simulation presented allows one not only to predict the spectrum of normal magnetization oscillation modes for composite

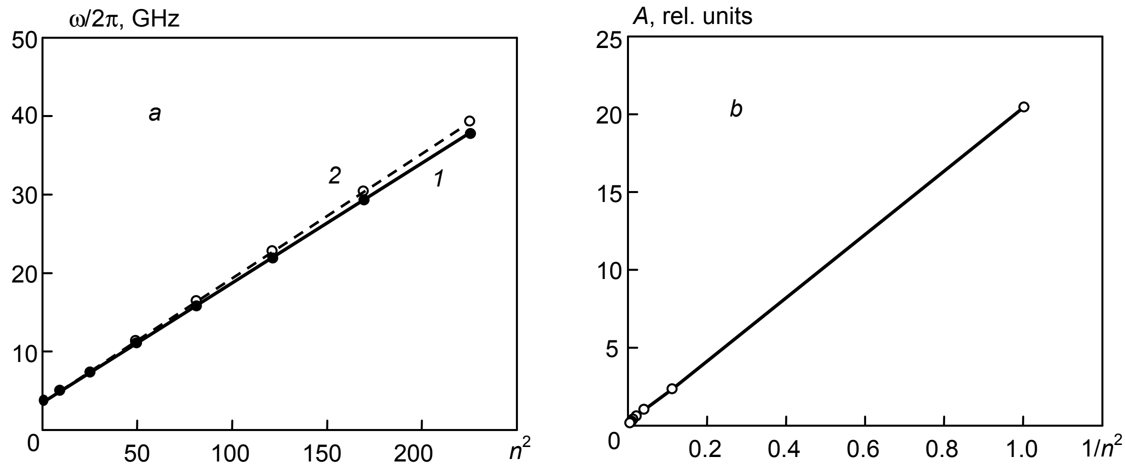

Fig. 2. Resonance frequencies versus mode number in the SWR spectrum (*a*) obtained by a numerical simulation of a micromagnetic model (curve *1*) and by the dispersion equation (20) (curve *2*) and the SWR peak intensity versus mode number *n* (*b*).

magnetic and nonmagnetic material specimens of any shape, but also to calculate the electromagnetic energy absorption spectrum both for a swept magnetic field and for a swept pump frequency. In particular, the method developed allows one to investigate frequency or field dependences of the permeability tensor components for complex multilayer structures, which are nowadays considered the best candidates for new microelectronic units.

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