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Kohn-Luttinger effect and anomalous pairing in repulsive Fermi-systems at low density (Review Article)

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We demonstrate the possibility of triplet p-wave pairing at low electron density a large number of models such as 3D and 2D Fermi-gas models with hard-core repulsion, 3D and 2D Hubbard models, and the Shubin-Vonsovsky model. The critical temperature for p-wave pairing can be considerably higher in the spin-polarized case or even in a two-band situation at low density and can reach experimentally observable values of 1–5 K. We also discuss briefly the d-wave pairing and high-T_c superconductivity with $T_c \sim 100$ K which arise in the extended Hubbard model and in the generalized t-J model when close to half-filling. [http://dx.doi.org/10.1063/1.4752091]

Introduction

One of the most important questions in the theory of HTSC is whether it is possible to switch the sign of the Coulomb interaction between electrons.[1] The first attempt to answer this question in a positive way was made by Kohn and Luttinger in 1965.[2] Unfortunately, their T_c was unrealistically small. Our answer is much more optimistic. We prove this statement in the limit of low density, far from AFM and structural instabilities. Moreover, in this limit we can develop a regular perturbation theory. The small parameter in the problem is a gas parameter ap_F (a is the scattering length, p_F is the Fermi momentum).

The T_c values which we obtain are not very low. In addition, our theory often even works for rather high densities because of the intrinsic nature of superconducting instabilities. In the latter case, the superconducting temperatures are reasonable.

The Fermi-gas model in three dimensions

The basic model for our theory is a Fermi-gas model. For a repulsive interaction between two particles in vacuum, the scattering length $a > 0$. However, effective interactions in matter, which involve polarization of a fermionic background, contain attractive p-wave harmonics, so that the system is unstable with respect to triplet p-wave superconductive pairing below a temperature[3–5]

$$T_{c1} \sim \varepsilon_F \exp\left\{-\frac{1}{(ap_F)^2}\right\}. \quad (1)$$

In the first two orders of perturbation theory the effective interaction in matter is given by

$$N_{3D}(0)V_{\rm eff}(p,k) = ap_F + (ap_F)^2 \prod(p+k), \quad (2)$$

where $\Pi(p+k)$ is an exchange diagram which coincides in the case of a short range interaction with the polarization operator, $N_{3D}(0) = mp_F/2\pi^2$ is the density of states in 3D.

The regular part also contains a Kohn's anomaly of the form (in the 3D case),

$$\prod\nolimits_{\sin g} \sim (\tilde{q} - 2p_F)\ln|\tilde{q} - 2p_F|, \quad (3)$$

where $\tilde{q} = |\mathbf{p} + \mathbf{k}|$ is the momentum transfer in a crossed channel. As a result we start from pure hard-core repulsion in vacuum and obtain a competition between repulsion and attraction in matter. The singular part of $V_{\rm eff}$ favors attraction and the regular part favors repulsion. S-wave superconductivity is suppressed by the hard core. However, for $l \neq 0$ the hard core is ineffective. Moreover, by $l = 1$ the attractive contribution is dominant. The exact solution yields[3–5]

$$T_{c1} - \varepsilon_F \exp\left\{-\frac{5\pi^2}{4(2\ln 2 - 1)(ap_F)^2}\right\} = \varepsilon_F \exp\left\{-\frac{13}{\lambda^2}\right\}, \quad (4)$$

where $\lambda = 2ap_F/\pi$ is the effective 3D gas-parameter of Galitskii.[6]

Two-dimensional Fermi-gas

In 2D, the effective interaction in the first two orders of the gas parameter is given by[7,8]

$$N_{2D}(0)V_{\rm eff}(q) \sim f_0 + f_0^2 \prod(\tilde{q}); \quad f_0 = \frac{1}{2\ln(p_F r_0)} \quad (5)$$

where f_0 is the 2D gas parameter of Bloom,[9] r_0 is the range of the potential, and $N_{2D}(0) = m/2\pi$ is the 2D density of states.

However, $V_{\sin g}(q) \sim f_0^2 \mathrm{Re}\sqrt{\tilde{q} - 2p_F} = 0$ for $q \le 2p_F$; i.e., the Kohn's anomaly is one-sided and is ineffective for superconductivity. SC appears only in the third order in f_0 (Refs. 7 and 8) where we have $f_0^3 \mathrm{Re}\sqrt{2p_F - \tilde{q}}$ for the singular contribution to $V_{\mathrm{eff}}(q)$. Exact calculation of all the third order diagrams yields[7,8]

$$T_{C1} \sim \varepsilon_F \exp\left\{-\frac{1}{6.1 f_0^3}\right\}. \tag{6}$$

3D and 2D Hubbard models. The Shubin-Vonsovsky model

The same results for p-wave critical temperature Eqs. (4) and (6) are valid for 3D and 2D Hubbard models[10] with repulsion. For the Hubbard model, the 3D gas-parameter of Galitskii[6] is given by $\lambda = 2dp_F/\pi$ (where d is intersite distance) and the 2D gas-parameter of Bloom,[9] by $f_0 = \frac{1}{2\ln[1/(2p_F d)]}$. In the 2D Hubbard model at low electron density with weak-coupling, d_{xy}-pairing also takes place.[11] We proved an existence of superconductivity in more than ten 2D and 3D models. In most of the models we obtained p-wave pairing including the most repulsive and the most unbeneficial for SC Shubin-Vonsovsky model.[12] The Hamiltonian for the Shubin-Vonsovsky model is

$$H = -t\sum_{\langle ij\rangle\sigma} c_{i\sigma}^{+} c_{j\sigma} + U\sum_i n_{i\uparrow} n_{i\downarrow} + \frac{V}{2}\sum_{\langle ij\rangle} n_i n_j, \tag{7}$$

where U is the on-site Hubbard repulsion, V is an additional Coulomb repulsion on neighboring sites, and t is hopping integral. One effective vacuum interaction for the Shubin-Vonsovsky model has the form shown in Fig. 1.

Even in the most repulsive strong-coupling limit of the model $U \gg V \gg W$ (W is the bandwidth; $W = 1-2t$ for a 3D simple cubic lattice; $W = 8t$ for a square lattice in 2D), we get the same critical temperatures for p-wave pairing (4) and (6) as in the absence of additional Coulomb repulsion (for $V = 0$) in both the 3D and 2D cases.

The additional Coulomb repulsion V changes only the preexponential factors in Eqs. (6) and (8).[13,14] This is an important result regarding the possible role of long-range screened Coulomb interactions for non-phonon mechanisms of SC first discussed in Refs. 15–17.

At higher electron densities there is Verwey localization[18,19] with a checkerboard charge-ordered state in the strong-coupling limit of the model for dimensionless electron density $n_{\mathrm{el}} = 1/2$ and Mott-Hubbard localization with an appearance of AFM-state[10,20–22] for $n_{\mathrm{el}} = 1$. We also have here extended regions of phase separation close to $n_{\mathrm{el}} = 1/2$ and $n_{\mathrm{el}} = 1$ (see Fig. 2 and Refs. 23–27). Thus, our arguments for homogeneous SC in strong-coupling case $U \gg V \gg W$ are valid up to densities $n_{\mathrm{el}} = 1/2 - \delta_c$, where for $V \gg t$,

$$\delta_c \sim \left(\frac{t}{V}\right)^{1/2} \text{ in 2D and } \delta_c \sim \left(\frac{t}{V}\right)^{3/5} \text{ in 3D.} \tag{8}$$

For $1/2 - \delta_c < n_{\mathrm{el}} < 1/2$ we have nano-scale phase-separation on small metallic clusters in the insulating checkerboard CO matrix (see Fig. 3).

At critical densities $n_{\mathrm{el}} = 1/2 - \delta_c$ the metallic clusters start to touch each other. As a result an infinite metallic cluster develops (the entire sample volume becomes metallic) for $n_{\mathrm{el}} < 1/2 - \delta_c$.

d-wave pairing in the extended Hubbard model close to half-filling

In the opposite Born case $W > U > V$, phase separation is absent in the model and we can construct an SC phase-diagram for p-wave, d_{xy}- and $d_{x^2-y^2}$-wave pairing for all the densities $0 < n_{\mathrm{el}} < 1$. The first results for this case were obtained for 2D in Refs. 16 and 17.

The main result of Kivelson *et al.*[16] is the following: if we just consider an extended Hubbard model with Hubbard repulsion U, nearest-neighbor hopping t, and next to nearest neighbor hopping t', then there are two maxima in the dependence of the effective interaction V_{eff} in the $d_{x^2-y^2}$-channel on the electron density n_{el}. The large central maximum in the $d_{x^2-y^2}$-channel corresponds to large densities $n_{\mathrm{el}} \sim (0.9 - 1)$, close to half-filling, while the second smaller maximum corresponds to lower densities. This maximum depends upon the details of the quasiparticle spectrum. For $t'/t \sim -0.3$ and $U \le W$ it is positioned at $n_{\mathrm{el}} \sim 0.6$ according to Kivelson *et al.*[16] In between the two maxima there is a local minimum at the position of the van Howe singularity. For $t'/t \sim -0.3$ this is at $n_{v.H.} \sim 0.7$.[16] Here V_{eff}^d is rather small in the d-wave channel. Here, evaluating the Kohn-Luttinger diagrams in the second order of perturbation theory (on the order of U^2/W) yields reasonable values of the main exponent for the d-wave critical temperature

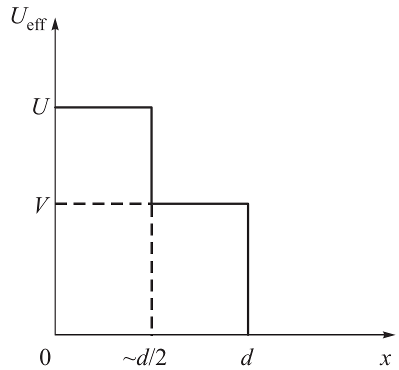

FIG. 1. Effective vacuum interaction in the Shubin-Vonsovsky model with Hubbard on-site repulsion U and additional Coulomb repulsion V at neighboring sites.

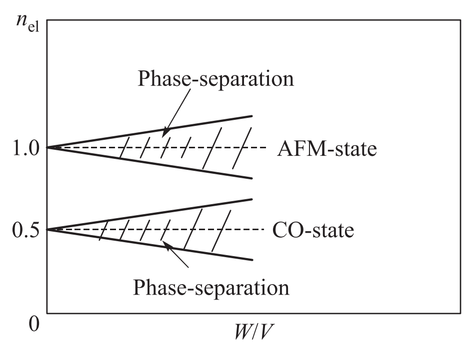

FIG. 2. Qualitative phase-diagram of the Shubin-Vonsovsky model in the strong coupling case. For $n_{\mathrm{el}} = 1$, an AFM state appears in the model, while for $n_{\mathrm{el}} = 1/2$ we have a checkerboard CO state. We have also extended regions of phase-separation close to $n_{\mathrm{el}} = 1/2$ and $n_{\mathrm{el}} = 1$.

FIG. 3. A phase-separated state for densities $1/2 - \delta_c < n_{el} < 1/2$ with nano-scale metallic clusters inside a CO checkerboard insulating matrix for $V \gg t$.

$$T_c \sim \varepsilon_F \exp\left\{-\frac{1}{|V^d_{\rm eff}|N_{2D}(0)}\right\}.$$

Namely in the $d_{x^2-y^2}$-channel for $t \sim 0.3\,\mathrm{eV}$ ($W = 8t$) and $U \sim 6t$, maximum values of $T_c \sim (80\text{–}100)$ K are obtained in this estimate for $n_{el} \sim (0.8\text{–}0.9)$.

Kagan, Val'kov, Korovushkin, and Mitskan are attempting[28] to check the stability of Kivelson results with respect to nonzero Coulomb repulsion on neighboring sites $V \neq 0$ (see Eq. (7) for the Hamiltonian of the Shubin-Vonsovsky model) and more distant hopping $t'' \neq 0$ for the uncorrelated quasiparticle spectrum

$$\varepsilon(p) - \mu = -2t(\cos p_x d + \cos p_y d) + 4t' \cos p_x d \cos p_y d \\ + 2t''(\cos 2p_x d + \cos 2p_y d)\,\mu.$$

That article[28] also investigates a dependence of the kernel of the integral Bethe-Salpeter equation for T_c on the intermediate Matsubara frequency (retardation effects), while here we proceed from a standard weak-coupling approach to a more sophisticated Eliashberg scheme. There the authors also attempt to evaluate the corrections to the main exponent and preexponential factor associated with the third and fourth order diagrams in U.

The possibility of increasing T_c even at low density

There are two possible ways to increase T_c even at low density:[29,30]

— applying an external magnetic field (or creating strong spin-polarization);[29]
— consider a two-band situation.[30]

In both cases the most important idea is that of separating the channels. In a magnetic field the Cooper pair is formed by two spins "up" while an effective interaction corresponds to two spins "down". As a result the Kohn's anomaly increases. For $H \neq 0$ it becomes

$$\prod\nolimits_{\sin g}(q) \sim (q_\uparrow - 2p_{F\downarrow})\ln|q_\uparrow - 2p_{F\downarrow}| = (\theta - \theta_c)\ln(\theta - \theta_c) \tag{9}$$

and θ_c differs from π in proportion to $(p_{F\uparrow}/p_{F\downarrow}) - 1$. Thus, already first derivative of $\Pi_{\sin g}$ and the effective interaction with respect to $(\theta - \theta_c)$ are divergent. Note that for $H = 0$ the Kohn's anomaly is given by $(\pi - \theta)^2 \ln(\pi - \theta)$ and only the second derivative of $V_{\rm eff}$ with respect to $(\pi - \theta)$ is divergent.

Unfortunately there is a competing process: namely the decrease of the density of states of the "down" spins: $N_\downarrow(0) = mp_{F\downarrow}/4\pi^2$. As a result of this competition $T_c^{\uparrow\uparrow}$ has reentrant behavior with a large maximum (see Fig. 4). This theory has been confirmed in experiments by the Frossati group in Leiden:[31] for ^{3}He $T_c^{\uparrow\uparrow}$ ($\alpha = 6\%$) = 3.2 mK while T_c ($\alpha = 0$) = 2.7 mK. As a result we obtain a 20% increase in the critical temperature. At the maximum $T_c^{\uparrow\uparrow} = 6.4T_c$ for ^{3}He and $T_c^{\uparrow\uparrow} = 10^5\,T_c$ for mixtures.[32]

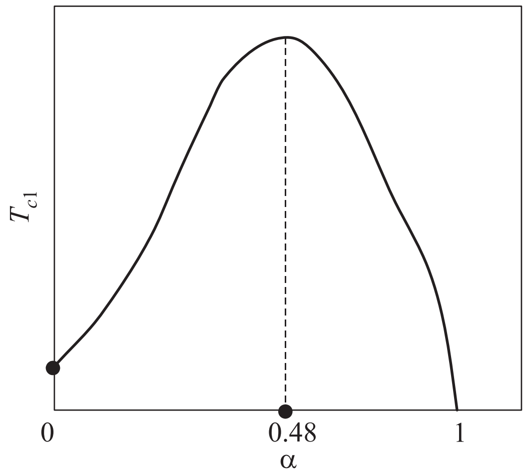

FIG. 4. Polarization dependence of T_c in 3D case.

In 2D films of ^{3}He in a magnetic field we have $\prod(q) \sim \mathrm{Re}\sqrt{q_\uparrow - 2p_{F\downarrow}}$ and the large 2D Kohn anomaly becomes effective for superconductivity. The maximum is broad and very large (see Fig. 5); it stretches from $\alpha = 0.1$ to $\alpha = 0.9$. At the maximum (for $\alpha = 0.6$),

$$T^{\uparrow\uparrow}_{c\,\max} = \varepsilon_F \exp\left\{-2\ln^2\left(\frac{1}{p_F r_0}\right)\right\}. \tag{10}$$

T_c at the maximum is 16 times bigger in exponent than T_c in 3D, $T^{\uparrow\uparrow}_{c\,\max} \to \varepsilon_F e^{-2}$ for $\ln(1\sqrt{p_F r_0}) \to 1$.

The same result could be obtained for a 2D electron gas in a parallel magnetic field.[33] The magnetic field does not change the in-plane motion of electrons here. The Meissner effect is suppressed. Hence, we have qualitatively the same situation as in uncharged (neutral) ^{3}He films (see Fig. 6) and reentrant superconductive behavior for T_c in a field. For $H \sim 15$ T and $\varepsilon_F \sim 30$ K $T_{c1} \sim 0.5$ K.

The two-band Hubbard model with one narrow band

When there are two bands the electrons in the first band play the role of spins "up" while the electrons in the second band play the role of spins "down." The bands are connected by interband an Coulomb interaction $U_{12}n_1n_2$. The following excitonic mechanism of superconductivity is possible: Cooper pairs are formed in one band due to polarization of the other.[30,34,35]

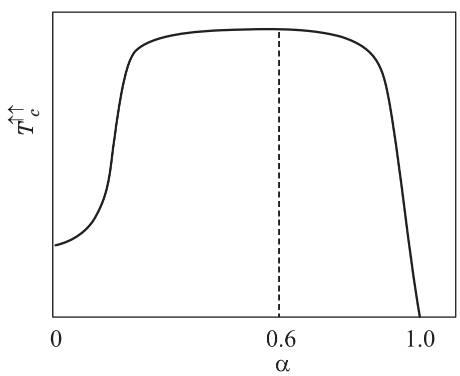

FIG. 5. Polarization dependence of T_c in 2D case.

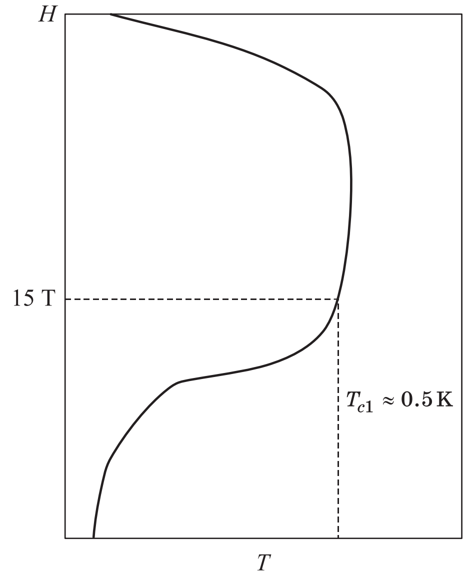

FIG. 6. *H-T* diagram for 2D electron gas in parallel magnetic field.

The relative filling of the bands n_1/n_2 determines the spin polarization α (see Fig. 7). If we consider a two-band Hubbard model with one narrow band, then the effective interaction is mostly governed by heavy-light repulsion (see Fig. 8) and

$$T_{c\,\max} = T_c\left(\frac{n_h}{n_L} \approx 4\right) = \varepsilon_F \exp\left\{-\frac{1}{2f_0^2}\right\} \tag{11}$$

where $n_1 = n_h$ for the heavy band, $n_2 = n_L$ for the light band, and $U_{12} = U_{hL}$ is the "heavy-light" interband Hubbard repulsion.

In the Born weak-coupling case

$$f_0^2 = \frac{m_h m_L}{4\pi^2} U_{hL}^2$$

depends on the interband Hubbard interaction U_{hL}.[30] In the strong-coupling case[34,35]

$$f_0^2 = \frac{m_h}{m_L} \frac{1}{\ln^2[1/(p_F^2 d^2)]}.$$

Finally, in the so-called unitary limit of screened Coulomb interaction $T_{c\,\max} \sim \varepsilon_{Fh}^* \exp(-2)$,[25,26] where the renormalized Fermi-energy $\varepsilon_{Fh}^* = p_{Fh}^2/(2m_h^*) \sim (30-50)$K and the enhanced heavy mass $m_h^* \sim 100 m_e$ owing to the many-body electron-polaron effect.[36,37] As a result, we can get $T_{c1} \sim 5$ K for the Fermi energies $\varepsilon_{Fh}^* \sim (30-50)$K typical of uranium-based HF compounds. Note that the electron-polaron effect which produces a large enhancement in the heavy mass in this model is related to the non-adiabatic part of the wave-function which describes a heavy electron dressed in the cloud of virtual electron-hole pairs of the light band (see Fig. 9).

If we collect the polaron exponent we get[36,37]

$$\frac{m_h^*}{m_h} = \left(\frac{m_h}{m_L}\right)^{\frac{b}{1-b}}, \tag{12}$$

where $b = 2f_0^2$ in 2D and $b = 2\lambda^2$ in 3D. Hence, for $f_0 = 1/2$ (unitary limit of the screened Coulomb interaction) $b = 1/2$; $b/(1-b) = 1$ and $m_h^*/m_h = m_h/m_L$. Correspondingly, $m_h^*/m_L = (m_h/m_L)^2$ and, if we start with $m_h/m_L \sim 10$ in the local density approximation (LDA scheme),[38] we can end up with with $m_h^* \sim 100\, m_e$ owing to the many-body electron-polaron effect and $T_{c1} \sim 5$ K.

Thus, we get an effective mass of heavy particles and superconductive temperatures realistic for uranium-based heavy fermion compounds.

This mechanism can be important in Bi- and Tl-based HTSC-materials. It can also produce superconductivity in superlattices (PbTe-SnTe) and dichalcogenides (CuS_2,

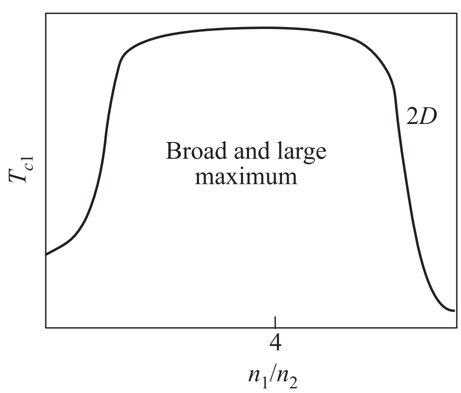

FIG. 7. T_c as a function of relative filling in the two band model.

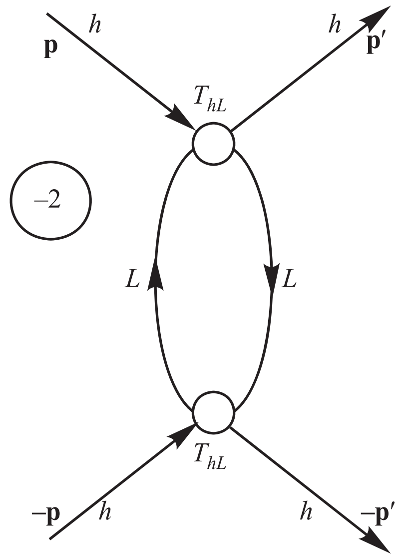

FIG. 8. The leading contribution to the effective interaction *V*eff for the *p*-wave pairing of heavy particles via polarization of light particles. The open circles stand for the vacuum *T*-matrix T_{hL}, which in Born case coincides with interband Hubbard interaction U_{hL}.

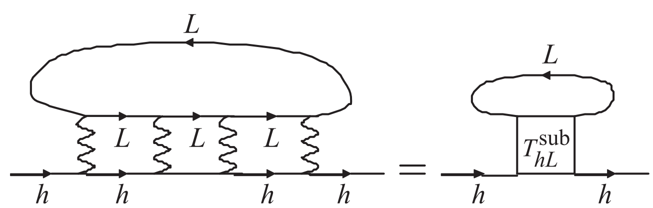

FIG. 9. The lowest order skeleton diagram for EPE in the self consistent *T*-matrix approximation. T_{hL}^{sub} stands for the T-matrix in matter.

$CuSe_2$) with geometrically separated layers. Note that the two bands also can belong to one layer. We have pointed out that this mechanism could be dominant in Sr_2RuO_4[13,34,35] and in fermionic ^{6}Li in magnetic traps.[39]

In the case of one heavy and one light band with $m_h \gg m_L$ and $n_h > n_L$, the critical temperature T_c is mostly governed by pairing of heavy electrons via polarization of light electrons (see Fig. 8). However, including the already infinitely small Geilikmann-Moskalenko-Suhl term $K\sum_{pp'} a_p^+ a_{-p}^+ \; b_{p'} b_{-p'}$[40–44] which rescatters the Cooper pair between the two bands, ensures opening of SC gaps in both the heavy and light bands at the same temperature.

Conclusion and discussion

Using a large variety of models, we have confirmed the existence of p-wave pairing in purely repulsive fermion systems. We demonstrated the possibility of increasing T_c up to experimentally feasible values $\sim$5 K even at low electrons densities in strongly spin-polarized or two-band systems. The systems where triplet p-wave pairing occurs or can be expected include superfluid ^{3}He, ultracold Fermi-gases in the regime of p-wave Feshbach resonance,[45] heavy fermion superconductors such as $U_{1-x}Th_xBe_{13}$ and the ruthenate Sr_2RuO_4, the organic superconductor α-(BEDT-TTF)$_2$I$_3$, layered dichalcogenides CuS_2-$CuSe_2$, semimetals, and the semimetallic superlattices InAs-GaSb, PbTe-SnTe. As for possible high-T_c superconductivity, we have obtained a simple estimate for attaining T_c in the range of 100 K for d-wave pairing $(d_{x^2-y^2})$ in the Born (weak coupling) approximation to the 2D extended Hubbard model close to half-filling. In the strong coupling approaches specified by the generalized $t-J$ model Kagan, Rice[46] (see also Emery *et al.*[47] and Plakida *et al.*[48,49]) have shown that we can also get a reasonable T_c in the range of 100 K for optimally doped high-T_c materials ($n_{el} \sim 0.85$, $J/t \sim (1/\ 2\ -1/\ 3)$). In underdoped high-T_c materials we can expect the spin-charge confinement predicted by Laughlin *et al.*,[50,51] and related to the creation of an AFM string (spin polaron or composite hole[52,53]) in the 3D and 2D cases. Here there is a strong bosonic contribution and we can consider superconductive pairing in terms of BCS-BEC crossover[54,55] for pairing of two composite holes (two AFM strings or spin polarons) in the $d_{x^2-y^2}$-channel.[48,49,54,56,57]

We have also analyzed the normal state of the basic models with repulsion at low electron density and find nontrivial corrections to Galitskii-Bloom Fermi-gas expansion owing to the presence of an antibound state[58] in the lattice models or a singularity in the Landau quasiparticle f-function at low density in 2D.[59] These corrections do not, however, invalidate the Landau Fermi-liquid picture in either the 3D or the 2D case.

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[1]P. W. Anderson, Science **235**, 1196 (1987).
[2]W. Kohn and J. M. Luttinger, Phys. Rev. Lett. **15**, 524 (1965).
[3]M. Yu. Kagan and A. V. Chubukov, JETP Lett. **47**, 525 (1988).
[4]D. Fay and A. Layzer, Phys. Rev. Lett. **20**, 187 (1968).
[5]M. A. Baranov, A. V. Chubukov, and M. Yu. Kagan, Int. J. Mod. Phys. B **6**, 2471 (1992).
[6]V. M. Galitskii, JETP **7**, 104 (1958).
[7]A. V. Chubukov, Phys. Rev. B **48**, 1097 (1993).
[8]D. V. Efremov, M. S. Mar'enko, M. A. Baranov, and M. Yu. Kagan, Physica B **284**, 216 (2000).
[9]P. Bloom, Phys. Rev. B **12**, 125 (1975).
[10]J. Hubbard, Proc. R. Soc. London A **276**, 238 (1963).
[11]M. A. Baranov and M. Yu. Kagan, Z. Phys. B Condens. Matter. **86**, 237 (1992).
[12]S. Shubin and S. Vonsovsky, Proc. R. Soc. London A **145**, 159 (1934).
[13]M. Yu. Kagan, D. V. Efremov, M. S. Mar'enko, and V. V. Val'kov, JETP Lett. **93**, 819 (2011).
[14]D. V. Efremov, M. S. Mar'enko, M. A. Baranov, and M. Yu. Kagan, JETP **90**, 861 (2000).
[15]A. S. Alexandrov and V. V. Kabanov, Phys. Rev. Lett. **106**, 136403 (2011).
[16]S. Raghu, S. A. Kivelson, and D. J. Scalapino, Phys. Rev. B **81**, 224505 (2010).
[17]S. Raghu, E. Berg, A. V. Chubukov, and S. A. Kivelson, Phys. Rev. B **85**, 024516 (2012).
[18]E. J. W. Verwey, Nature **144**, 327 (1939).
[19]E. J. W. Verwey and P. W. Haayman, Physica **8**, 979 (1941).
[20]N. F. Mott and E. A. Davis, *Electronic Processes in Non-Crystalline Materials* (Oxford University Press, 1979).
[21]Yu. A. Izumov, F. A. Khassan-Ogly, and Yu. N. Skryabin, *Field Theoretical Methods in the Theory of Ferromagnetism* (Science, Moscow, 1974).
[22]V. V. Val'kov and S. G. Ovchinnikov, *Quasiparticles in Strongly-Correlated Systems* (Publishing House of Siberian Branch of RAS, Novosibirsk, 2001).
[23]M. Yu. Kagan, K. I. Kugel, and D. I. Khomskii, JETP **93**, 415 (2001).
[24]M. Yu. Kagan and K. I. Kugel, Sov. Phys. Usp. **171**, 577 (2001).
[25]M. Yu. Kagan, A. V. Klaptsov, I. V. Brodsky, K. I. Kugel, A. O. Sboychakov, and A. L. Rakhmanov, J. Phys. A **35**, 9155 (2003).
[26]K. I. Kugel, A. L. Rakhmanov, A. O. Sboychakov, M. Yu. Kagan, I. V. Brodsky, and A. V. Klaptsov, JETP **98**, 572 (2004).
[27]A. O. Sboychakov, A. L. Rakhmanov, K. I. Kugel, M. Yu. Kagan, and I. V. Brodsky, JETP **95**, 753 (2002).
[28]M. Yu. Kagan, V. V. Val'kov, M. M. Korovushkin, and V. A. Mitskan, JETP Lett. (to be published).
[29]M. Yu. Kagan and A. V. Chubukov, JETP Lett. **50**, 517 (1989).
[30]M. Yu. Kagan, Phys. Lett. A **152**, 303 (1991).
[31]S. A. J. Wiegers, T. Hata, P. G. Van de Haar, L. P. Roobol, C. M. C. M. Van Woerkens, R. Jochemsen, and G. Frossati, Physica B **165**, 733 (1990).
[32]M. Yu. Kagan, Sov. Phys. Usp. **37**, 69 (1994).
[33]M. A. Baranov, D. V. Efremov, and M. Yu. Kagan, Physica C **218**, 75 (1993).
[34]M. Yu. Kagan and V. V. Val'kov, Fiz. Nizk. Temp. **37**, 84 (2011) [Low Temp. Phys. **37**, 69 (2011)]; *A Lifetime in Magnetism and Superconductivity: A Tribute to Professor David Schoenberg* (Cambridge Scientific, 2011).
[35]M. Yu. Kagan and V.V. Val'kov, JETP **113**, 156 (2011).
[36]Yu. Kagan and N. V. Prokof'ev, JETP **66**, 211 (1987).
[37]Yu. Kagan and N. V. Prokof'ev, JETP **63**, 1276 (1986).
[38]W. Kohn and L. J. Sham, Phys. Rev. A **140**, 1133 (1965).
[39]M. A. Baranov, M. Yu. Kagan, and Yu. Kagan, JETP Lett. **64**, 301 (1996).
[40]H. Suhl, T. B. Matthias, and L. R. Walker, Phys. Rev. Lett. **3**, 552 (1959).
[41]B. T. Geilikman, JETP **21**, 796 (1965).
[42]B. T. Geilikman, Sov. Phys. Usp. **88**, 327 (1966).
[43]B. T. Geilikman, Sov. Phys. Usp. **109**, 65 (1973).
[44]M. A. Baranov and M. Yu. Kagan, JETP **75**, 165 (1992).
[45]M. Yu. Kagan and D. V. Efremov, JETP **110**, 426 (2010).
[46]M. Yu. Kagan and T. M. Rice, J. Phys.: Condens. Matter **6**, 3771 (1994).
[47]V. J. Emery, S. A. Kivelson, and H. Q. Lin, Phys. Rev. Lett. **64**, 475 (1990).
[48]N. M. Plakida, L. Anton, S. Adam, and G. Adam, JETP **124**, 367 (2003).
[49]N. M. Plakida, JETP Lett. **74**, 36 (2001).
[50]R. B. Laughlin, Phys. Rev. Lett. **60**, 2677 (1988).
[51]A. L. Fetter, C. B. Hanna, and R. B. Laughlin, Phys. Rev. B **79**, 9679 (1989).
[52]L. N. Bulaevskii, E. L. Nagaev, and D. I. Khomskii, JETP **54**, 1562 (1968).
[53]W. F. Brinkman and T. M. Rice, Phys. Rev. B **2**, 1324 (1970).

[54] M. Yu. Kagan, I. V. Brodsky, A. V. Klaptsov, R. Combescot, and X. Leyronas, Sov. Phys. Usp. **176**, 1105 (2006).
[55] R. Combescot, X. Leyronas, and M. Yu. Kagan, Phys. Rev. A **73**, 023618 (2006).
[56] V. I. Belinicher, A. L. Chernyshev, and V. A. Shubin, Phys. Rev. B **56**, 3381 (1997).
[57] V. I. Belinicher, A. L. Chernyshev, A. V. Dotsenko, and O. P. Sushkov, Phys. Rev. B **51**, 6076 (1995).
[58] M. Yu. Kagan, V. V. Val'kov, and P. Woelfle, Fiz. Nizk. Temp. **37**, 1046 (2011) [Low Temp. Phys. **37**, 834 (2011)].
[59] M. A. Baranov, M. Yu. Kagan, and M. S. Mar'enko, JETP Lett. **58**, 709 (1993).

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