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METALS

Bulk Plasma Waves in a Randomly Inhomogeneous Conductor

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Abstract—The modification of the spectrum and damping of bulk plasma waves due to three-dimensional random inhomogeneities of the density of a degenerate electron gas in a conductor have been investigated using the averaged Green's function method. The dependences of the frequency and damping of the averaged plasma waves, as well as the position ν_m and width $\Delta\nu$ of the peak of the imaginary part of the Fourier transform of the averaged Green's function, on the wave vector **k** have been determined in the self-consistent approximation, which makes it possible to take into account multiple scattering of plasma waves by inhomogeneities. It has been found that, in the long-wavelength region of the spectrum, the decrease revealed in the frequency of the plasma waves is caused by the inhomogeneities, which agrees qualitatively with the behavior of the position of the peak ν_m. In the range of large values of the correlation length of inhomogeneities and small values of k, the damping of the plasma waves tends to zero, whereas the width of the peak $\Delta\nu$ remains finite, which is due to the nonuniform broadening. A comparison with the data of numerical calculations has been performed.

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1. INTRODUCTION

Investigation of plasma waves is of interest from the viewpoint of fundamental problems in the physics of conductors and the prospects of their technical applications. In recent years, considerable progress has been achieved in the development of research in plasmonic gratings [1–3], which are promising for the design and fabrication of miniature optical signal control devices. In this regard, it is important to perform the investigation of plasma waves in conductors and, in particular, to elucidate the influence exerted by inhomogeneities of the medium on plasma waves, because real materials, including metal films, which have been used in the fabrication of plasmonic gratings, are not perfect. In films, a significant role is played by random inhomogeneities arising during preparation and treatment of samples (fluctuations of the thickness and composition, surface roughness, polycrystallinity, etc.); moreover, inhomogeneities can also be created artificially. In plasmonic gratings, there have been used surface plasma waves that are sensitive to random roughnesses of the surface of conductors (see, for example, [4, 5]). Surface plasma waves do not interact with inhomogeneities in the bulk of conductors, but they are coupled with bulk plasma waves, which are sensitive to such inhomogeneities. Therefore, from the point of view of solving problems of surface plasma waves, it is also of interest to investigate bulk plasma excitations in a randomly inhomogeneous conductor.

Random inhomogeneities in a conductor lead to a modification of the spectrum of bulk plasma waves and contribute to the wave damping (see, for example, [6–15]). In these works, it was shown, in particular, that the most significant changes in the spectrum of bulk plasma waves due to the occurrence of random inhomogeneities are observed in the long-wavelength region. Therefore, the influence of random inhomogeneities on the plasma waves can be investigated using the hydrodynamic approximation [16, 17]. In earlier theoretical studies [11–14], the analysis of plasma waves in randomly inhomogeneous conductors was restricted to second-order perturbation theory; i.e., allowance was made for double scattering of plasma waves by inhomogeneities. In this work, bulk plasma waves in a randomly inhomogeneous gas of conduction electrons have been investigated within the framework of the hydrodynamic approach taking into account multiple scattering of plasma waves by electron density fluctuations.

2. MODEL AND THE WAVE EQUATION

Bulk plasma waves in a degenerate inhomogeneous gas of conduction electrons are described in the hydrodynamic approximation using the linearized equation (see, for example, [17])

$$\frac{3}{5}v_F^2(\mathbf{x})\nabla^2 n_1 - \frac{4\pi e^2 n_0(\mathbf{x})}{m}n_1 - \frac{\partial^2 n_1}{\partial t^2} = 0, \qquad (1)$$

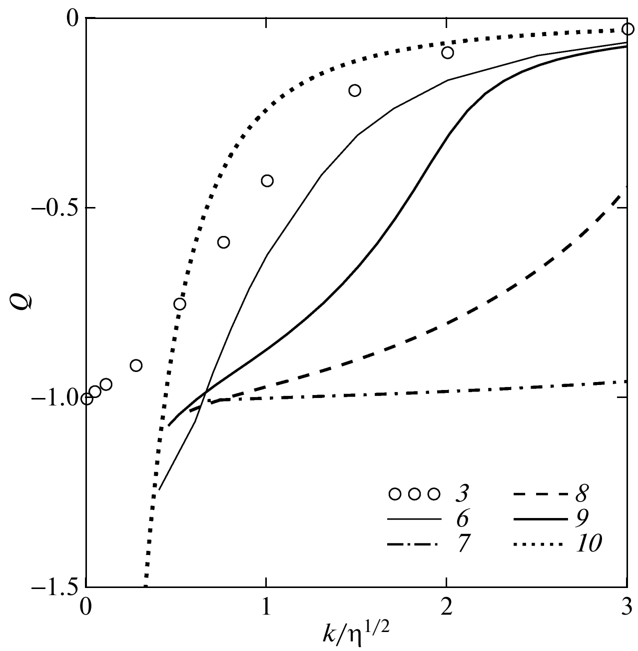


Fig. 1. Spectrum ν' of plasma waves and the position ν_m of the peak of the function $\bar{G}''(k, \nu)$: $Q = (\nu_m - k^2)/\eta$ (curves *3*, *6*) and $Q = (\nu' - k^2)/\eta$ (curves *7–10*). (*3*) The value of the peak position ν_m is taken from in [20]; (*6*) ν_m is obtained from formulas (6), (13), and (17); (*7–9*) ν' is determined using formula (19); and (*10*) ν' is calculated according to formula (10). Curve *7* is plotted for the parameter $k_c/\sqrt{\eta} = 0.1$; curve *8* for the parameter $k_c/\sqrt{\eta} = 0.3$; and curves *3*, *6*, *9*, and *10* for the parameter $k_c/\sqrt{\eta} = 0.5$. Numbering of the curves corresponds to that used in Fig. 2.

where $n_1 \equiv n_1(\mathbf{x}, t)$ is the electron density determined by the plasma waves; t is the time; $\mathbf{x} = \{x, y, z\}$ are the spatial variables; and $n_0(\mathbf{x})$, $v_F(\mathbf{x})$, e, and m are the density, the Fermi velocity, the charge, and the effective mass of conduction electrons, respectively ($n_0(\mathbf{x}) \gg |n_1(\mathbf{x}, t)|$). In order to simplify the model, we assume that, in the linearized equation (1), the electron Fermi velocity is a homogeneous parameter, and the electron density undergoes spatial fluctuations:

$$n_0(\mathbf{x}) = n_0[1 + \gamma\rho(\mathbf{x})], \quad (2)$$

where $\rho(\mathbf{x})$ is the centered ($\langle\rho(\mathbf{x})\rangle = 0$) and normalized ($\langle\rho^2(\mathbf{x})\rangle = 1$) functions of the coordinates; the angle brackets here indicate the averaging over an ensemble of realizations of the random function $\rho(\mathbf{x})$, and γ is the relative root-mean-square fluctuation of the electron density.

After the Fourier transformation of the linearized equation (1) with respect to the time, we obtain

$$\nabla^2 n_1(\mathbf{x}, \omega) + [\nu - \eta\rho(\mathbf{x})]n_1(\mathbf{x}, \omega) = 0, \quad (3)$$

where $\nu = 5(\omega^2 - \omega_p^2)/3v_F^2$, ω is the frequency of the wave, $\omega_p^2 = 4\pi e^2 n_0/m$ is the plasma frequency, and $\eta = 5\gamma\omega_p^2/3v_F^2$. In these designations, from expression (3) for the spectrum of a plasma wave in a homogeneous electron gas ($\gamma = 0$), we obtain

$$\nu(k) = k^2, \quad (4)$$

where $\mathbf{k}$ is the wave vector, $k = |\mathbf{k}|$; $n_1 \propto \exp[i(\mathbf{kx} - \omega t)]$. This dispersion relation in Fig. 1 is shown by the straight line $Q = 0$. Expression (4) leads to the well-known formula for the spectrum of plasma waves in the long-wavelength approximation ($k \gg \omega_p/v_F$):

$$\omega^2 = \omega_p^2 + \frac{3}{5}v_F^2 k^2. \quad (5)$$

In order to investigate plasma waves in a randomly inhomogeneous electron gas ($\gamma \neq 0$), we use the Kraichnan approximation [18], which takes into account multiple scattering of waves by inhomogeneities. This approximation is also known as the self-consistent approximation [19] or the coherent potential approximation, which takes into account the correlations of inhomogeneities [20]. According to the approach described in the aforementioned papers, the Fourier transform of the averaged Green's function corresponding to expression (3) has the form

$$\bar{G}(\mathbf{k}, \nu) = \frac{(2\pi)^{-3}}{\nu - k^2 - \Sigma(\mathbf{k}, \nu)}, \quad (6)$$

where the mass operator $\Sigma(\mathbf{k}, \nu)$ obeys the integral equation

$$\Sigma(\mathbf{k}, \nu) = \eta^2 \int \frac{S(\mathbf{k} - \mathbf{k}_1)d\mathbf{k}_1}{\nu - k_1^2 - \Sigma(\mathbf{k}_1, \nu)}. \quad (7)$$

Here, $S(\mathbf{k})$ is the spectral density related by the Fourier transform to the correlation function of the inhomogeneities $K_\rho(\mathbf{r}) = \langle\rho(\mathbf{x})\rho(\mathbf{x} + \mathbf{r})\rangle$; $k_1 = |\mathbf{k}_1|$. By setting the denominator of the function $\bar{G}(\mathbf{k}, \nu)$ equal to zero, we obtain the equation

$$\nu(\mathbf{k}) - k^2 = \Sigma(\mathbf{k}, \nu(\mathbf{k})). \quad (8)$$

The solution to this equation determines $\nu(\mathbf{k})$, i.e., the dispersion law of the averaged wave.

The equations similar to expressions (3), (6), and (7), but with other physical quantities as the unknowns and parameters, describe the propagation of waves of different nature in various randomly inhomogeneous media. For example, in [20], the numerical solution to the integral equation (7) was obtained for spin waves in a ferromagnet with a random anisotropy parameter for the exponential decay of the correlations of three-dimensional random inhomogeneities:

$$K_\rho(\mathbf{r}) = e^{-k_c r}, \quad S(\mathbf{k}) = \frac{k_c}{\pi^2(k_c^2+k^2)^2}, \qquad (9)$$

where k_c is the correlation wave number of random inhomogeneities; $r = |\mathbf{r}|$. The function $\rho(\mathbf{x})$ and the decay of the correlations of three-dimensional random inhomogeneities are assumed to be sufficiently smooth (the correlation radius is $r_c = 1/k_c \gg a_0$; a_0 is the interatomic distance). In [20], the authors determined the dependences of the position ν_m and width $\Delta\nu$ of the peak of the function $\bar{G}''(k, \nu) = \mathrm{Im}\bar{G}(k, \nu)$ on the wave vector k, which are shown respectively in Fig. 1 (the sequence of points *3*) and Fig. 2 (the sequence of points *1–3*); the width of the peak was calculated at its half-height. It was found that the position ν_m of the peak of the function $\bar{G}''(k, \nu)$ is shifted toward lower frequencies as compared to the peak position obtained for the propagation of waves in a homogeneous medium ($\nu_m - k^2 < 0$). In this case, the quantities $|\nu_m - k^2|$ and $\Delta\nu$ are maximum for small values of the wave vector ($k \ll \eta$) and the correlation wave number ($k_c \ll \eta$).

The dependences $\nu_m(k)$ and $\Delta\nu(k)$, which appear when the numerical solution of the integral equation (7) is used in expression (6), cannot be unequivocally identified with the spectrum $\nu' = \mathrm{Re}\,\nu(k)$ and the damping $\nu'' = -\mathrm{Im}\,\nu(k)$ of the averaged plasma wave, where $\nu(k)$ is determined by expression (8). Indeed, as was noted in [20], the Green's function calculated in the self-consistent approximation, in addition to the dissipative contribution introduced by the waves, has a nondissipative component, which is determined by the stochastic spread in the values of the randomly inhomogeneous parameter and is known as the nonuniform (fluctuation) broadening. In this connection, the search for the dispersion relation of the waves is of additional interest, because, with the knowledge of this relation, it is possible to evaluate the contribution from the nonuniform broadening to the position and shape of the resonance lines.

3. SPECTRUM OF PLASMA WAVES AND THEIR DAMPING

In order to determine the spectrum and damping of bulk plasma waves, we find an approximate solution to the integral equation (7) in the analytical form. For this purpose, it is common practice to use simplifying assumptions for the calculation of the integral on the right-hand side of this equation. The criterion for the admissibility of this simplification should be a correspondence between the numerical and analytical results. In [20], it was proposed to use an approximate method for solving the integral equation (7), which consists in replacing the mass operator $\Sigma(\mathbf{k}_1, \nu)$ with the mass operator $\Sigma(k, \nu)$ in the integrand. There is also the widely known Bourret approximation [21], which follows from the integral equation (7) provided that, on the right-hand side in the integrand, we set $\Sigma(\mathbf{k}_1, \nu) = 0$. Both of these approximations at $k \ll \eta$ and $k_c \ll \eta$ give the displacement of the position ν_m of the peak of the function $\bar{G}''(k, \nu)$ toward higher frequencies, which contradicts the numerical calculations performed in [20] (Fig. 1, the sequence of points *3*). However, if in the integral equation (7), in addition to the requirement $\Sigma(\mathbf{k}_1, \nu) = 0$ (Bourret approximation), we use the equality $\nu = k^2$, then, for the mass operator after calculating the integral, we come to an expression that is independent of ν. After substituting this expression into equation (8), we obtain the dispersion relation of the averaged plasma waves in the form

$$\nu(k) - k^2 = -\frac{\eta^2}{k_c^2 + 4k^2} - 2i\eta^2\frac{k}{k_c(k_c^2+4k^2)}. \qquad (10)$$

It should be noted that, although dispersion relation (10) differs from the dispersion relation of the plasma waves, which was obtained previously [12] on the basis of the kinetic equation, in the limiting cases $k \ll k_c$ and

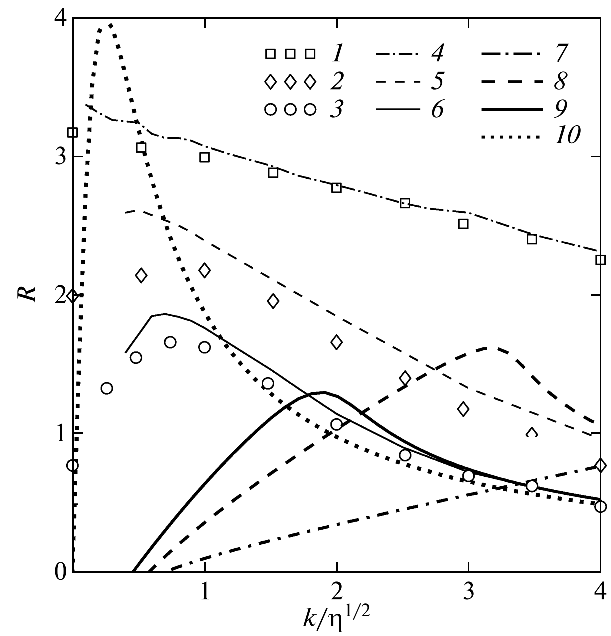


Fig. 2. Damping ν'' of plasma waves and the width $\Delta\nu$ of the peak of the function $\bar{G}''(k, \nu)$: $R = \Delta\nu/\eta$ (curves *1–6*) and $R = 2\nu''/\eta$ (curves *7–10*). (*1–3*) The value of $\Delta\nu$ is taken from [20]; (*4–6*) $\Delta\nu$ is obtained from formulas (6), (13), and (17); (*7–9*) ν'' is determined using formula (19); and (*10*) ν'' is calculated according to formula (10). Curves *1*, *4*, and *7* are plotted for the parameter $k_c/\sqrt{\eta} = 0.1$; curves *2*, *5*, and *8* for the parameter $k_c/\eta = 0.3$; and curves *3*, *6*, *9*, and *10* for the parameter $k_c/\sqrt{\eta} = 0.5$.

$k \gg k_c$ these expressions coincide to within a constant factor. A formula similar to the dispersion relation (10) was obtained in [22] in the second-order perturbation theory for the spectrum and damping of spin waves in a ferromagnet with a random anisotropy parameter. The modification of the spectrum of the plasma waves, which follows from formula (10), agrees qualitatively with the position ν_m of the peak of the function $\bar{G}''(k, \nu)$, which was obtained from expressions (6) and (7) ($\nu' - k^2 < 0$ and $\nu_m - k^2 < 0$); however, when $k \ll k_c < \sqrt{\eta}/2$, it turns out that $|\nu' - k^2| \gg |\nu_m - k^2|$. A more obvious manifestation of the limitation of expression (10) for the description of the dispersion relation of the plasma waves is that, when $k \approx k_c < \sqrt{\eta}/2$, the double wave damping obeys the inequality $2\nu'' \gg \Delta\nu$. At the same time, the quantity $\Delta\nu$, which is determined by both the dissipative contribution and the nonuniform (nondissipative) broadening, should not at least be less than $2\nu''$.

Let us now obtain the approximate analytical solution to the integral equation (7). For this purpose, we will use the approach proposed earlier [23] in the study of electromagnetic waves in a randomly inhomogeneous Josephson junction with one-dimensional random inhomogeneities. This approach lies in the fact that the angular dependence of $\Sigma(\mathbf{k}_1, \nu)$ is ignored and the denominator of the integrand on the right-hand side of the integral equation (7) in the vicinity of the point $k_1 = k$ is expanded into the power series

$$\nu - k_1^2 - \Sigma(k_1, \nu) = g - \left[2k + \frac{d\Sigma(k, \nu)}{dk}\right](k - k_1) - \left[1 + \frac{1}{2}\frac{d^2\Sigma(k, \nu)}{dk^2}\right](k - k_1)^2 - \dots, \tag{11}$$

where $g = \nu - k^2 - \Sigma(k, \nu)$. In the cases where the spectral density $S(\mathbf{k} - \mathbf{k}_1)$ and the other factors in the numerator of the integrand of the integral equation (7) form a function of the integration variable k_1, which, in the vicinity of the point $k_1 = k$, has a well-defined maximum, we can restrict ourselves in the denominator of the integrand to the first two or three terms of expansion (11). By using expression (9) for the spectral density on the right-hand side of the integral equation (7) and integrating over the angular variables of the spherical coordinate system with the polar axis along the wave vector $\mathbf{k}$, we obtain

$$\Sigma(k, \nu) = \eta^2 \frac{4k_c}{\pi} \int_0^{\infty} \frac{1}{\nu - k_1^2 - \Sigma(k_1, \nu)} \times \frac{k_1^2}{(k_c^2 + k^2 + k_1^2)^2 - 4k^2k_1^2} dk_1, \tag{12}$$

where the second cofactor in the integrand has a maximum at $k_1 = k_{1n} \equiv \sqrt{k^2 + k_c^2}$; whence, for $k > k_c$, it follows that $k_{1n} \approx k$. Then, we substitute expansion (11) into expression (12), restrict ourselves to the first two terms of the expansion, and integrate using the method of residues. As a result, we obtain

$$\tilde{\Sigma}(k, \nu) = \frac{A_1}{\pi K Y (a^2 + K_c^2)[(2K + a)^2 + K_c^2]}, \tag{13}$$

where

$$A_1 = 4KK_c(K + a)^2 \ln \frac{K + a}{\sqrt{K^2 + K_c^2}} + 2[(K^2 - K_c^2)(K + a)^2 - (K^2 + K_c^2)^2] \arctan \frac{K}{K_c} + \pi K (K + a)(a^2 + 2aK - K_c^2) - 4i\pi K K_c (K + a)^2 \tag{14}$$

and the following dimensionless quantities are introduced:

$$\tilde{\Sigma}(k, \nu) = \frac{\Sigma(k, \nu)}{\eta}, \quad K = \frac{k}{\sqrt{\eta}}, \quad K_c = \frac{k_c}{\sqrt{\eta}}, \quad Y = \frac{2k + d\Sigma(k, \nu)/dk}{\sqrt{\eta}}, \quad a = \frac{g}{\eta Y}. \tag{15}$$

The differential equation in $\Sigma(k, \nu)$, which follows from expression (13), at present cannot be solved; therefore, this equation can be considered as a transcendental equation and the derivative $d\Sigma(k, \nu)/dk$ involved in it can be obtained by differentiating expression (12):

$$\frac{d\Sigma(k, \nu)}{dk} = \eta^2 \frac{4k_c}{\pi} \int_0^{\infty} \frac{k_1^2}{\nu - k_1^2 - \Sigma(k_1, \nu)} \times \frac{d}{dk} \frac{1}{(k_c^2 + k^2 + k_1^2)^2 - 4k^2k_1^2} dk_1. \tag{16}$$

The integral on the right-hand side of this equation can be calculated by performing the differentiation over the parameter k and the integration using the method of residues over the integration variable k_1. We perform the latter integration using expansion (11) and restrict ourselves, as was done above, to the first two terms. As a result, we have

$$Y = 2K - \frac{\tilde{\Sigma}}{K} - \frac{1}{\pi K Y}\left\{\frac{2K_c}{K_c^2 + K^2} + (K + a)\frac{d}{dK_c}\frac{A_2}{(a^2 + K_c^2)[(2K + a)^2 + K_c^2]}\right\}, \tag{17}$$

where $\tilde{\Sigma} \equiv \tilde{\Sigma}(k, \nu)$,

$$A_2 = 2(K+a)(a^2+2aK+K_c^2)\left(\ln\frac{K+a}{\sqrt{K^2+K_c^2}} - i\pi\right) - 4KK_c(K+a)\arctan\frac{K}{K_c} - \pi K_c[(K+a)^2+K^2+K_c^2]. \quad (18)$$

Expressions (13) and (17) form a system of two transcendental equations in the unknowns $\tilde{\Sigma}$ and Y. After substituting the values of the mass operator $\Sigma(k, \nu)$, which were obtained by the numerical solution of this system of equations ($\mathrm{Im}\,a > 0$), into expression (6), we determine the position ν_m and width $\Delta\nu$ of the peak of the function $\bar{G}''(k, \nu)$, which are shown, respectively, in Fig. 1 by curve *6* and in Fig. 2 by curves *4–6*. It can be seen that these curves with the sequences of points *1–3* are in qualitative agreement and, in some cases, coincide with each other. The closeness of the numerical results obtained for the Green's function in the framework of the approach developed in this work and in [20] makes the use of expressions (13) and (17) justified for the determination of the spectrum of the averaged plasma waves and their damping, as well as indicates that, using expansion (11) in the integral equation (7), we are left in the main within the scope of the self-consistent approximation and take into account the multiple scattering of plasma waves by the inhomogeneities.

In the case when equality (8) is satisfied, the system of equations, which is formed by expressions (13) and (17), is simplified ($a = 0$). As a result, we can obtain the solution of this system of equations in the analytical form. Thus, for the dispersion relation of the averaged plasma waves, we have

$$X = \frac{2K^3\ln\frac{K^2}{K_c^2+K^2} - 2K_c(K_c^2+3K^2)\arctan\frac{K}{K_c} - \pi K^2(K_c+4iK)}{\pi K_cK^2(K_c^2+4K^2)+\sqrt{F}}, \quad (19)$$

where $X = [\nu(k) - k^2]/\eta$,

$$F = \pi^2K^4[K_c^2(4K^2+K_c^2)^2 + 2K_c^2 - 8K^2 + 16iK_cK] - 2\pi KK_c(K_c^4+6K^2K_c^2+8K^4) - 8\pi K_cK^5\ln\frac{K^2}{K_c^2+K^2} + 2\pi[K_c^4(7K^2+K_c^2) + 2K^4(3K_c^2-4K^2)]\arctan\frac{K}{K_c}. \quad (20)$$

The spectrum ν' and the double damping $2\nu''$ of the averaged plasma wave, which follow from these expressions, are shown in Figs. 1 and 2 by lines *7–9*, respectively. Thus, it has been found that the frequency of the averaged plasma waves in a randomly inhomogeneous electron gas decreases in comparison with the frequency of these waves in a homogeneous medium. This correlates with the shift of the position ν_m of the peak of the function $\bar{G}''(k, \nu)$ toward lower frequencies, which was determined in [20] (the sequence of points *3* in Fig. 1) and calculated according to formulas (6), (13), and (17) (curve *6* in Fig. 1).

Figure 2 demonstrates that, in the case when $k < k_\omega$, where k_ω is the solution of the equation $\mathrm{Im}X = 0$ (when $k = k_\omega$, curves *7–9* intersect the horizontal axis), expression (19) does not describe the averaged plasma wave. In order to obtain an analytical estimate of the quantity k_ω, we consider the limiting cases. First and foremost, we note that, when $\gamma \longrightarrow 0$, the right-hand side of expression (19) vanishes; as a result, we arrived to formulas (4) and (5) for the spectrum of plasma waves in a homogeneous medium. For $\gamma \neq 0$, $K_c \ll 1$, and $K > K_c$, from expression (19) in the vicinity of the point $\mathrm{Im}X = 0$, we obtain

$$X \approx -1 - \frac{3}{4}K_c^2 + \frac{1}{2}K^2K_c^2 + 2iK_c\left(\frac{1}{\sqrt{2}} - \frac{K_c}{\pi} - K\right). \quad (21)$$

From this expansion, using definitions (15) and changing over to the dimensional quantities, we have

$$\omega^2 \approx \omega_p^2(1-\gamma) + \frac{3}{5}k^2v_F^2\left(1 + \frac{3}{10}\frac{k_c^2v_F^2}{\omega_p^2\gamma}\right) + \frac{6}{5}ik_cv_F\left(\sqrt{\frac{5\gamma}{6}}\omega_p - \frac{k_cv_F}{\pi} - kv_F\right), \quad (22)$$

where $\omega' = \omega' - i\omega''$. Expression (22) describes the spectrum ω' and the damping ω'' of the averaged plasma waves for the wave vector

$$k \geq \sqrt{\frac{5\gamma}{6}}\frac{\omega_p}{v_F} - \frac{k_c}{\pi} \approx k_\omega \quad (23)$$

under the condition $\mathrm{Im}\,\omega \leq 0$. This expression approximates the parameter k_ω and shows that, with a decrease in the parameters γ and r_c, the lower boundary of the existence of the averaged plasma waves is shifted toward the long-wavelength range.

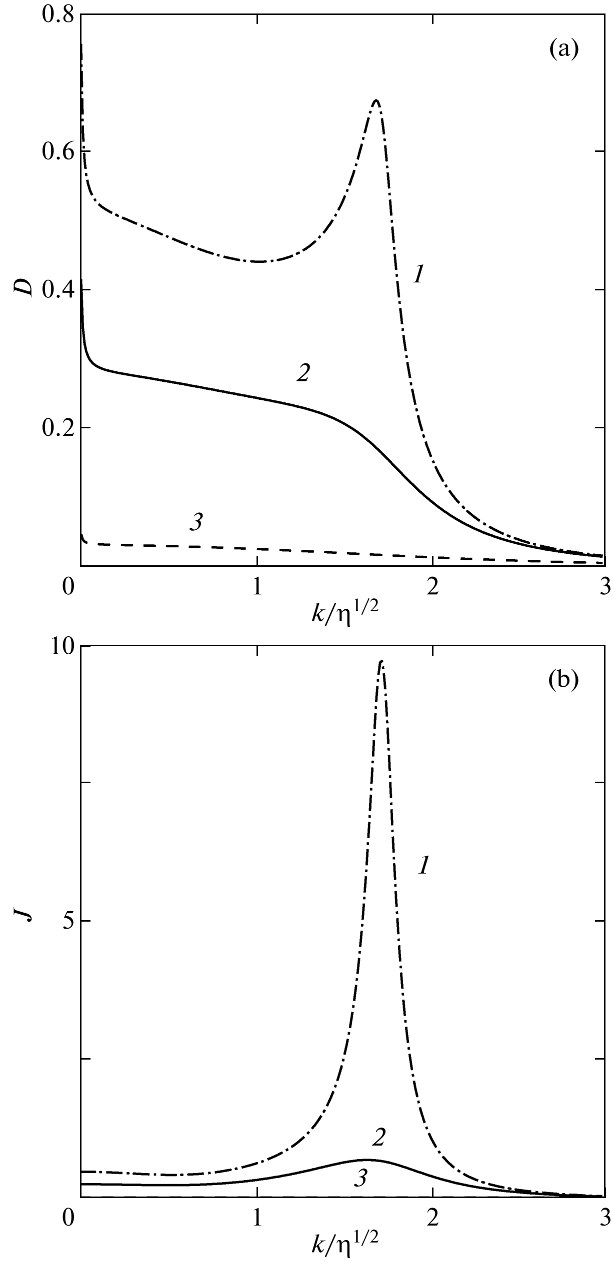


Fig. 3. Normalized derivatives of the mass operator: (a) $D = |d\Sigma(k,\ \nu)/dk|/2k$ and (b) $J = |d^2\Sigma(k,\ \nu)/dk^2|/2$. The curves are plotted for the parameters $k_c/\sqrt{\eta}$ = (*1*) 0.1, (*2*) 0.5, and (*3*) 2.0.

By using expression (19) for $K \gg 1$ and definitions (15) and changing over to the dimensional quantities, we have

$$\omega^2 \approx \omega_p^2\left[1 - \gamma^2\frac{5\omega_p^2}{6v_F^2}\left(\frac{1}{k^2} + i\frac{1}{k_c k}\right)\right] + \frac{3}{5}v_F^2 k^2. \qquad (24)$$

It can be seen that, in this limit, the damping of plasma waves is inversely proportional to the wave vector k and coincides with the asymptotics of the imaginary part of expression (10) for $k \gg k_c$. Thus, according to expression (19), the dependence of the damping of plasma waves on the wave vector k changes with an increase in the value of k from linear to inversely proportional and passes through a maximum at the point $k \approx \eta/k_c$. This pattern of the change in the double wave damping $2\nu''$ with an increase in the wave vector k is displayed in Fig. 2 (curves *8* and *9*). From this figure, it also follows that the parameters $\Delta\nu$ and $2\nu''$ have different dependences on the wave vector k. This behavior is associated with the fact that the quantity $\Delta\nu$ calculated in the self-consistent approximation, as was noted above, is determined not only by the damping of plasma waves but also by the nonuniform broadening. The sequences of points *1*–*3* and curves *4*–*6* in Fig. 2 reflect the combined influence of these two mechanisms on the width $\Delta\nu$ of the peak of the function $\bar{G}''(k,\ \nu)$. In order to estimate the contribution from each of these mechanisms, we use the obtained damping of the averaged plasma wave, which follows from expression (19). From a comparison of the quantity $\Delta\nu$ with the double wave damping $2\nu''$ (Fig. 2), we can conclude that, for small values of k_c, the width $\Delta\nu$ of the peak of the function $\bar{G}''(k,\ \nu)$ in the range $k \ll \sqrt{\eta}$ is predominantly determined by the nonuniform broadening, and in the range $k \gg \sqrt{\eta}$, the contribution from the damping of the plasma wave becomes dominant.

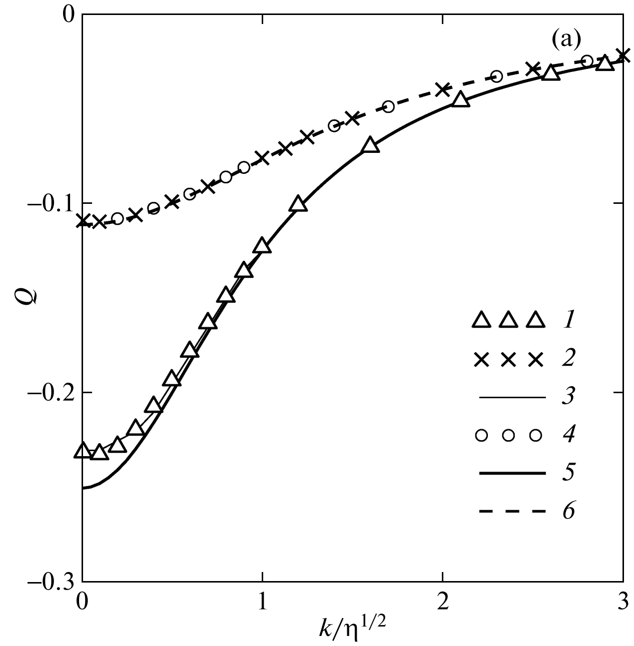
With an increase in k_c, the spectral density in the integral equation (7) becomes a more smooth function of k_1; consequently, there is a need to use in this equation three terms of expansion (11). However, when $k_c \gg \sqrt{\eta}$, the inequalities $|d^2\Sigma(k,\ \nu)/dk^2| \ll 2$ and $|d\Sigma(k,\ \nu)/dk| \ll 2k$ are satisfied (Figs. 3a and 3b), which allows us to disregard the derivatives of $\Sigma(k,\ \nu)$ in expansion (11). As a result, this expansion takes the form

$$\nu - k_1^2 - \Sigma(k_1, \nu) \approx \nu - k_1^2 - \Sigma(k, \nu). \qquad (25)$$

The use of this formula in the denominator of the integrand in expression (7) leads to the approximation proposed in [20]. Taking into account expression (25) and performing the integration on the right-hand side of the integral equation (7), we obtain the following equation:

$$\Sigma(k, \nu) = -\eta^2\frac{k_c^2 - g + 2ik_c\sqrt{g + k^2}}{(g + k_c^2)^2 + 4k^2k_c^2}, \qquad (26)$$

which leads to the algebraic equation of the fourth degree investigated in [20]. By substituting the value of the mass operator $\Sigma(k,\ \nu)$ obtained by the numerical solution of equation (26) into expression (6), we determine the position ν_m and width $\Delta\nu$ of the peak of the



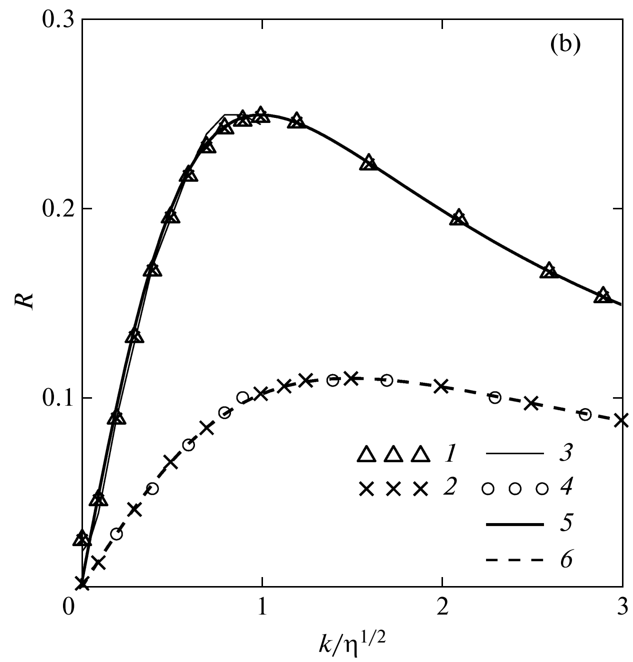


Fig. 4. (a) Spectrum ν' of plasma waves and the position ν_m of the peak of the function $\bar{G}''(k, \nu)$: $Q = (\nu_m - k^2)/\eta$ (curves *1–4*) and $Q = (\nu' - k^2)/\eta$ (curves *5* and *6*). (b) The damping ν'' of plasma waves and the width $\Delta\nu$ of the peak of the function $\bar{G}''(k, \nu)$: $R = \Delta\nu/\eta$ (curves *1–4*) and $R = 2\nu''/\eta$ (curves *5* and *6*). (*1*, *2*) The values of ν_m and $\Delta\nu$ are obtained from formulas (6) and (7); (*3*, *4*) ν_m and $\Delta\nu$ are determined using formulas (6) and (26); and (*5*, *6*) ν' and ν'' are calculated according to formula (10). Curves *1*, *3*, and *5* are plotted for the parameter $k_c/\sqrt{\eta} = 2$, and curves *2*, *4*, and *6* for the parameter $k_c/\sqrt{\eta} = 3$.

function $\bar{G}''(k, \nu)$, which are shown respectively in Figs. 4a and 4b (curves *3* and sequences of points *4*). It can be seen that they almost completely coincide with the sequences of points *1* and *2* obtained using the numerical solution of the integral equation (7) in expression (6).

In order to determine the dispersion relation of the plasma waves, we will use equality (8), according to which $g = 0$. Then, formula (10) for the spectrum and damping of the excitations follows from expression (26). This means that, when $k_c \geq 2\sqrt{\eta}$, the double scattering of plasma waves by inhomogeneities becomes dominant, and the right-hand side of expression (10) well approximates the modification of the dispersion relation of the plasma waves, which was calculated in the self-consistent approximation. The spectrum and damping of the plasma waves, which follow from expression (10), are shown respectively in Figs. 4a and 4b (curves *5* and *6*), i.e., $\nu' \approx \nu_m$ and $2\nu'' \approx \Delta\nu$. In this case, the mass operator is weakly dependent on ν, and for the spectrum and damping of the plasma waves in accordance with expression (8), we have $\nu' - k^2 \approx \Sigma'(k)$ and $\nu'' \approx -\Sigma''(k)$. In particular, the right-hand side of expression (10), in addition to the modification of the spectrum and damping of the waves, describes the mass operator.

It should be noted that the derivatives of $\Sigma(\nu, k)$ in the second and third terms of expansion (11) can be disregarded for any values of k_c when $k \gg \sqrt{\eta}$ (see, for example, Figs. 3a and 3b). Therefore, in this limit, equation (26) for $\Sigma(\nu, k)$ holds true, and the spectrum of plasma waves is determined by expression (10). This makes it possible to understand the coincidence between the characteristics of plasma waves calculated in the self-consistent approximation and those obtained in terms of the perturbation theory.

4. CONCLUSIONS

The dispersion relation of the bulk plasma waves has been investigated in a simple model of a degenerate randomly inhomogeneous gas of conduction electrons in the hydrodynamic approximation using the averaged Green's function method. Equation (3), which describes bulk plasma waves in a randomly inhomogeneous medium, has been analyzed using the self-consistent approximation that takes into account multiple scattering of plasma waves by inhomogeneities and according to which the Fourier transform (6) of the averaged Green's function is expressed in terms of the mass operator that obeys the integral equation (7). The solution of this equation describes the contribution made to the Green's function by both the plasma waves and the stochastic spread in the values of the randomly inhomogeneous parameter $n_0(\mathbf{r})$. This spread leads to an increase in the width $\Delta\nu$ of the peak of the function $\bar{G}''(k, \nu)$, which is known as the nonuniform (fluctuation, nondissipative) broadening. The dispersion relation for the averaged plasma waves has been obtained using the simplification of the integral equation (7), which provides a small violation of self-consistency. The criterion for the admissibility of this violation is the correspondence of the position ν_m of the peak of the function $\bar{G}''(k, \nu)$ and its width $\Delta\nu$, which were obtained numerically from expressions (6) and (7) and determined using the simplifying assumptions in the integral equation (7). The use of the first two terms in the expansion of the denominator of the integrand of this equation into the power series (11) in the vicinity of the point $k_1 = k$ made it possible to achieve a qualitative agreement between the dependences $\nu_m(k)$ and $\Delta\nu(k)$ for $k_c \leq \sqrt{\eta}/2$ and $k > k_c$, which were determined in [20] and obtained from expressions (6), (13), and (17). These correspondences, together with expressions (13) and (17), made it possible to determine the spectrum ν' and the damping ν'' of the averaged plasma waves (see formula (19)). We can note a qualitative agreement between the dependences $\nu'(k)$ and $\nu_m(k)$ obtained from expression (19) and formulas (6), (13), (17), respectively (curves *9* and *6* in Fig. 1), and also their coincidence when $k \gg \sqrt{\eta}$. At the same time, the dependences $\nu''(k)$ and $\Delta\nu(k)$ obtained from expression (19) and formulas (6), (13), (17), respectively, as well as those determined in [20], are significantly different at $k < \eta/k_c$. In this range of variations in values k, the width of the peak $\Delta\nu$ remains finite (curves *4–6* in Fig. 2), whereas the wave damping ν'' is equal to zero at $k = k_\omega$, increases linearly with an increase in the value of k, and passes through a maximum at $k \approx \eta/k_c$ (curves *8* and *9* in Fig. 2). The same expression for the position of the maximum of the wave damping was obtained in [23].

Expression (19) allows one to estimate the contribution to the width of the peak $\Delta\nu$ from the damping of the averaged wave. From a comparison of the quantities $\Delta\nu$ (sequences of points *1–3* in Fig. 2) and $2\nu''$ (curves *7–9* in Fig. 2), it can be concluded that the width of the peak $\Delta\nu$ for $k_c \leq \sqrt{\eta}/2$ in the range $k_\omega < k \ll \eta/k_c$ is determined by the nonuniform broadening. At $k \gtrsim \eta/k_c$, the contribution from the wave damping to the width of the peak $\Delta\nu$ becomes dominant.

With an increase in the correlation wave number k_c, the spectral density (9) is smoothed, and there is a need to use three terms of expansion (11) in the integral equation (7). Furthermore, when $k_c \gg \sqrt{\eta}$, the derivatives in expression (11) become negligible (see Figs. 3a and 3b); as a result, we obtain the approximate equation for the mass operator, which was proposed in [20].

It should be noted that the results obtained in the present work are more applicable to “bad” conductors (degenerate semiconductors, amorphous metals, etc.); however, apparently, they give a qualitatively correct picture for the spectrum and damping of plasma waves in randomly inhomogeneous materials, such as microcrystalline metals and amorphous semiconductors.

Equation (3) is isomorphic to the equations describing spin waves in ferromagnets with a random anisotropy parameter, elastic and electromagnetic waves in the scalar approximation with density fluctuations of the medium and its dielectric constant, respectively, as well as electromagnetic waves in randomly inhomogeneous Josephson junctions, etc. Therefore, the results obtained can be useful in investigating other excitations in a number of materials with three-dimensional random inhomogeneities.

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