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The General Relativity with Conformal Units[1]

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Abstract—General Relativity rewritten in conformal units identifies conformal intervals with the real observational distances. This identification gives a base to explain all epochs of the Universe evolution including Ia supernova luminosity long distance-redshift relation by the dominance of the Casimir vacuum energy of all physical fields. A set of arguments is discussed in favor that SNe Ia data in the conformal units can be an evidence of the conformal twistor structure of the space-time as a nonlinear realization of the affine group, like the nonlinear realization of chiral symmetry and phenomenological Lagrangian is evidence of the quark structure of hadrons.

Keywords: General Relativity, Cosmology

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1. INTRODUCTION

The conformal symmetry as a basis of the construction of the General Relativity (GR) was independently introduced by Deser and Dirac [1, 2]. In particular, Dirac formulated the conformal-invariant approach to the GR [2] as a new variational principle for the Hilbert action introducing a dilaton (scalar) field, in addition to the metric components $g_{\mu\nu}$.

The conformal treatment of gravity is supported by the Ogievetsky theorem [3] according to which the GR-diffeomorphism group Diff$R^{(1,3)}$ can be obtained as the closure of two finite-dimensional groups: the 15-parameter conformal group and the 20-parameter affine group having the Poincaré group as a common subgroup. Further it was shown [4] that in the case of the dynamical affine symmetries the method of nonlinear realization of symmetry groups [5] leads to the Hilbert action of Einstein's gravitational theory expressed in terms of the Fock simplex components [6]. Conformal invariance not only picks out the Einstein GR among several appropriated invariants of nonlinearly realized affine symmetry, but also it establishes the conformal units which have been introduced before by several researches including Dirac [2].

In the present paper we discuss a set of observational data and facts that follow from the conformal units [1, 2] in comparison with the Einstein ones. In our conformal version of the GR (CGR), the conformal symmetry breaking happens due to the presence of the Casimir vacuum energy [7] in a finite volume of the Universe. In our approach the Casimir vacuum energy substitutes the dark energy. It provides a good description of SNe Ia data [8] within the conformal cosmology [9, 10]. We found that the Universe horizon and the Planck least action postulate lead to the Planck scale hierarchy and the instance of the primordial particle creation from vacuum, with the Casimir vacuum energy being the source of the creation.

2. CONFORMAL GENERAL RELATIVITY

The Conformal General Relativity (CGR) is a nonlinear realization of joint conformal and affine $A(4)$ symmetries in the factor space $A(4)/L$ with the Lorentz subgroup L of the stable vacuum (here we use the concepts of the theory [5]). Recall that the affine group $A(4)$ is the group of all linear transformations of the four-dimensional manifold $x^\mu \to \tilde{x}^\mu = x^\mu + y^\mu + L_{[\mu\nu]}x^\nu + R_{\{\mu\nu\}}x^\nu$, where y^μ is a shift of coordinate and $L_{[\mu\nu]}$ and $R_{\{\mu\nu\}}$ are antisymmetric and symmetric matrices respectively. A nonlinear realization of $A(4)$ is based on finite transformations $G = e^{iPx}e^{iRh}$ defined by means of the shift operator P, proper affine transformation R and the following Goldstone modes: four coordinates x_μ and ten gravitational fields h Further it was shown [4] that in the case of the dynamical affine sym-

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metries the method of nonlinear realization of symmetry groups [5] leads to the Hilbert action of Einstein's gravitational theory expressed in terms of the Fock simplex components as $W_E = -(M_{\rm Pl}^2/16)\int d^4x\sqrt{-g}\,R^{(4)}$, where $R^{(4)}$ is the curvature. Taking into account the hidden conformal symmetry associated with a dilaton (scalar) field D and, consequently, transforming $g_{\mu\nu}$ [2]

$$g_{\mu\nu} = \tilde{g}_{\mu\nu}e^{2D}, \tag{1}$$

we obtain the conformal-invariant action:

$$W_{\rm C} \equiv -M_{\rm C}^2\frac{3}{8\pi} \times \int d^4x\left[\frac{\sqrt{-\tilde{g}}}{6}R^{(4)}(\tilde{g})e^{-2D} - e^{-D}\partial_\mu\left(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu e^{-D}\right)\right], \tag{2}$$

where the $M_{\rm C}$ is a scale unit. It is defined in the Riemannian space-time where the conformal interval

$$ds_{\rm C}^2 = \tilde{g}_{\mu\nu}dx^\mu dy^\nu = \tilde{\omega}_{(0)}\otimes\tilde{\omega}_{(0)} - \tilde{\omega}_{(b)}\otimes\tilde{\omega}_{(b)} \equiv \eta_{(\alpha)(\beta)}\tilde{\omega}_{(\alpha)}\otimes\tilde{\omega}_{(\beta)} \tag{3}$$

is identified with the measurable one, instead of the Einstein interval

$$ds_{\rm E}^2 = g_{\mu\nu}dx^\mu dy^\nu. \tag{4}$$

If $D = 0$, one obtains $W_{\rm C} \equiv W_{\rm E}$ and $ds_{\rm C} \equiv ds_{\rm E}$. Thus, the GR model based on the conformal and aine symmetry principles (described by the action (2)) differs from the original Einstein–Hilbert action $W_{\rm E}$ by the following elements and treatments. Namely:

(1) Action (2) deals with the conformal geometrical interval (3) $ds_{\rm C}^2$ instead of the Einstein one (4) $ds_{\rm E}^2 = g_{\mu\nu}dx^\mu dx^\nu < ds_{\rm C}^2$.

(2) The cosmological evolution in the CGR can be provided by the mean field dynamics of the dilaton zeroth mode instead of the homogeneous approximation [11] (see below).

(3) The CGR contains the Newton coupling constant ($G_N = M_{\rm C}^{-2}e^{2D} = M_{\rm Pl}^{-2}$) as the present day value of the dilaton field D. We recall that the standard GR contains the effective Newtonian coupling constant as the absolute fundamental parameter of the equations of motion. In the CGR the relation of the coupling constant to the Early Universe is clarified below.

In order to establish a relation between physical scales relevant for the Early Universe, we assume that there is a common source of the conformal symmetry breaking. We suppose that the Casimir vacuum energy of the Empty Universe could be naturally associated with this source (see below).

Hereafter, we use the *natural units*:

$$M_{\rm Pl}\sqrt{3/(8\pi)} = c = \hbar = 1. \tag{5}$$

Taking into account Eqs.(3), (4), the simplex components $[\tilde{\omega}_{(0)}, \tilde{\omega}_{(b)}]$ can be written as

$$\tilde{\omega}_{(0)} = e^{-2D}Ndx^0, \tag{6}$$

$$\tilde{\omega}_{(b)} = \mathbf{e}_{(b)i}dx^i + N_{(b)}dx^0, \tag{7}$$

where $N_{(b)} = N^j\mathbf{e}_{j(b)}$ are the shift vector components, and $N(x^0, x^j)$ is the lapse function. Here $\tilde{\omega}_{(b)}$ are the linear forms defined via the triads $\mathbf{e}_{(b)i}$ with a unit spatial metric determinant $[\tilde{g}_{ij}^{(3)}] = 1$ known as the Lichnerowicz gauge [12]. This gauge fixes the scalar dilaton field D as the logarithm of the conformal factor:

$$D = -(1/6)\ln\left|g_{ij}^{(3)}\right|. \tag{8}$$

3. THE DILATON SCALAR FIELD

The group of invariance of the GR for the Dirac-ADM foliation is known as the kinemetric subgroup of the general coordinate transformation [13]:

$$x^0 \to \tilde{x}^0 = \tilde{x}^0(x^0), \tag{9}$$

$$x^k \to \tilde{x}^k = \tilde{x}^k(x^0, x^1, x^2, x^3). \tag{10}$$

This group admits the decomposition of the dilaton into the sum of the zeroth and nonzeroth harmonics:

$$D(x^0, x^1, x^2, x^3) = \langle D\rangle(x^0) + \bar{D}(x^0, x^1, x^2, x^3). \tag{11}$$

The introduction of the zeroth mode $\langle D\rangle(x^0)$ is consistent with the Einstein cosmological principle of averaging of all scalar fields of the theory over a finite volume $V_0 = \int_{V_0} d^3x$ [14] so that

$$\langle D\rangle(x^0) = V_0^{-1}\int_{V_0} d^3x D(x^0, x^1, x^2, x^3), \tag{12}$$

Note that the zeroth dilaton harmonics coincides by construction with the cosmological scale factor logarithm [11]

$$\langle D\rangle = -\ln a = \ln(1+z). \tag{13}$$

Thus in the finite volume V_0 (taking into account Eqs.(11), (13)), we have the following action:

$$W_{\rm C} = \underbrace{W_{\rm Universe}}_{=0 \text{ for } V_0=\infty} + W_{\rm graviton} + W_{\rm potential}, \tag{14}$$

$$W_{\rm Universe}[\langle D\rangle, N_0] = -V_0\int_{\tau_I}^{\tau_0}\underbrace{dx^0N_0}_{=d\tau}\left[\left(\frac{d\langle D\rangle}{N_0dx^0}\right)^2 + \rho_{\rm Cas}(\langle D\rangle)\right], \tag{15}$$

$$W_{\rm graviton} = \int d^4x\frac{N}{6}\left[v_{(a)(b)}v_{(a)(b)} - e^{-4D}R^{(3)}(\mathbf{e})\right], \tag{16}$$

$$W_{\rm potential} = \int d^4xN\left[-v_{\bar{D}}^2 - \underbrace{\frac{4}{3}e^{-7D/2}\Delta^{(3)}e^{-D/2}}_{\text{Newtonian potentials}}\right], \tag{17}$$

where

$$v_{\overline{D}} = \frac{1}{N}\Big[(\partial_0 - N^l\partial_l)\overline{D} + \partial_l N^l/3\Big], \quad (18)$$

$$v_{(a)(b)} = \frac{1}{N}[\omega^R_{(a)(b)}(\partial_0 - N^l\partial_l) + \partial_{(a)}N^{\perp}_{(b)} + \partial_{(b)}N^{\perp}_{(b)}], \quad (19)$$

are the velocities of the metric components and fields, $\Delta = \partial_i[\mathbf{e}^i_{(a)}\mathbf{e}^j_{(a)}\partial_j]$ is the Beltrami–Laplace operator, and $R^{(3)}(\mathbf{e})$ is the three-dimensional spatial curvature expressed in terms of triads $e_{(a)i}$. Here, we have introduced in action (15) the additional term $\rho_{\rm Cas}(\langle D\rangle)$. The introduction of the finite volume $V_0 = \int_{V_0} d^3x < \infty$ creates a dimensional parameter, and therefore, it breaks the conformal symmetry. According to the general wisdom [5], this breaking leads to appearance of a Goldstone mode [15, 16]. It is just the zeroth harmonic $\langle D\rangle$ that can notL be defined in the infinite volume. Note however that the Hamiltonian dynamics governed by the equations of motion must obey the conformal symmetry (see below). We will show that this source could be associated with the Casimir energy of the Universe giving a non-zero density contribution $\rho_{\rm Cas}(\langle D\rangle) \neq 0$.

The choice of the zeroth dilaton mode $\langle D\rangle$ as an evolution parameter has two consequences in the Hamiltonian approach. First, the zeroth dilaton mode canonical momentum density

$$P_{\langle D\rangle} = \frac{2}{V_0}\int_{V_0} d^3x\sqrt{-g}g^{00}\frac{d}{dx^0}\langle D\rangle$$
$$\equiv 2\frac{d}{d\tau}\langle D\rangle = 2v_{\langle D\rangle} = \text{Const.} \neq 0 \quad (20)$$

can be treated as a generator of the Hamiltonian evolution in the field space of events [17, 18]. We stress that the scale-invariance ($D \to D + \Omega$) admits only a constant $P_{\langle D\rangle}$. In virtue of Eqs. (11), (12), the Dirac Hamiltonian theory provides the orthogonality condition

$$\int_{V_0} d^3x\overline{D}(x^0, x^1, x^2, x^3) \equiv 0. \quad (21)$$

This condition enables us to consider the zeroth and nonzeroth components as independent ones.

The second consequence of the orthogonality condition (21) is that the nonzeroth harmonics $\overline{D}\,(x_0, x_1, x_2, x_3)$ do not depend on the evolution parameter. Therefore, one can consider these components as gravitational Newton-type potentials due to the condition for the canonical momentum of dilaton nonzeroth modes

$$P_{\overline{D}}/2 = v_{\overline{D}} = \Big[(\partial_0 - N^l\partial_l)\overline{D} + \partial_l N^l/3\Big]/N = 0. \quad (22)$$

This result fixes the longitudinal shift vector component.

As a result, we have

$$\int d^3x v_{\langle D\rangle} \cdot v_{\overline{D}} = 0, \quad (23)$$

that follows from of Eqs.(11), (12), and (21). The orthogonality conditions (21) and (23) preserve the definite metrics in the Hilbert space of states [16].

4. CONFORMAL CASIMIR ENERGY AND UNIVERSE HORIZON

Let us consider the Early Universe. We assume that at the instance of creation the world was empty and finite in size. Therefore its energy can be associated only with the quantum Casimir energy of all physical fields in the given space. We will treat all those field as massless since $m(a) \xrightarrow{a\to 0} 0$ in the Early Universe epoch.

The Casimir energy of a massless field f

$$\mathrm{H}^{(f)}_{\rm Cas} = \sum_{\mathbf{k}}\frac{\sqrt{\mathbf{k}}^2}{2} = \frac{\tilde{\gamma}^{(f)}}{d_{\rm Cas}(a)}. \quad (24)$$

depends on the geometry, size $d_{\rm Cas}$, topology, boundary conditions, and spin (in particular, for a sphere of diameter $d_{\rm Cas}$ the number of $\tilde{\gamma} \sim 0.1$–0.03) [16]. For simplicity we assume that the Universe has a spherical volume limited by the horizon.

It is natural to suggest that the energy of a massless field is proportional to the inverse visual size of the Universe $d_{\rm Cas}(a)$. Assuming the same dependence for all fields, we define the total Casimir energy density of the Universe summing over all fields

$$\rho_{\rm Cas} = \sum_f \mathrm{H}^{(f)}_{\rm Cas}/V_0 = \frac{C_0}{d_{\rm Cas}(a)}. \quad (25)$$

The key assumption of our model is that the Casimir dimension $d_{\rm Cas}(a)$ is equal to the Universe visual size (its horizon)

$$d_{\rm Cas}(a) \equiv d_{\rm hor}(a) = 2C_0^{-1/2}\int_{a_I\to 0}^{a} d\bar{a}\, d^{1/2}_{\rm Cas}. \quad (26)$$

Eq. (26) has the solution

$$d^{1/2}_{\rm Cas}(a) = [C_0]^{-1/2}a \to d_{\rm Cas}(a) = \frac{a^2}{C_0}. \quad (27)$$

Comparing Eq. (27) with the horizon

$$d_{\rm hor}(a) = \frac{a^2}{H_0}. \quad (28)$$

one obtains

$$C_0 = H_0. \quad (29)$$

Thus, in our approach, parameter C_0 is equal to the Hubble parameter H_0 which can be determined from observations.

The hierarchy law of the cosmological scales in GeV ($M^*_{Pl} = \sqrt{3/(8\pi)}\, M_{Pl}$)

$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
$H_0 \simeq 1.4 \times 10^{-42}$	$R^{-1}_{\rm Cel.S.} \simeq 1.2 \times 10^{-27}$	$k_{0\rm CMB} \simeq 10^{-12}$	$\phi_0 \simeq 3 \times 10^2$	$M^*_{Pl} \simeq 4 \times 10^{18}$

5. HIERARCHY OF COSMOLOGICAL SCALES

Let us consider the Early Universe at the rigid state horizon (28). A hypothetical observer measures the conformal horizon $d_{\rm hor} = 2r_{\rm hor}(z)$ as the distance that a photon covers within its light cone. The latter is determined by the zero interval equation $d\eta^2 - dr^2 = 0$ during the photon lifetime in the homogeneous Universe, which is the subject of the condition $\eta_{\rm hor} = r_{\rm hor}(z) = 1/[2H_0(1 + z)^2]$, in accordance with Eq. (27). This means that the four-dimensional space-time volume restricted by the horizon is equal to

$$V^{(4)}_{\rm hor} = \frac{4\pi}{3} r^3_{\rm hor}(z)\eta_{\rm hor}(z) = \frac{4\pi}{3 \times 16 H_0^4 (1+z)^8}. \quad (30)$$

It is natural to assume that at the instance of the Universe origin the world was essentially quantum. Therefore, we claim that action (15) is the subjected of the *Planck's least action postulate* so that

$$W_{\rm Universe} = \rho_{\rm cr} V^{(4)}_{\rm hor}(a_{Pl}) = \frac{M^2_{Pl}}{H^2_0}\frac{(1+z_{Pl})^{-8}}{32} = 2\pi. \quad (31)$$

Using the present day ($\tau = \tau_0$) observational data for the Planck mass and the Hubble parameter at $h \simeq 0.7$

$$M_C e^{\langle D\rangle(\tau_0)} = M_{Pl} = 1.2211 \times 10^{19}\,{\rm GeV}, \quad \langle D\rangle(\tau_0) = 0,$$
$$\frac{d}{d\tau}\langle D\rangle(\tau_0) = H_0 = 2.1332 \times 10^{-42}\,{\rm GeV} \cdot h \quad (32)$$
$$= 1.4332 \times 10^{-42}\,{\rm GeV},$$

we obtain from (31) the primordial redshift value

$$a^{-1}_{Pl} = (1 + z_{Pl}) \approx [M_{Pl}/H_0]^{1/4}[4/\pi]^{1/8}/2 \simeq 0.85 \times 10^{15}. \quad (33)$$

In other words, the Plank mass and the present day Hubble parameter value (the main cosmological scales) are related to each other by the age of the Universe expressed in terms of the cosmological scale factor.

In field theories, characteristic scales associated with physical states are classified according to the Poincaré group representation [19]. In our approach the Poincaré classification of energies arises from the decomposition of the mean particle energy $\omega_\tau = a^2\sqrt{\mathbf{k}^2 + a^2 M_0^2}$ conjugated to the dilaton time interval. We express this decomposition in the form

$$\langle\omega\rangle^{(n)}(a) = (a/a_{Pl})^{(n)} H_0, \quad (34)$$

based on the primordial redshift value (33). This equation enables one to introduce the conformal weights $n = 0, 2, 3, 4$ which correspond to: the dilaton velocity $v_D = H_0$, the massless energy $a^2\sqrt{\mathbf{k}^2}$, the massive one $M_0 a^3$, and the Newtonian coupling constant $M_{Pl}a^4$ (31), respectively. One can also include in these classification the scale of the nonrelativistic particle $H_0 = a^1_{Pl} \times 10^{-13}$ cm^{-1} with the unit conformal weight of its energy $\omega^{\rm nonr.}_\tau = a^1\mathbf{k}^2/M_0$. As a result, the redshift leads to a hierarchy law of the present day ($a = 1$) cosmological scales

$$\omega^{(n)}_0 \equiv \langle\omega\rangle^{(n)}(a)\big|_{(a=1)} = (1/a_{Pl})^{(n)} H_0 \quad (35)$$

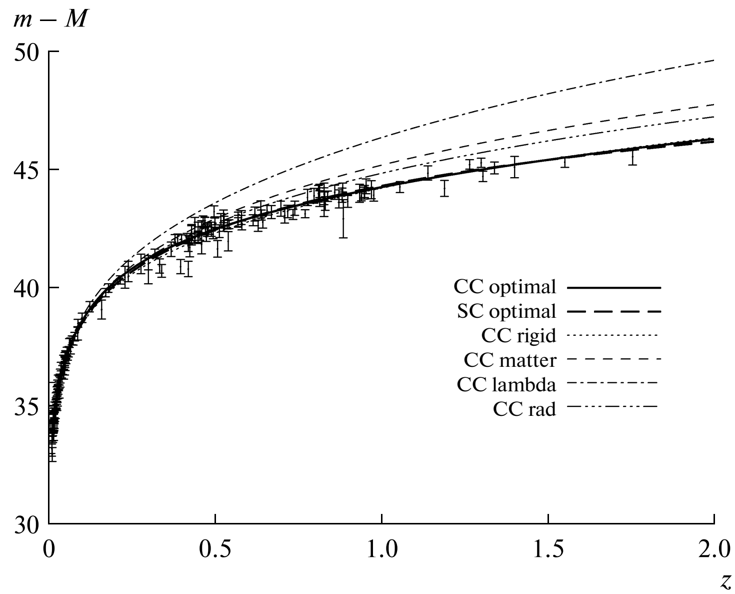
shown in table.

Table 1 contains the scales corresponding to the Hubble parameter ($n = 0$), the Celestial System size ($n = 1$), the Cosmic Microwave Background mean wave-momentum ($n = 2$), the electroweak scale of the SM ($n = 3$), and the Planck mass ($n = 4$). We conclude that the observational data testify that the cosmic evolution (34) of all these mean energies with conformal weights ($n = 0, 1, 2, 3, 4$) have a common origin which could be assosiated with the Casimir vacuum energy, see [22].

6. SNE IA DATA AS THE EVIDENCES OF LONG CONFORMAL UNITS

A particular conformal cosmological model, based on the ideas discussed above, has been developed in papers [9, 10, 20, 21]. It was shown that the model leads to a viable cosmology being in agreement with observations. For example, a good description of the modern supernovae type Ia (SNe Ia) data was constructed [9, 10]. In the present paper we show that the Casimir vacuum effect in a finite-size Universe could provide both the scale invariance breaking and the rigid state dominance, required in our model to describe the SNe Ia data.

Since the end of the last century distant supernovae data is a widespread test for all theoretical cosmological models in spite of the fact the correctness of the hypothesis about SNe Ia as the perfect standard candles is still not proven [23]. Conformal cosmological models [24–26], where all observables are identified with the scale-invariant quantities of GR introduced yet by Lichnerowicz [12], are also discussed among other possibilities [27].



$\mu(z)$—dependence for cosmological models in SC and CC. The data points include 186 SN Ia (the "gold" and "silver" sample) used by the cosmological supernova HST team. For a reference we use the best fit for the flat standard cosmology model with $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$ (the thick dashed line), the best fit for CC is shown with the thick solid line. For this CC model we do not put any constraints on Ω_m.

Assuming that supernovae type Ia are standard candles one could use them to test cosmological theories. The Hubble Space Telescope team analyzed 186 SNe Ia [28] to test the Standard Cosmological model (SC) associated with expanded lengths in the Universe and evaluate its parameters. We use the same sample to determine parameters of Conformal Cosmological model (CC) with relative reference units of intervals, so that conformal quantities of General Relativity are interpreted as observables. We concluded, that really the test is extremely useful and allows to evaluate parameters of the model. From a formal statistical point of view the best fit of the CC model is almost the same quality approximation as the best fit of SC model with $\Omega_\Lambda = 0.72$, $\Omega_m = 0.28$. As it was noted earlier, for CC models, a rigid matter component could substitute the Λ-term (or quintessence) existing in the SC model. We note that a free massless scalar field can generate such a rigid matter. We describe results of our analysis for more recent "gold" data (for 192 SNe Ia).

7. SUMMARY

Any science is based on information. The units of information is a bit (1,0), The units of the quantum information is the 2-dimensional twistor $(\cos\theta, \sin\theta e^{i\delta})$ as fundamental representation of the conformal (C) group in the Penrose twistor program [29]. In according with this twistor program the four parametric matrix of the space-time coordinates $\hat{x}_{A\dot{A}} = x_0\hat{I}_{A\dot{A}} + x_j\sigma^j_{A\dot{A}}$ is constructed from the 2-dimensional twistor fundamental representations like mesons (as a joint representation of $SU(2)$) are constructed from the two dimensional quark fundamental representation of $SU(2)$.

The next step in this analogy with the hadron physics (that is beyond of the Penrose twistor program) is a nonlinear realization of the affine and conformal symmetries $A(4) \otimes C$ in the factor-space $K = A(4)/L$ in conformal units. This step is just like nonlinear realizations of the finite-parameter $SU(2) \times SU(2)$ group over the vacuum stability subgroup $SU(2)$. Recall that this step leads to the effective chiral hadron Lagrangians constructed via the Maurer–Cartan linear forms without any reference to the underlying QCD theory. In this analogy the Plank mass $M_{\rm Pl}$ plays the role of the weak decay constant parameter $F_\pi \simeq$ 93 MeV as a specific scale of hadron low energy physics. One can see that the twistor dissociation (like QCD parton-type deep-inelastic scattering processes) can happen in the Quantum Theory of Space-Time when energy is greater than the Plank mass $M_{\rm Pl}$. The question is what is the QCD analogy of such a twistor dissociation?

This hadron-like chain of the Quantum Theory of Space-Time is evidence that the supersymmetric unification can be based on the finite-parameter geometrization of all interactions via nonlinear realization of this super-affine group $A[(2_b + 2_f)x(2_b + 2_f)] = A[8_b + 8_f]$, where the role of twistors as fundamental representation of the conformal group can be played by the supertwistors proposed in paper [30] together with the commutation-relation algebra of operators "superspace" conformal transformations associated with these supertwistors. This algebra forms super-affine group and its non-linear realization as the $8_b + 8_f$ space-time. Following to the hadron analogy one can obtain the nonlinear realization, where super-curvature $R^{8_b + 8_f}$ is basis of a unified supersymmetric theory. These programs are supported by last results in paper [31] where the gravi-electroweak and strong interactions was obtained by the unification of an 8-dimensional theory by compactification of four extra space dimensions in the theory with the curvature $R^{(8)}$.

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