
OPTICAL
PROPERTIES

Transmission of Light through a Plane-Parallel Plate of a Two-Dimensional Resonant Photonic Crystal

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Abstract—The transmission spectra of two-dimensional resonant photonic crystals of two types have been investigated. Crystals of one type consist of dielectric cylinders forming a square lattice filled by a resonant gas with mercury atoms, and crystals of the other type consist of cylindrical holes filled with a gas and forming a square lattice in a dielectric matrix. It has been established that characteristics of the spectrum of additional transmission arising in the band gap of the photonic crystal can be changed significantly by varying the gas pressure and the angle of incidence. It has been demonstrated that the calculated features in the transmission spectrum of the photonic crystal are stable with respect to a significant increase in the width of the atomic resonance.

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1. INTRODUCTION

Photonic crystals, which are structures with a spatial modulation of dielectric properties on a scale of the order of the wavelength, have a band-type spectrum. Owing to the presence of photonic band gaps and regions of an anomalous increase in the density of photon states in the spectrum of electromagnetic waves, photonic crystals have attracted enhanced interest that is aimed at searching for effective methods of controlling light fluxes [1–3]. This interest is primarily associated with the wide possibilities of using photonic crystal structures for the design of element base for optoelectronic devices and information technologies.

It should be noted that the first theoretical works were concerned with the study of photonic crystals prepared from nonabsorbing and dispersionless materials, whereas photonic crystals involving materials characterized by the absorption and/or frequency dispersion have been widely investigated in the subsequent works. The manifestation of the dispersion leads to an additional significant change in the spectral properties of photonic crystals. However, these changes manifest themselves only in a narrow frequency range in the vicinity of the resonant frequency, so that these crystals are referred to as the resonant photonic crystals. The resonant photonic crystals in which optical resonances of materials are close to the Bragg frequencies of the lattice are most interesting. The band structure and the optical properties of two-dimensional resonant photonic crystals formed by

cylindrical or square rods from ionic materials, which are characterized by the polariton permittivity, were studied in [4, 5] and [6, 7], respectively. The resonant photonic crystals based on exciton resonances have been investigated actively (see, for example, [8–11]). In particular, Poddubnyi [9] studied the exciton–polariton band structure and the light reflection and diffraction spectra of the two-dimensional resonant photonic crystal formed by semiconductor cylinders located at the sites of a square lattice and placed in a dielectric matrix.

Lin et al. [12] proposed and implemented an idea to use two-dimensional photonic band gap structures for the design of a new class of ultrarefractive prisms in the radio-frequency range. The experimental studies [13] of optical limiting in two-dimensional photonic band gap structures prepared from nanochannel glass plates, which make it possible to provide the formation of the band gap in the optical range, have stimulated the extension of the idea regarding ultrarefractive photonic band gap prisms to the optical region. New possibilities for implementing this idea are provided by a combination of the intrinsic dispersion of a photonic band gap structure produced from spatially periodic nanochannels in glass plates and the dispersion of a resonant medium placed inside a periodic structure.

The simplest implementation of the structure with a combined dispersion is an infinite layered medium in which one of the alternating isotropic layers represents a resonant gas [14]. It has been shown that a combination of the resonant dispersion of the gas with the dis-

persion of the photonic crystal structure leads to a qualitative change in the spectra of photonic crystals: there appear narrow transparency bands in the photonic band gap and additional band gaps in the continuous transmission spectrum of the photonic crystal structure. An increase in the dispersion of the photonic band gap structure filled with the gas allows one to substantially improve the spectral resolution as compared to conventional prisms filled with a resonant gas or to spectral devices designed on the basis of photonic band gap prisms [14]. The specific features of the transmission spectrum of a resonantly absorbing one-dimensional photonic crystal were investigated in [15]. The band structure of an infinite two-dimensional resonant photonic crystal was studied in our earlier work [16].

In the present work, the transmission spectrum of two-dimensional structures filled by a resonantly absorbing gas with photonic band gaps has been investigated using the Pendry transfer matrix technique [17]. As in [16], we examine two types of samples of resonant photonic crystals in the form of a plate that is infinite in two directions but has a finite thickness: (a) elements of the crystal represent identical dielectric cylinders of infinite length that form a square lattice embedded in a resonant gas and (b) elements of the crystal are hollow cylinders of infinite length that form a square lattice in a dielectric matrix and are filled with a resonant gas.

2. RESULTS OF CALCULATIONS AND DISCUSSION

First and foremost, we consider the results of the calculation of the transmission spectrum for a plate of a two-dimensional resonant photonic crystal of the *a* type. The structure of the resonant photonic crystal is characterized by the permittivity of rods ε_1 and the permittivity of the resonant gas $\varepsilon_2(\omega)$. The permittivity of the gas in the Lorentz model is given by the expression

$$\varepsilon_2(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 + i\gamma\omega}, \quad (1)$$

where $\omega_p^2 = 4\pi NFe^2/m$, e is the elementary charge, m is the electron mass, N is the density of resonant atoms, F is the oscillator strength, γ is the line width, ω_0 is the central resonant frequency, and ω is the radiation frequency.

We assume that the axis of cylinders is perpendicular to the *xy* plane and parallel to the *z* axis. In the *xy* plane, the centers of the cross sections of dielectric cylinders form the square lattice filled with the reso-

nant gas. In order to calculate the transmission spectrum of *s*-polarized electromagnetic waves propagating in the *xy* plane with the dielectric vector parallel to the *z* axis, we use the transfer matrix formalism developed for one-dimensional layered media and extended by Pendry to the case of two- and three-dimensional photonic crystals [17–20]. The transfer matrix relates the amplitude of plane waves at the output of the medium to the amplitudes of waves at the input of the medium. This matrix is formed by the multiplication of the matrices relating the amplitudes of plane waves in neighboring layers into which the sample is divided. However, in photonic crystals with the dimension higher than one, the transfer matrix is unstable and diverges exponentially, because its eigenvalues increase in a geometric progression with an increase in the number of layers. In order to eliminate this singularity, the transfer matrix is transformed into a scattering matrix, which allows one to eliminate numerical instabilities and to calculate transmittances.

The calculations were performed for the resonant photonic crystal in which the permittivity of the cylinder is close to the corresponding value for a glass $\varepsilon_1 = 3.24$, the structure period was $a = 130$ nm, and the plate thickness in the *x* direction was $L = 20a$. The filling factor, i.e., the fraction of the dielectric or the resonant gas in the photonic crystal for models of the *a* and *b* types, respectively, is represented by the expression $f = \pi r^2/a^2$ (where r is the cylinder radius) and is $f = 24\%$ for the *a* model. The parameters of the resonant gas were taken identical to those used in the study of the band structure of the infinite resonant photonic crystal [16]. The line width and the plasma frequency of the resonant gas, which are close to the corresponding values for mercury vapors [14], are $\gamma = 1.4 \times 10^{-7}\omega_1$ and $\omega_p^2 = 3.8 \times 10^{-8}\omega_1^2$, where $\omega_1 = \pi cn_1/a$ is the characteristic frequency of the photonic band gap and $n_1 = \sqrt{\varepsilon_1}f + (1-f)$ is the averaged refractive index of the medium. The line width $\gamma = 1.2$ GHz corresponds to the resonance of mercury atoms at the wavelength $\lambda_0 = 253.7$ nm.

Figure 1 shows the seed transmission spectrum of the photonic crystal plate of thickness $L = 20a$ for normal incidence of light from vacuum at $\varepsilon_1 = 3.24$ and $\varepsilon_2 = 1$ when all other parameters of the system remain unchanged. The widths of the band gaps in the transmission spectrum of the plate agree with the widths of the gaps along the *x* direction of the Brillouin zone [16]. For the filling factor $f = 24\%$, the first photonic band gap has a maximum width, which in the transmission spectrum lies in the frequency range ω/ω_1 from 0.843 to 1.084. The resonant frequency $\omega_0 =$

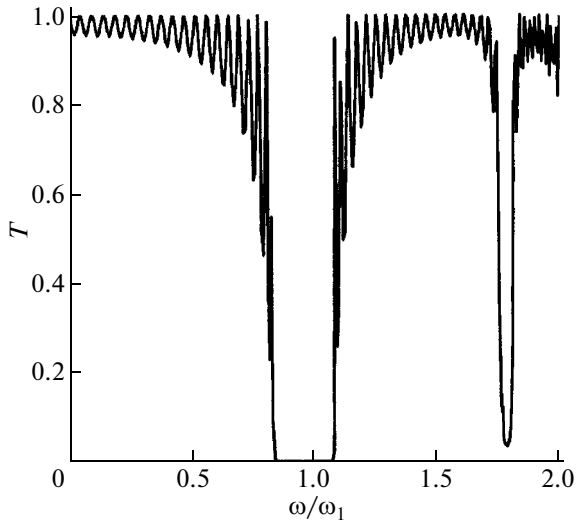


Fig. 1. Transmittance as a function of the frequency for *s*-polarized waves propagating in a plate of thickness $L = 20a$ in the *x* direction. The permittivities of the photonic crystal are $\varepsilon_1 = 3.24$ and $\varepsilon_2 = 1$, and the factor of filling by the dielectric is $f = 24\%$.

$1.214\omega_1$ lies in the continuous spectrum near the high-frequency edge of the band gap.

The inclusion of the frequency dispersion of the permittivity leads to qualitative changes in the structure of the transmission spectrum. These changes that occur near the high-frequency edge of the photonic band gap are shown in Fig. 2 in comparison with the seed transmission spectrum. It can be seen from this figure that a combination of the dispersion of the photonic crystal structure with the dispersion of the gas (relationship (1)) leads to the appearance of additional narrow band gaps in the continuous spectrum of the seed photonic crystal (Fig. 2a) or narrow transmission bands in the band gap of the photonic crystal (Fig. 2b), which are undistinguishable on the scale of Fig. 1. The additional band gap is shown in Fig. 2a for the angle of incidence $\theta = 29.65^\circ$, when the resonant frequency ω_0 coincides with the frequency of the first side maximum of the continuous transmission spectrum. For angles smaller than 29.65° , the frequency ω_0 can lie between two neighboring transmission maxima. In this case, the additional band gap in the transmission spectrum is strongly distorted and is of no practical interest. If ω_0 lies in the photonic band gap near its edge, there arise additional transmission bands. Figure 2b shows the additional transmission band in the photonic band gap for an angle of 30° . This effect of the splitting of the band gap was considered in [11] on the basis of the one-dimensional optical lattice formed by absorbing atoms and also in [14] by using the model of the one-dimensional layered structure in which one of the alternating layers was the resonant gas. The reason for

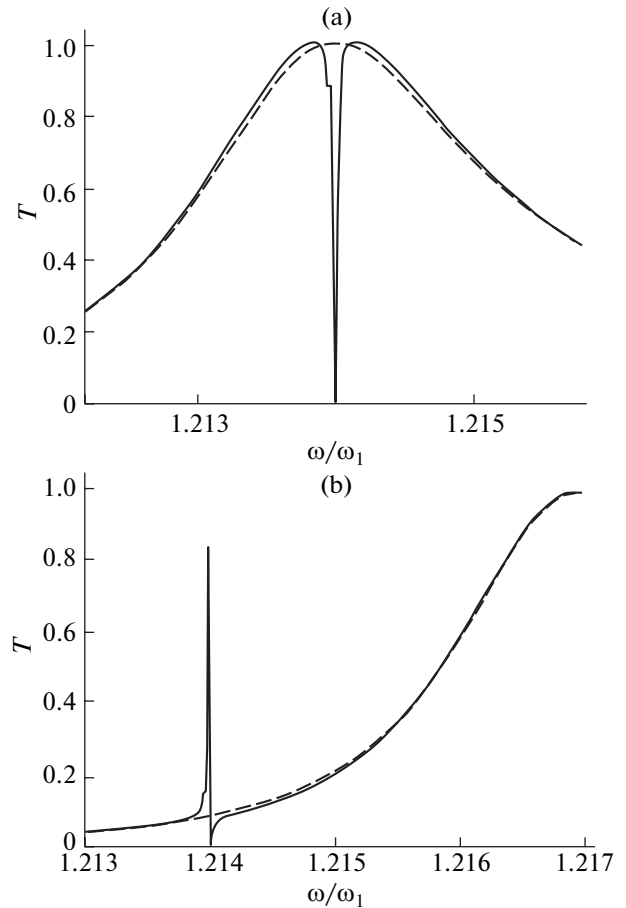


Fig. 2. Frequency dependences of the transmittance near the high-frequency edge of the band gap. Dashed and solid lines indicate the results of the calculations for the seed photonic crystal with $\varepsilon_2 = 1$ and the resonant photonic crystal with $\varepsilon_2(\omega)$ defined by expression (1), respectively: (a) the angle of incidence is equal to 29.65° , and the resonant frequency ω_0 lies at the edge of the continuous spectrum and (b) the angle of incidence is equal to 30.00° , and the resonant frequency ω_0 lies in the band gap. The other parameters are the same as in Fig. 1.

these qualitative changes in the transmission spectrum is mixing of photon modes with the resonance mode. For normal incidence of radiation on the plate, when the resonant frequency lies in the continuous spectrum far from the band gap edge, the above changes in the additional transmission spectrum disappear.

The additional transmittance is very sensitive to a change in the angle of incidence θ , when the band gap edge is close to the resonant frequency of the gas ω_0 . With an increase in θ , the band edge in accordance with the Bragg condition moves away from the frequency ω_0 and the intensity of the transmission band at the maximum decreases sharply. Moreover, the transmission spectrum of the resonant photonic crystal depends significantly on the line width associated with the atomic resonance.

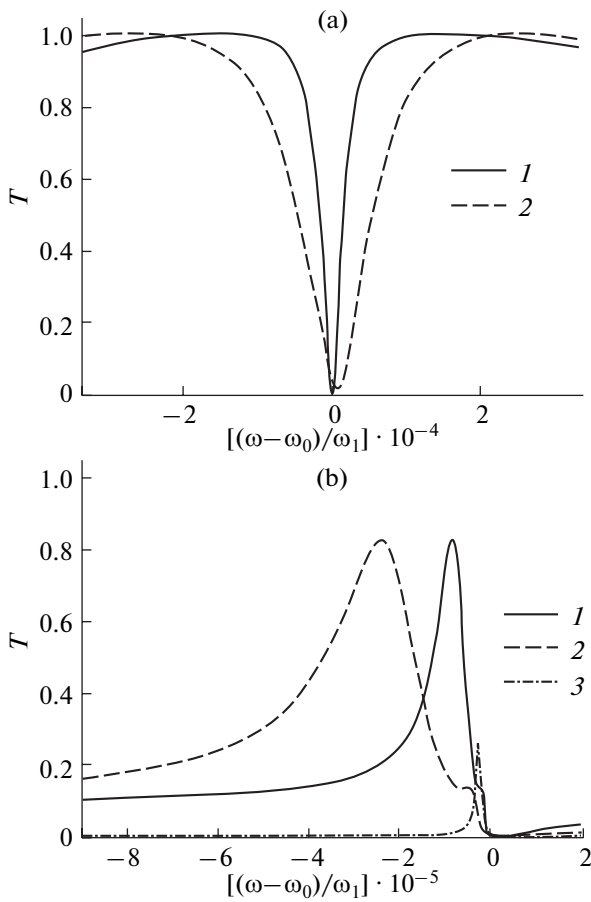


Fig. 3. Transmittance of the resonant photonic crystal as a function of the detuning of the frequency from the resonant frequency of the gas: (a) (1, 2) lines of the additional band gap according to the calculations for (1) $N = 4 \times 10^{14} \text{ cm}^{-3}$, $\gamma = 1.4 \times 10^{-7} \omega_1$ and (2) $N_1 = 3N$, $\gamma_1 = 3\gamma$ at $\theta = 29.65^\circ$ for the resonant frequency lying near the edge of the photonic band gap and (b) (1, 2) lines of the additional transmission band according to the calculations for (1) $N = 4 \times 10^{14} \text{ cm}^{-3}$, $\gamma = 1.4 \times 10^{-7} \omega_1$, (2) $N_1 = 3N$, $\gamma_1 = 3\gamma$ at $\theta = 30.00^\circ$, and (3) for the same quantities N and γ at $\theta = 30.70^\circ$ for ω_0 lying in the photonic band gap. The other parameters are the same as in Fig. 2.

The dependence of the transmittance of the resonant photonic crystal on the detuning of the frequency ω from the resonant frequency of the gas is shown in Fig. 3. It can be seen from Fig. 3a that the line width of the additional band gap is one order of magnitude larger than the width of the resonance line γ . A comparison of the transmission curves for angles of 30.7° and 30° (Fig. 3b) demonstrates that an increase in the angle of incidence from 30° to 30.7° , i.e., by 0.7° , leads to a decrease in the intensity of the additional transmission band at the maximum by three times from 80% to 25%. The line width of additional transmission with a decrease in θ increases: for $\theta = 30.7^\circ$, it is one order of magnitude larger than the width of the resonance line γ and additionally increases by a factor

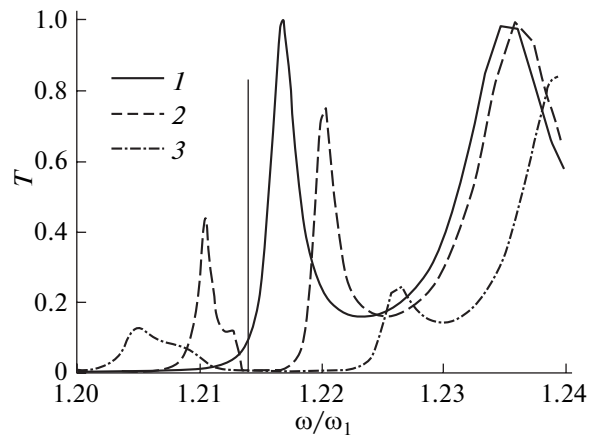


Fig. 4. Frequency dependences of the transmittance for different values of the line width γ : (1) $N = 4 \times 10^{14} \text{ cm}^{-3}$, $\gamma = 1.4 \times 10^{-7} \omega_1$, (2) $N_1 = 10^3 N$, $\gamma_1 = 10^3 \gamma$, and (3) $N_2 = 5 \times 10^3 N$, $\gamma_2 = 5 \times 10^3 \gamma$ at $\theta = 30^\circ$. The other parameters are the same as in Fig. 3.

of three at $\theta = 30^\circ$. When the density of the resonant gas increases by a factor of three, the damping also increases by a factor of three in the case of collision mechanism of broadening. In this case, the width of the additional band gap corresponding to $\theta = 29.65^\circ$ also increases by a factor of three (Fig. 3a). The spectrum of the transmission band corresponding to $\theta = 30^\circ$ is shifted from the resonance toward lower frequencies, the band width increases by a factor of three, and the transmittance of the band at the maximum remains almost unchanged (Fig. 3b).

The aforementioned changes in the transmission spectrum of the two-dimensional resonant photonic crystal with a variation in the angle of incidence and the width of the resonance line are similar to those calculated in the model of the one-dimensional crystal [15]. In order to determine the stability of the calculated features in the transmission spectrum of the photonic crystal with respect to an increase in the width of the resonance, we calculated the transmission spectrum for line widths considerably larger than the line width γ . For comparison, Fig. 4 shows the transmission curves of the resonant photonic crystal for the angle $\theta = 30^\circ$ at different values of the line width and the density of the gas. The solid line in this figure corresponds to the case where the line width γ associated with the resonance of mercury atoms is approximately equal to 1 GHz at a pressure of 6.3 Torr, which corresponds to the density of resonance atoms $N = 4 \times 10^{14} \text{ cm}^{-3}$ [14, 21]. When the density and, correspondingly, the resonance width increase by 5×10^3 times, the intensity of the additional transmission band at the maximum decreases by one order of magnitude, whereas, for small values of damping, it remains almost unchanged (Fig. 3b). However, the

notion of the line width does not lose meaning, and it, as before, considerably lower than the frequency at the maximum of transmission. The edge of the band gap shifts toward the high-frequency range when γ increases, whereas the transmittance corresponding to the band edge decreases. For the resonance width $\gamma_z = 5 \times 10^{-3}\gamma$, the band edge becomes significantly distorted and the transmission intensity at the edge decreases two times as compared to the initial intensity. The decrease in the intensity of the transmission curves at the maximum with the increase in γ near the resonant frequency ω_0 agrees with the frequency dependence of the imaginary part of the permittivity $\text{Im}\varepsilon(\omega)$, which is defined by expression Eq. (1). Indeed, for the exact resonance $\omega = \omega_0$, the absorption is maximum. For larger γ , it decreases near the resonant frequency ω_0 less sharply and, thus, causes the decrease in the intensity at the transmission maxima. Here, it should be noted that the approximation taking into account only the real part of the permittivity $\text{Re}\varepsilon(\omega)$ is not correct. Our calculations show that, in this case, the structure of the transmission spectrum changes qualitatively and four additional transmission bands appear in the band gap. This number of bands is obviously associated with the mixing of the resonance mode with the photon modes, which correspond to the nearest maxima in the continuous transmission spectrum. When both the real part $\text{Re}\varepsilon(\omega)$ and the imaginary part $\text{Im}\varepsilon(\omega)$ of the permittivity are taken into account, the fine structure of the transmission spectrum is not observed, but its trace manifests itself as a shoulder in the profiles of the lines of additional transmission (Fig. 4). The width of the line corresponding to the additional band gap shown in Fig. 3a increases with an increase in the width of resonance to a value of $5 \times 10^3\gamma$, and the steepness of the band edges decreases.

Now, let us discuss the results of the calculations for the resonant photonic crystal sample in the form of a plate of the *b* type, in which the elements are infinite hollow cylinders that are filled with the resonant gas and form the square lattice in the dielectric matrix.

For the filling factor $f = 79.5\%$ and the parameters $\omega_2 = \pi c n_2 / a$, where $n_2 = f + \sqrt{\varepsilon_1(1-f)}$, $\varepsilon_2 = 1$, $\varepsilon_1 = 3.24$, and $a = 130$ nm, this photonic crystal has a maximum width of first band gap, which in the transmission spectrum lies in the frequency range ω/ω_2 from 0.854 to 1.076. The width of the first band gap in the transmission spectrum of the photonic crystal plate, as for a sample of the *a* type, agrees with the width of the corresponding gap in the *x* direction of the Brillouin zone [16]. The resonant frequency $\omega_0 = 1.213\omega_2$ lies in the continuous spectrum near the high-frequency edge of the first band gap. The results of the calculations

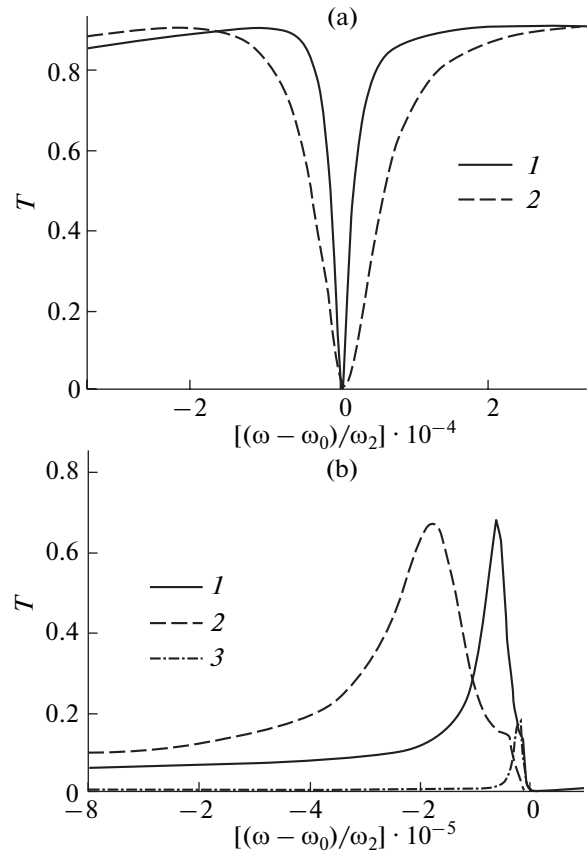


Fig. 5. Frequency dependences of the transmittance for the factor of filling by the dielectric $f = 79.5\%$: (a) (1, 2) lines of the additional band gap according to the calculations for (1) $N = 4 \times 10^{14} \text{ cm}^{-3}$, $\gamma = 1.4 \times 10^{-7}\omega_2$ and (2) $N_1 = 3N$, $\gamma_1 = 3\gamma$ at $\theta = 29.5^\circ$ (ω_0 is the frequency of the first side maximum of the continuous spectrum) and (b) (1, 2) lines of the additional transmission band according to the calculations for (1) $N = 4 \times 10^{14} \text{ cm}^{-3}$, $\gamma = 1.4 \times 10^{-7}\omega_2$, (2) $N_2 = 3N$, $\gamma_2 = 3\gamma$ at $\theta = 30.0^\circ$, and (3) for the same quantities N and γ at $\theta = 30.7^\circ$ (ω_0 lies in the band gap). The other parameters are the same as in Fig. 2.

tions of the additional band gaps and the transparency bands in the transmission spectrum for different angles of incidence and densities of the resonant gas are presented in Fig. 5. A comparison of Figs. 3 and 5 demonstrates that the structures of the transmission spectra of both types of resonant photonic crystals are similar to each other, which is apparently caused by the closeness of their factors of filling with the dielectric. An increase in the width of the atomic resonance leads to the appearance of specific features similar to those described above for a crystal of the *a* type.

3. CONCLUSIONS

Thus, we calculated the transmission spectra for two-dimensional resonant photonic crystals consist-

ing of dielectric cylinders that form a square lattice filled with a resonant gas or, by contrast, hollow cylinders that are filled with a resonant gas and form a square lattice in a dielectric matrix. For close factors of filling of the photonic crystals with the dielectric, the transmission spectra obtained for both types of photonic crystals do not differ substantially. We have analyzed how the structure of the transmission spectrum of the resonant photonic crystal varies with an increase in the line width γ associated with resonance of mercury atoms. For the resonance width $\gamma = 5 \times 10^3$ GHz, the edge of the band gap is strongly distorted, and the intensity of the maximum of the curve of additional transmission in the band gap of the resonant photonic crystal decreases significantly. For a small width of atomic resonances ($\gamma \approx 1$ GHz), the calculated transmission spectra are similar to the transmission spectra of the one-dimensional photonic crystals, which represent layered structures consisting of alternating isotropic dielectric layers and layers of the resonant gas [15]. It should be noted that the specific features obtained in the transmission spectra of the resonant photonic crystals can be calculated using other resonances of atoms or molecules with other geometric sizes of photonic band gap structures. The resonant photonic crystals under consideration can hold promise for the design of narrow-band filters with a high contrast of the filtering of optical radiation and spectral prisms with an increased dispersion. It has been demonstrated that the transmission parameters of the photonic crystals can be controlled by varying the gas pressure and the angle of incidence.

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