# Small-angle light scattering and transmittance of polymer film, containing liquid crystal droplets with inhomogeneous boundary conditions 

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## A R T I C L E I N F O

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#### Abstract

Light scattering by a monolayer of oriented liquid crystal (LC) droplets in polymer matrix is considered. A method for modeling transmittance and angular distribution of scattered light by a monolayer of LC droplets with cylindrical symmetry of the director configuration is developed. It is based on the anomalous diffraction and interference approximations. The internal structures of nematic LC droplets are calculated on base of the free energy minimization problem solution using the relaxation method. The results for monolayers of spherical LC droplets with homogeneous and inhomogeneous boundary conditions are obtained. It is shown that for monolayers containing droplets with inhomogeneous boundary conditions of the "tangential-normal" type there is an asymmetry of the angular structure of the transmitted light over the polar angle.


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## 1. Introduction

Polymer dispersed liquid crystal (PDLC) films are composite materials consisting of liquid-crystal droplets dispersed in a solid polymer matrix. They are used as light modulators for photonics and optoelectronics applications. Controlled light scattering from liquid crystal (LC) materials is achieved by using electrically, magnetically, or thermally induced change in director orientation and molecular configuration to manipulate their optical properties [1-3].

It is possible to create multi-functional devices with controllable parameters on the basis of these films, such as: intensity and phase modulators of light radiation, polarizers and polarization converters of light, lenses, filters, reflectors, flexible displays, etc.

Recently it was proposed and implemented a new method for controlling the structure of liquid crystal droplets [4] in the polymer matrix by an electric field. It

[^0]is based on a modification of the boundary conditions by ionic surfactants. It creates heterogeneity of the boundary conditions and reduces value of the electric field intensity in comparison with the ordinary PDLC films [1,2].

Since liquid crystals are optically anisotropic materials, scattering problems are more difficult to solve for single LC droplet and droplet arrays as compared to optically isotropic particles [5-10]. Typically the single-scattering approximation is used [1,2]. The major results are obtained under the Rayleigh-Gans approximation (RGA) and the anomalous diffraction (AD) approximation. Some results are obtained under the discrete-dipole approximation (DDA) and by the Wentzel-Kramers-Brillouin (WKB) method [1,2,11-16]. Typically concentration of droplets is not small and the ordering of droplets conditioned by their concentration should be taken into account.

We develop a method for analyzing transmittance and angular distribution of light scattered by a monolayer PDLC film with droplet size larger than the light wavelength in the polymer matrix. The method is based on the anomalous diffraction approximation and the interference approximation, which takes into account cooperative
scattering effects at high concentration of droplets [ $1,2,17$ ]. A monolayer of anisotropic spherical LC droplets with cylindrical symmetry of the director configuration and inhomogeneous boundary conditions is considered.

## 2. Basic relations

Let a PDLC monolayer is illuminated by a normally incident linearly polarized plane wave. The laboratory coordinate system ( $x y z$ ) is defined by light illumination direction ( $x$ axis), and the film plane ( $y z$ ).

Here we consider the monolayers of large droplets. Ordinary refractive index of liquid-crystal and refractive index of polymer are nearly equal. Typical value of birefringence is about 0.2 [2,3]. Under these conditions, multiple scattering is negligible, and we can use interference approximation which takes into account the far-field interference of waves scattered by the droplets. The multiple-scattering contribution decreases with the droplet size increase as more light is scattered forward. Taking into account the far-field interference of waves scattered by the droplets, we write expressions for the intensities of the $v v$ - and $v h$-components of the incoherent light as follows [17]:

$$
\begin{align*}
I_{\mathrm{inc}}^{v \nu}= & \frac{E_{i}^{2} N}{k^{2} R^{2}} \sum_{l=1}^{m} P_{l}\left|f_{l}^{\nu v}\left(\mathbf{k}_{s}\right)\right|^{2} \\
& +\frac{E_{i}^{2} N}{k^{2} R^{2}} \sum_{l, l^{\prime}=1}^{m} P_{l} P_{l^{\prime}} f_{l}^{v v}\left(k_{s}\right) f_{l^{\prime}}^{v * *}\left(\mathbf{k}_{s}\right)\left(S_{l l^{\prime}}\left(\mathbf{k}_{s}\right)-1\right),  \tag{1}\\
I_{\mathrm{inc}}^{v h}= & \frac{E_{i}^{2} N}{k^{2} R^{2}} \sum_{l=1}^{m} P_{l}\left|f_{l}^{\nu h}\left(\mathbf{k}_{s}\right)\right|^{2} \\
& +\frac{E_{i}^{2} N}{k^{2} R^{2}} \sum_{l, l^{\prime}=1}^{m} P_{l} P_{l^{\prime}} f_{l}^{v h}\left(\mathbf{k}_{s}\right) f_{l^{\prime}}^{v h *}\left(\mathbf{k}_{s}\right)\left(S_{l l^{\prime}}\left(\mathbf{k}_{s}\right)-1\right) . \tag{2}
\end{align*}
$$

The $v v$ - and $v h$-components correspond to the geometries of parallel and crossed polarizer and analyzer. The first one ( $v v$ ) corresponds to the component of the light which is parallel to the polarization plane of the incident wave, the second one ( $v h$ ) corresponds to the component of the light which is perpendicular to the polarization plane of the incident wave.

In Eqs. (1) and (2) $R$ is the distance from the coordinate origin to the observation point, $\mathbf{k}_{s}$ is the wave vector of the scattered wave, $N$ is the number of droplets within the illuminated area $A$. Functions $S_{l l^{\prime}}\left(\mathbf{k}_{s}\right)$ are the partial structure factors:
$S_{l l}\left(\mathbf{k}_{s}\right)=1+\Lambda \int_{A}\left(W_{l l^{\prime}}(\mathbf{r})-1\right) \exp \left(i \mathbf{k}_{s} \mathbf{r}\right) \mathrm{d} \mathbf{r}$.
In the expressions above, $E_{i}$ is electric field amplitude of the incident wave; $k$ is the wavenumber in the polymer; subscripts $l$ and $l^{\prime}$ refer to LC droplet types that differ in terms of shape, size, internal structure, etc.; $m$ is the number of LC droplet types; $P_{l}, P_{l^{\prime}}$ denote the partial surface concentrations of droplets of types $l$ and $l^{\prime} ; \Lambda$ is the mean surface concentration of LC droplets; the pair distribution function $W_{I l^{\prime}}(\mathbf{r})$ describes the probability that droplets of types $l$ and $l^{\prime}$ are separated by the relative position vector $\mathbf{r}$ in the $x y$ plane; $f_{l}^{v v}\left(\mathbf{k}_{s}\right)$ and $f_{l}^{v h}\left(\mathbf{k}_{s}\right)$ are the
$v v$ - and $v h$-components of the scattering matrix in the $\mathbf{k}_{s}$ direction for LC droplets of type $l$; and the asterisk denotes the complex conjugat. According to expressions (1)-(3), to analyze the angular distribution of scattered light, the scattering-matrix components $f_{l}^{v v}\left(\mathbf{k}_{s}\right)$ and $f_{l}^{\nu h}\left(\mathbf{k}_{s}\right)$ should be determined by solving the scattering problem for single LC droplets, and the partial structure factors $S_{l l}\left(\mathbf{k}_{s}\right)$ should be found by calculating the pair distribution functions $W_{l l^{\prime}}(\mathbf{r})$. General solution of these problems is a formidable task because of the complexity of external effects on the molecular configuration inside LC droplet. This motivates using of approximate methods to obtain simplified solutions relating the angular distribution of light scattered by a PDLC monolayer to the orientational structure of the layer.

The general consideration of the problem is cumbersome, so for the simplicity, we write equations for coherent and incoherent transmitted light for monolayer of oriented monodisperse spherical LC droplets. The numerical results are presented for the monodisperse droplets [18-20]. The data of Fig. 9 display the results for polydisperse droplets with gamma distribution $[17,21]$ of sizes.

Equation for coherent transmittance for $v v-\left(T_{c}^{v v}\right)$ and $v h-\left(T_{c}^{v h}\right)$ components can be written as follows [17]:
$T_{c}^{v v}=\left|T_{a}^{v v}\right|^{2}=1-\frac{4 \pi}{k^{2} \sigma} \eta \operatorname{Re} f_{v v}^{0}+\frac{4 \pi^{2}}{k^{4} \sigma^{2}} \eta^{2}\left|f_{v v}^{0}\right|^{2}$,
$T_{c}^{v h}=\left|T_{a}^{v h}\right|^{2}=\frac{4 \pi^{2}}{k^{4} \sigma^{2}} \eta^{2}\left|f_{v h}^{0}\right|^{2}$.
Here, $\eta$ is the filling factor of the PDLC monolayer, which is numerically equal to the ratio of the section area of LC droplets by the monolayer plane to the area where they are distributed, $k=2 \pi n_{p} / \lambda$ is the wave number, $n_{p}$ is the refractive index of the polymer matrix, $\lambda$ is the wavelength of incident light, $f_{v v}^{0}$ and $f_{v h}^{0}$ are the $v v$ - and $v h$ - components of the vector amplitude scattering function $[6,22]$ in the forward direction $\left(\theta_{s}=0\right), \sigma$ is the cross section of a droplet.

Coherent transmission coefficient of PDLC monolayer under the normal illumination by linearly polarized light, at the absence of polarization devices, is the sum of Eqs. (4) and (5). We write the result as follows:
$T_{c}^{p}=1-Q_{p} \eta+\frac{Q_{p}^{2} L_{T}}{2} \eta^{2}$.
Here $Q_{p}$ is the extinction efficiency factor of linearly polarized light of a single droplet:
$Q_{p}=\frac{4 \pi}{k^{2} \sigma} \operatorname{Re} f_{v v}^{0}$,
$L_{T}=\frac{1}{2}\left(1+\frac{\operatorname{Im}^{2} f_{v v}^{0}}{\operatorname{Re}^{2} f_{v v}^{0}}\right)\left(1+\frac{\left|f_{v h}^{0}\right|^{2}}{\left|f_{v v}^{0}\right|^{2}}\right)$.
To consider the angular structure of radiation scattered by monolayer of droplets we use the interference approximation [13,21,22]. At this approximation for the parallel $\left(I_{v v}\right)$ and perpendicular $\left(I_{v h}\right)$ components of the incoherently
scattered intensity, we obtain:
$I_{v v}\left(\theta_{s}, \varphi_{s}\right)=C \frac{\eta}{\sigma k^{2}}\left|f_{v v}\left(\theta_{s}, \varphi_{s}\right)\right|^{2} S\left(\theta_{s}\right)$,
$I_{v h}\left(\theta_{s}, \varphi_{s}\right)=C \frac{\eta}{\sigma k^{2}}\left|f_{v h}\left(\theta_{s}, \varphi_{s}\right)\right|^{2} S\left(\theta_{s}\right)$.
Here $C=A E_{i}^{2} / R^{2}$ is the normalization constant, $f_{v v}\left(\theta_{s}, \varphi_{s}\right)$ and $f_{v h}\left(\theta_{s}, \varphi_{s}\right)$ are the $v v$ - and $v h$-components of the vector amplitude scattering function in the direction of the wave vector of the scattered wave $\boldsymbol{k}_{s}=\left(k \cos \theta_{s}\right.$, $\left.k \sin \theta_{s} \cos \varphi_{s}, k \sin \theta_{s} \sin \varphi_{s}\right), S\left(\theta_{s}\right)$ is the structure factor of the monolayer $[10,17,22], \theta_{s}$ and $\varphi_{s}$ are the polar and azimuthal scattering angles, respectively. For a statistically isotropic ensemble of droplets in the form of spheres or oblate spheroids, it does not depend on the azimuthal angle $\varphi_{s}$. At low concentration of LC droplets single scattering of light is implemented. In this case the structure factor $S\left(\theta_{s}\right) \approx 1$ for all values of the polar scattering angle. The difference of the structure factor from unity indicates the degree of influence of interference effects in angular structure of scattered light.

Components of the amplitude function in the expressions (9) and (10) are defined in terms of the elements of the amplitude scattering matrix $S_{j}(j=1,2,3,4)[5,6,17]$ as follows:

$$
\begin{align*}
f_{v v}\left(\theta_{s}, \varphi_{s}\right)= & S_{2}\left(\theta_{s}, \varphi_{s}\right) \cos ^{2}\left(\alpha-\varphi_{s}\right)+S_{1}\left(\theta_{s}, \varphi_{s}\right) \sin ^{2}\left(\alpha-\varphi_{s}\right) \\
& +\frac{1}{2}\left(S_{3}\left(\theta_{s}, \varphi_{s}\right)+S_{4}\left(\theta_{s}, \varphi_{s}\right)\right) \sin 2\left(\alpha-\varphi_{s}\right), \tag{11}
\end{align*}
$$

$$
\begin{align*}
f_{v h}\left(\theta_{s}, \varphi_{s}\right)= & S_{3}\left(\theta_{s}, \varphi_{s}\right) \sin ^{2}\left(\alpha-\varphi_{s}\right)-S_{4}\left(\theta_{s}, \varphi_{s}\right) \cos ^{2}\left(\alpha-\varphi_{s}\right) \\
& +\frac{1}{2}\left(S_{2}\left(\theta_{s}, \varphi_{s}\right)-S_{1}\left(\theta_{s}, \varphi_{s}\right)\right) \sin 2\left(\alpha-\varphi_{s}\right) . \tag{12}
\end{align*}
$$

For light scattered in the direction of the incident wave, the off-diagonal elements of the scattering matrix are zero $\left(S_{3}=S_{4}=0\right)$. In this case the components of the vector amplitude scattering function can be written as follows:
$f_{v v}^{0}=S_{2}^{0} \cos ^{2} \alpha+S_{1}^{0} \sin ^{2} \alpha$,
$f_{v h}^{0}=\frac{1}{2}\left(S_{2}^{0}-S_{1}^{0}\right) \sin 2 \alpha$.
Here $S_{2,1}^{0}$ are the diagonal elements of the amplitude scattering matrix at zero scattering angle ( $\theta_{s}=0$ ); $\alpha$ is the polarization angle of the incident light. This is the angle between polarization vector of the incident light and $y$-axis.

In the anomalous diffraction approximation the field scattered by particle in the far zone is considered as the result of diffraction on the amplitude-phase screen. Complex transmission matrix of the screen is given by the projection of droplet onto a plane orthogonal to the direction of the incident light [6]. When droplet is illuminated along the $x$ axis, using the known results [23] for the amplitude scattering matrix elements, we get:
$S_{1}\left(\theta_{s}, \varphi_{s}\right)=\frac{k^{2} \sigma}{2 \pi} \int_{\sigma}\left(1-T_{1}(\xi)\right) \exp \left(-i \mathbf{k}_{s} \xi\right) \mathrm{d} \xi$,
$S_{2}\left(\theta_{s}, \varphi_{s}\right)=\frac{k^{2} \sigma}{2 \pi} \cos \theta_{s} \int_{\sigma}\left(1-T_{2}(\xi)\right) \exp \left(-i \mathbf{k}_{s} \xi\right) \mathrm{d} \xi$,
$S_{3}\left(\theta_{s}, \varphi_{s}\right)=-\frac{k^{2} \sigma}{2 \pi} \cos \theta_{s} \int_{\sigma} T_{3}(\xi) \exp \left(-i \mathbf{k}_{s} \xi\right) \mathrm{d} \xi$,
$S_{4}\left(\theta_{s}, \varphi_{s}\right)=-\frac{k^{2} \sigma}{2 \pi} \int_{\sigma} T_{4}(\xi) \exp \left(-i \mathbf{k}_{s} \xi\right) \mathrm{d} \xi$.
Here $\xi$ is the radius-vector in the cross section $\sigma, T_{j}(\xi)$ are the elements of the $2 \times 2$ transmission matrix of the equivalent amplitude-phase screen $(j=1,2,3,4)$. It depends on the internal structure of the LC droplet orientation [24]. Eqs. (15)-(18) allow us to calculate the amplitude matrix elements of the scattering at homogeneous and inhomogeneous boundary conditions on the surface of the droplet based on the configuration of the director (the distribution of local optical axis) in the droplet. The configuration of the director [25-28] in the droplet with given boundary conditions and applied voltage was found by solving the problem of minimizing the bulk free energy density by using results published in [29].

## 3. Results and discussion

Schematic presentation of the surface of liquid crystal droplet with inhomogeneous boundary conditions is displayed in Fig. 1. The structure is symmetric with respect to the $y$-axis. There is normal and tangential orientation of liquid crystal molecules at the droplet-polymer interface. They are indicated in the figure by short lines, which are normal and parallel to the surface, respectively. Parameter $W$ (see figure) characterizes the fraction of the surface layer of droplet with normal boundary conditions. The value of $W$ is determined by the ratio of the height of segment surface of the droplet with normal boundary conditions to droplet diameter. The parameter values $W=0 \%$ and $W=100 \%$ correspond to a fully homogeneous tangential and normal surface adhesion of LC molecules at the interface LC-polymer, respectively. When parameter


Fig. 1. Schematic presentation of the surface of liquid crystal droplet with inhomogeneous boundary conditions.


Fig. 2. Textures of LC droplets in crossed polarizers with homogeneous ( $W=0$ and $W=100 \%$ ) and inhomogeneous boundary conditions (IBC) at different values of parameter $W$. The value of $W$ is determined by the ratio of the height of segment surface of the droplet with normal boundary conditions to droplet diameter as it is shown in Fig. 1. No applied field. Left picture shows a droplet with bipolar director configuration, right picture shows a droplet with radial director configuration.


Fig. 3. Coherent transmission coefficient for polarized light $T_{c}^{p}$ of a monolayer of spherical LC droplets on the diffraction parameter $\rho$ at different values of parameter $W(W=0 \%, W=25 \%, W=50 \%, W=75 \%$, and $W=100 \%$ ). No applied field. The wavelength of incident light $\lambda=0.633 \mu \mathrm{~m}$. Polarization angle of the incident light $\alpha=0$. Azimuthal scattering angle $\varphi_{s}=0$. Refractive indices of the LC $n_{o}=1.531, n_{e}=1.717$. Refractive index of the polymer $n_{p}=1.53$. Filling factor of the monolayer $\eta=0.5$.
$W=0 \%$ there is a cylindrically symmetric bipolar configuration of the LC droplet with rigidly fixed poles. At $W=100 \%$ the internal structure of the LC droplets orientation is radially symmetric. Pay attention that Fig. 1 displays only a general director structure at the surface. In reality transition from the tangential anchoring to the homeotropic can be not sharp [26,27].

The director configuration of the LC droplet is calculated in a Cartesian coordinate system by finding the distribution of directors of the elementary volumes of liquid crystal in the droplet using the relaxation method of the bulk free energy density minimization problem solution [29]. Textures of the considered LC droplets with homogeneous and inhomogeneous boundary conditions (at different values of parameter $W$ ) at the absence of the applied field in crossed polarizers are displayed in Fig. 2. Such pictures are observed in the polarization microscope when the directions of analyzer and polarizer are crossed [26-29].


Fig. 4. Degree of light polarization for coherent transmitted light at different values of parameter $W(W=0 \%, W=25 \%, W=50 \%, W=75 \%$, and $W=100 \%$ ). No applied field. $\lambda=0.633 \mu \mathrm{~m}, n_{o}=1.531, n_{e}=1.717$, $n_{p}=1.53, \eta=0.6, \alpha=0 . \varphi_{s}=0$.


Fig. 5. Dependence of $I_{v v}$-component of the light intensity scattered by a monolayer of spherical LC droplets with homogeneous tangential ( $W=0 \%$ ) and normal ( $W=100 \%$ ) boundary conditions as function of the polar scattering angle $\theta_{s}$ with value of the azimuthal scattering angle $\varphi_{s}=0$. No applied field. Droplet radius $a=4 \mu \mathrm{~m} . \lambda=0.633 \mu \mathrm{~m}, n_{o}=1.531$, $n_{e}=1.717, n_{p}=1.532, \eta=0.5, \alpha=0 . \varphi_{s}=0$.

Dependence of the coherent light transmission $T_{c}^{p}$ calculated by the Eq. (6) for the layer with spherical droplets of LC on the diffraction parameter $\rho=k a$ (where $a$ is the radius of the droplets) is shown in Fig. 3. The calculations are performed in the absence of an external electric field, the polarization angle of incident light $\alpha=0$, the ordinary and extraordinary refractive indices of the LC are $n_{o}=1.531$ and $n_{e}=1.717$, respectively; incident light wavelength $\lambda=0.633 \mu \mathrm{~m}$, the refractive index of the polymer $n_{p}=1.53$, the monolayer filling factor $\eta=0.5$. The results displayed in Fig. 3 show that at linearly polarized light illumination with polarization angle $\alpha=0$, the monolayer with radial internal structure of droplets ( $W=100 \%$ ) has larger values of the coherent transmission coefficient $T_{c}^{p}(\rho)$ virtually in the entire range of considered values of diffraction parameter $\rho$ (from 5 to 80) compared to the bipolar configuration of LC droplets ( $W=0 \%$ ).


Fig. 6. Dependence of $I_{v v}$-component of the light intensity scattered by a monolayer of spherical LC droplets with inhomogeneous boundary conditions at different values of parameter $W$ ( $W=25 \%, W=50 \%$, and $W=75 \%$ ). No applied field. $\lambda=0.633 \mu \mathrm{~m}, n_{o}=1.531, n_{e}=1.717$, $n_{p}=1.532, \eta=0.5, a=4 \mu \mathrm{~m}, \alpha=0 . \varphi_{s}=0$.

When the monolayer is illuminated by a linearly polarized light with polarization angle $\alpha=\pi / 2$, the values of $T_{c}^{p}(\rho)$ for layer with tangential boundary conditions (and the bipolar configuration of the LC droplets) are greater than the values $T_{c}^{p}(\rho)$ for the layer with normal boundary conditions (and the radial configuration of the droplets).

The degree of polarization $P$ of the transmitted light on the droplet size at the absence of the external applied electric field at different values of $W$ is shown in Fig. 4. It is determined by the relation:
$P=\frac{T_{c}^{\|}-T_{c}^{\perp}}{T_{c}^{\|}+T_{c}^{\perp}}$.
Here
$T_{c}^{\|}=\left|T_{a}^{v v}(\alpha=0)\right|^{2}$,
$T_{c}^{\perp}=\left|T_{a}^{v v}(\alpha=\pi / 2)\right|^{2}$
$T_{a}^{v v}$ is the amplitude transmittance of layer for $v v$-polarization of the transmitted light for polarization of the incident light parallel $(\alpha=0)$ and perpendicular ( $\alpha=\pi / 2$ ) to the $y$-axis of the lab system, respectively.

The results for the angular structure of $I_{v v}$ component of the transmitted light on the polar scattering angle $\theta_{s}$ at $\varphi_{s}=0$ are shown in Figs. 5 and 6 at the absence of the applied field. The calculations are presented in relative units (value of the normalization constant $C=1$ ). They are performed for the droplets radius $a=4 \mu \mathrm{~m}$ and the refractive index of the polymer matrix $n_{p}=1.532$. The values of other parameters are indicated in figure captions. Fig. 5 shows dependence of $I_{v v}$ components of the scattered light on the polar scattering angle $\theta_{s}$ for monolayer of droplets with homogeneous boundary conditions ( $W=0 \%$ and $W=100 \%$ ). The data are calculated at the absence of the applied electric field. Scattering plane coincides with the principal plane $(y x)$. The value of the azimuthal scattering angle $\varphi_{s}=0$ and the light incident angle is zero. The results for intensity of scattered light $I_{v v}\left(\theta_{s}\right)$ in the
b


Fig. 7. Dependence of $I_{v v}$-component of the light intensity scattered by a monolayer of spherical LC droplets with inhomogeneous boundary conditions at different values of filling factor $\eta=0.01,0.02,0.03$ (a), $\eta=0.1,0.2,0.4$ (b). No applied field. $W=50 \% . \lambda=0.633 \mu \mathrm{~m}, n_{o}=1.531, n_{e}=1.717, n_{p}=1.532, \eta=0.5$, $a=5 \mu \mathrm{~m}, \alpha=0 . \varphi_{s}=0$.


Fig. 8. Dependence of $I_{v v}$-component of the light intensity scattered by a monolayer of spherical LC droplets with inhomogeneous boundary conditions ( $W=50 \%$ ) at weak ( $E_{y}=2$ ) and strong ( $E_{y}=100$ ) normalized electric field $E_{y}$ applied to the layer parallel to the optical axes of the droplets (along $y$-axis). $\lambda=0.633 \mu \mathrm{~m}, n_{o}=1.531, n_{e}=1.717, n_{p}=1.532$, $\eta=0.5, a=4 \mu \mathrm{~m}, \alpha=0 . \varphi_{s}=0$.
layers containing droplets with inhomogeneous boundary conditions are presented in Fig. 6.

For a layer of droplets with homogeneous boundary conditions, the values of $I_{v v}\left(\theta_{s}\right)$ are identical for the same deviations in the polar scattering angle $\theta_{s}$ in different directions relative to the normal to the layer $\left(I_{v v}\left(\theta_{s}\right)=\right.$ $\left.I_{v v}\left(-\theta_{s}\right)\right)$ at $\varphi_{s}=0$. For a layer of droplets with inhomogeneous boundary conditions, such as "tangential-normal", there is an asymmetry of the angular structure of scattered light over the polar angle $\left(I_{v v}\left(\theta_{s}\right) \neq I_{v v}\left(-\theta_{s}\right)\right)$. This effect is most pronounced at parameter $W=50 \%$, i.e., at equal share of the tangential and normal boundary conditions on the droplet surface.

Analysis of the vh-component of the scattered light intensity shows that this component also has asymmetry in the scattering of light in polar angle, but it is weaker in comparison with the $v v$-component.

Influence of droplet concentration is illustrated by Fig. 7. At the considered parameters one can see the asymmetry in angular structure for transmitted light for a wide range of concentrations. The angular structure of


Fig. 9. Components $I_{v v}$ (left) and $I_{v h}$ (right) of light scattered in small angles at different concentrations of polydisperse LC droplets for monolayer of droplets with homogeneous director configuration in the droplet in a strong applied field in the monolayer plane along $y$-axis in a monolayer. Filling factor $\eta=0.3$ (top), $\eta=0.7$ (bottom). The mean radius of droplets $\langle a\rangle=2.5 \mu \mathrm{~m}$. Variation coefficient of droplet radii is 0.2 . $\lambda=0.589 \mu \mathrm{~m}, n_{o}=1.5183$, $n_{e}=1.7378, n_{p}=1.524, \alpha=0$. The mean value of the director of droplets is situated in the plane of monolayer along $y$-axis $\left(\left\langle\theta_{d}\right\rangle=\pi / 2,\left\langle\varphi_{d}\right\rangle=0\right)$. Droplet directors are distributed uniformly relative to the mean direction of droplets in 12 degrees of polar and azimuthal angles.


Fig. 10. Schematic representation of the orientation of the LC droplet directors in the monolayer. Vector $\langle\mathbf{d}\rangle$ is the average direction of orientation of the directors of the droplets; $\left\langle\theta_{d}\right\rangle$ and $\left\langle\varphi_{d}\right\rangle$ are the averaged polar and azimuthal angles defining the orientation of the vector $\langle\mathbf{d}\rangle ; \theta_{d}$ and $\varphi_{d}$ are the polar and azimuthal angles defining the orientation of the directors $\mathbf{d}_{j}$ of individual droplets; $2 \varphi_{m}$ is the opening angle of the cone of directors of the droplets in the plane $(\langle\mathbf{d}\rangle, z), 2 \theta_{m}$ is the opening angle of the cone of directors of the droplets in the plane $(\langle\mathbf{d}\rangle, x)$.
concentrated layer has asymmetry determined by the specific of scattering diagram of the separate anisotropic droplet and ordering of droplets due their high concentration.

Influence the electric field (normalized to the threshold value) applied parallel to the optical axes of the droplets on the angular structure of light scattering at $W=50 \%$ is illustrated by Fig. 8. At weak applied field the angular structure of light scattering is asymmetric. In a strong field the internal structure of droplet becomes uniformly oriented along the applied field and the angular structure of the scattered light becomes symmetric.

Angular structure of $I_{v v}\left(\theta_{s}, \varphi_{s}\right)$ and $I_{v h}\left(\theta_{s}, \varphi_{s}\right)$ components of the scattered light intensity in the angles $0<\theta_{s} \leq 8^{\circ}, 0<\varphi_{s} \leq 360^{\circ}$ for monolayer of droplets with homogeneous director configuration in the droplet in a strong applied field in the monolayer plane along $y$-axis is shown in Fig.9. These results are obtained for monolayer of polydisperse droplets with gamma distribution of sizes [17]. We suppose that the droplet directors $\mathbf{d}_{j}$ are distributed uniformly (Fig. 10) relative to the average direction of the directors of the droplets $\langle\mathbf{d}\rangle$ over the polar $\theta$ and azimuthal $\varphi$ angles in the range of angles $\pm \theta_{m}$ and $\pm \varphi_{m}$, where $\theta_{m}$ and $\varphi_{m}$ are maximal polar and azimuthal angles, respectively. The values $y=\sin \theta_{s} \cos \varphi_{s}$ and $z=\sin \theta_{s} \sin \varphi_{s}$ are plotted on the vertical and horizontal axes, respectively. The $I_{v v}$ and $I_{v h}$ components are presented in relative units. The greater is the intensity of scattered light, the brighter is the area in the figures. They display the intensities of the scattered light in the far zone on the screen. concentrations of polydisperse LC droplets in a monolayer.

## 4. Conclusion

A method for modeling and computing of light propagation via a monolayer of liquid-crystal droplets dispersed in polymer matrix is developed. Light scattering by films containing droplets with homogeneous and
inhomogeneous adhesion of liquid crystal molecules on the interface polymer-liquid crystal is investigated.

The effect of asymmetry in polar scattering angle for the light scattered in forward hemisphere for polymer films containing droplets with inhomogeneous boundary conditions has been revealed. This effect should be observed for the light scattered in the backward hemisphere.

The method provides a tool for examining the liquid crystal droplet configuration. It can be used to study fieldand temperature-induced phase transitions in LC droplets with cylindrical symmetry (bipolar, axial, etc.) by analyzing the transmittance, polarization, and angular distribution of forward-scattered light.

The obtained results can be used to analyze transmittance, angular structure, and polarization characteristics of light scattered by a monolayer of liquid crystal droplets with homogeneous and inhomogeneous boundary conditions.

The results can be applied in developing new types of various liquid crystal devices, for example, amplitude and phase modulators, polarization converters, displays, etc.

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