Symmetry breaking in photonic crystal waveguide coupled with the dipole modes of a nonlinear optical cavity

Evgeny N. Bulgakov[1,2] and Almas F. Sadreev[1,*]

[1]*L.V. Kirensky Institute of Physics, Krasnoyarsk 660036, Russia*
[2]*Siberian State Aerospace University, Krasnoyarsk 660014, Russia*
**Corresponding author: almas@tnp.krasn.ru*

Received June 14, 2012; revised September 8, 2012; accepted September 8, 2012;
posted September 10, 2012 (Doc. ID 170685); published September 27, 2012

We present stable symmetry breaking solutions in a nonlinear optical cavity with dipole eigenmodes embedded into the propagation band of a directional photonic crystal waveguide for symmetric injecting condition. We demonstrate how this phenomenon can be exploited for all-optical switching of light transmission from the one side of the waveguide to another by application of input pulses. When the light injected to both sides of the waveguide has equal intensities but different phases, we reveal a wealth of new solutions. © 2012 Optical Society of America

OCIS codes: 190.3270, 190.4360, 190.5890.

1. INTRODUCTION

Symmetry breaking (SB) is a counterintuitive physical effect that describes the appearance of asymmetric states while the structure under study, and its excitation, is completely symmetric. Different nonlinearities can instigate the breaking, but we employ the instantaneous Kerr nonlinearity, where the index depends on the intensity of the local electric field. That field being expanded over the appropriate eigenmodes leads to the coupling of the modes. Therefore, it is possible that the symmetric situation is no longer stable at a certain input power. Then the system will drift to a situation where even and odd modes are excited, and thus an asymmetric state arises. That mechanism of the SB was studied in nonlinear optics [1,2], in particular in a system of two nonlinear optical cavities [3–7] and in a nonlinear dual-core directional fiber [8–10]. In the present paper we consider SB in the directional photonic crystal (PhC) waveguide with a single nonlinear defect made of Kerr media when the defect is presented by two dipole modes. The SB allows access to both dipole modes that are not readily accessible in symmetric configurations. Thus, a symmetrical injecting condition can end with asymmetrical steady state, and these systems could provide key functions, such as flip-flop operations [4,7] in all-optical systems.

The single linear cavity with two degenerated dipole modes has been already considered in the cross waveguide in a seminal paper by Johnson *et al.* [11]. They demonstrated fundamental role of the symmetry selection rules for cross talk. Yanik *et al.* [12] considered a nonlinear cavity of elliptic shape with two dipole modes at the center of the cross PhC waveguide. They have shown that, due to nonlinearity of the cavity, the transmission over the x direction can be reversibly switched on/off by a control power over the y direction to realize an all-optical transistor in the x-shaped waveguide. Recently it was shown that the nonlinear cavity with two degenerated dipole modes positioned at the center of the directional waveguide can operate in two regimes [13]. In the first conventional regime, the ingoing wave excites only that dipole mode whose parity coincides with the parity of the wave. In the second regime, both modes with opposite parities are resonantly excited due to a Kerr effect. That results in a giant vortex for the Poynting vector. In the present paper we explore this property of the nonlinear dipole cavity to spontaneously excite both dipole modes to give rise to SB for the symmetric injecting condition.

2. COUPLED-MODE THEORY

The one-dimensional linear PhC waveguide formed by removal of the single row of dielectric rods is shown in Fig. 1. The waveguide supports a single band of guided TM mode spanning from the bottom band edge 0.315 to the upper one 0.41 in terms of $2\pi c/a$ with the electric field directed along the rods [6]. By choice of dielectric constant ϵ_0 or radius of the defect rod, one can fit two dipole eigenfrequencies of the defect cavity into the propagation band of the waveguide [14] while other modes remain beyond. The corresponding dipole modes $E_1(\mathbf{x})$ and $E_2(\mathbf{x})$ obtained by numerical solution of the Maxwell equations are shown in Fig. 1 where the first mode is even and the second mode is odd relative to the mirror reflection left/right. Henceforth, we call such a defect cavity as a dipole defect. Furthermore, we remove two rows of the rods shown in Fig. 1 by open dashed circles. That forms the directional waveguide. However, we leave two rods around the defect rod to decrease the coupling of the dipole eigenmodes with continua of the waveguide [12]. As the result, the dipole states become the extended resonant ones.

Following [3–5], we apply light to both ends of the waveguide with the same amplitude $E_{\rm in}$, however with different phases. In the resonant approximation, we can write the electric field within the dipole cavity as $E(x,y) \approx A_1E_1(x,y) + A_2E_2(x,y)$. Let us take for a while that

 © 2012 Optical Society of America

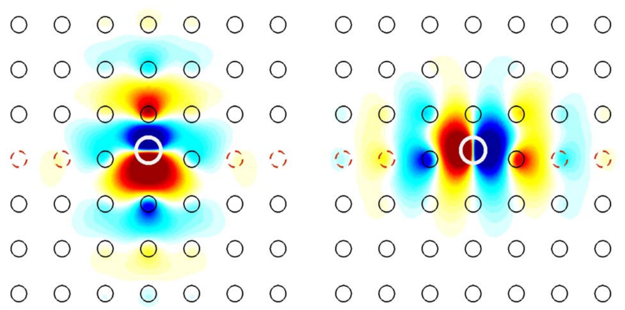

Fig. 1. (Color online) Two cavity dipole TM eigenmodes (space profiles of the electric field directed parallel to the rods) with the eigenfrequencies $\omega_1 a/2\pi c = 0.371$ and $\omega_2 a/2\pi c = 0.367$ in the two-dimensional square lattice PhC consisting of GaAs dielectric rods with radius $0.18a$ and dielectric constant $\epsilon = 11.56$, where $a = 0.5$ μm is the lattice unit. These rods are shown by black open circles. The defect shown by open white circle has the radius $0.288a$ and $\epsilon_0 = 12$. It is shifted relative to the waveguide center line by the distance $0.2a$.

the dipole cavity is linear. Then one can write the following coupled-mode theory (CMT) equations for the amplitudes A_m, $m = 1, 2$ [15]:

$$i\dot{A}_1 = [\omega_1 - i\gamma_1]A_1 + i\sqrt{\gamma_1}E_{\text{in}}e^{-i\omega t}(1 + e^{i\theta}),$$
$$i\dot{A}_2 = [\omega_2 - i\gamma_2]A_2 + i\sqrt{\gamma_2}E_{\text{in}}e^{-i\omega t}(1 - e^{i\theta}), \tag{1}$$

where E_{in} is the amplitude of the injected light with the frequency ω, θ the phase difference between waves ingoing to the left and to the right ends of the waveguide. Here the coupling matrix of the dipole modes with ingoing waves takes the form $\begin{pmatrix}\sqrt{\gamma_1} & \sqrt{\gamma_2}\\ \sqrt{\gamma_1} & -\sqrt{\gamma_2}\end{pmatrix}$ and the decay matrix $\Gamma = \begin{pmatrix}\gamma_1 & 0\\ 0 & \gamma_2\end{pmatrix}$ [15]. For symmetrical position of the defect at the center line of the waveguide, we have $\gamma_1 = 0$ and the first dipole mode could not be excited. If $\theta = 0$, the second dipole mode is not excited for the symmetrical injecting condition. Thus, the dipole defect positioned symmetrically in the waveguide is *completely invisible* for light injected from both sides with equaled phases. In what follows we consider the dipole defect shifted relative to the center line of the waveguide with $\gamma_1 \neq 0$.

Next, let the dipole defect rod be made from a Kerr media. Then we have to modify the CMT Eq. (1) to account for the nonlinear contributions. That could be done by the use of the remarkable analogy between electrodynamics in dielectric media and quantum mechanics [14]. We consider that the instantaneous Kerr change of the dielectric constant,

$$\delta\epsilon(\vec{r}) = \frac{n_0 c n_2 |E(\vec{r})|^2}{4\pi} \approx \frac{n_0 c n_2 |A_1 E_1(\vec{r}) + A_2 E_2(\vec{r})|^2}{4\pi}, \tag{2}$$

is small compared to the linear dielectric constant $\epsilon_0 = n_0^2$. Here n_0 and n_2 are the linear and nonlinear refractive indexes of the dipole defect, respectively; c is the light velocity. Then we can use the perturbation theory developed in [6] and modify the CMT Eq. (1) as follows:

$$i\dot{A}_1 = [\omega_1 + V_{11} - i\gamma_1]A_1 + V_{12}A_2 + i\sqrt{\gamma_1}E_{\text{in}}e^{-i\omega t}(1 + e^{i\theta}),$$
$$i\dot{A}_1 = [\omega_2 + V_{22} - i\gamma_2]A_2 + V_{21}A_1 + i\sqrt{\gamma_2}E_{\text{in}}e^{-i\omega t}(1 - e^{i\theta}), \tag{3}$$

where

$$\langle m|V|n\rangle = -\frac{(\omega_m + \omega_n)}{4N_m}\int d^2\vec{r}\,\delta\epsilon(\vec{r})E_m(\vec{r})E_n(\vec{r}), \tag{4}$$

$$N_m = \int d^2\vec{r}\,\epsilon(\vec{r})E_m^2(\vec{r}) = \frac{a^2}{cn_2}, \tag{5}$$

is the normalization constant of the eigenmodes with $\epsilon(\vec{r})$ as the dielectric constant of whole defectless PhC. After substitution of Eqs. (4) and (5) and $A_m = A_m e^{-i\omega t}$, we write the stationary CMT Eq. (3) in the dimensionless form,

$$[\omega - \omega_1 + \lambda_{11}I_1 + \lambda_{12}I_2 + i\gamma_1]A_1 + 2\lambda_{12}\,\mathrm{Re}(A_1^* A_2)A_2$$
$$= i\sqrt{\gamma_1}E_{\text{in}}(1 + e^{i\theta}),$$
$$2\lambda_{12}\,\mathrm{Re}(A_1^* A_2)A_1 + [\omega - \omega_2 + \lambda_{22}I_2 + \lambda_{12}I_1 + i\gamma_2]A_2$$
$$= i\sqrt{\gamma_2}E_{\text{in}}(1 - e^{i\theta}), \tag{6}$$

where we introduced $I_m = |A_m|^2$ as the intensities of the dipole modes and dimensionless constants of nonlinearity

$$\lambda_{mn} = \frac{(\omega_m + \omega_n)n_0 c^2 n_2^2}{16\pi a^2}\int_\sigma E_m^2(x,y)E_n^2(x,y)d^2\vec{r},$$

with σ as the cross section of the defect rod. Respectively, the transmission amplitude from the left to the right and from the right to the left equal [15]:

$$t_L = \sqrt{\gamma_1}A_1 + \sqrt{\gamma_2}A_2 - E_{\text{in},}$$
$$t_R = \sqrt{\gamma_1}A_1 - \sqrt{\gamma_2}A_2 - E_{\text{in}}e^{i\theta}, \tag{7}$$

respectively.

3. NUMERICAL RESULTS

We take the Kerr nonlinear refractive index for the defect rod $n_2 = 2 \times 10^{-13}$ cm^2/W. Other material parameters are listed in the caption of Fig. 1. Substituting numerically calculated eigen dipole modes shown in Fig. 1 into Eqs. (2) and (4) and taking into account Eq. (5), we obtained $\lambda_{11} = 1.163 \times 10^{-3}$, $\lambda_{22} = 1.24 \cdot 10^{-3}$, $\lambda_{12} = 4.25 \cdot 10^{-4}$. The resonant widths γ_m were obtained directly from resonant transmission in PhC waveguide with a linear dipole defect. The transmission was calculated by use of the Maxwell equations for the TM mode propagation. As a result we obtained $\gamma_1 = 3.6 \cdot 10^{-4}$, $\gamma_2 = 1.8 \cdot 10^{-3}$.

First, we present the self-consistent solutions of Eqs. (6) and (7) for the symmetric injected condition $\theta = 0$, which are shown in Fig. 2. There are two types of solutions. In the first, the symmetry-preserving solution, the even dipole mode E_1 is excited only. The intensity of excitation shown in Fig. 2(a) by the dashed lines has resonance frequency behavior typical for the single nonlinear mode [14,16]. The transmission amplitudes given by Eq. (7) would have the same resonance behavior if the input power were applied to only one side of the waveguide. However, for the symmetrical injecting condition, we have the equal transmissions $T_L = T_R$ as seen from Eq. (7). We normalized the transmissions as $T_L + T_R = 1$, where $T_L = |t_L|^2/E_{\text{in}}^2$, $T_R = |t_R|^2/E_{\text{in}}^2$. Therefore, for the symmetry-preserving solution $T_L = T_R = 0.5$ as shown in Figs. 2(b) and 2(c) by the dashed line.

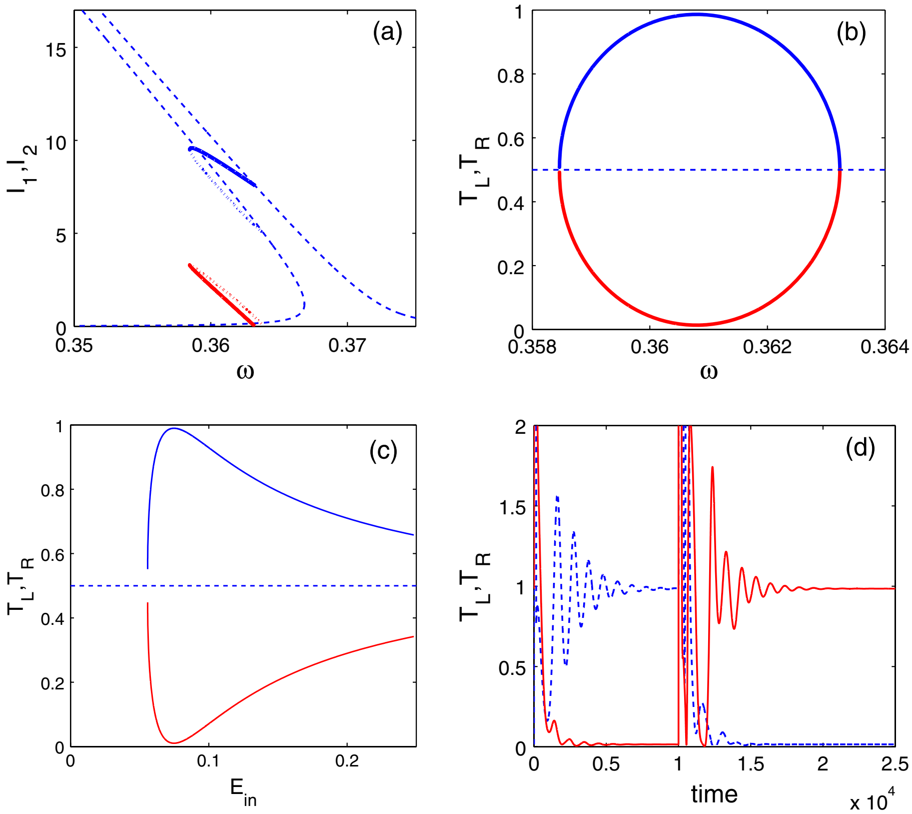

Fig. 2. (Color online) Frequency behavior of (a) intensities of dipole modes and (b) transmissions to the left T_L and to the right T_R for light injection with $E_{\rm in} = 0.08$, $\theta = 0$ onto both sides of the waveguide. In (a), blue lines show the intensity of even dipole mode $I_1 = |A_1|^2$, while red lines show the intensity of odd dipole mode $I_2 = |A_2|^2$. The parameters are given in the beginning of Section 3. In (b) and (c), red lines show T_R, blue solid lines show T_L. (c) Transmissions as dependent on the input amplitude $E_{\rm in}$ for $\omega = 0.361$. In (a)–(c), dashed lines show the symmetry-preserving solution, while solid lines show the SB solution. The thicker lines mark stable solutions. (d) Time dependence of the transmissions to the left (blue lines) and to the right (red lines), which follow the impulses of the input light. The first and second impulses have amplitudes 2 and 5 and durations 150 and 200, respectively (are not shown). In (b) and (c), only stable solutions are presented.

One of our main results is that the off-diagonal nonlinear terms in Eq. (6) can provoke excitement of the second odd dipole mode spontaneously in some finite frequency region provided that the input power exceeds the threshold as shown in Fig. 2 by solid lines. Then a participation of the odd mode in the light transmission breaks the symmetry of light transmission as shown in Figs. 2(b) and 2(c). Moreover, for the frequency $a\omega/2\pi c = 0.361$, the light output to the right is completely blocked, while the output to the left is fully opened as seen from Fig. 2(b). Figure 2(c) shows that the SB exists only above of critical value of the input amplitude $E_{\rm in}$, which depends on frequency. There is also the equivalent stable SB solution where the left and the right are inverted. That solution is not shown in Fig. 2. Also, we do not show unstable solutions in Figs. 2(b) and 2(c).

Maes *et al.* [4] have demonstrated the switching phenomenon through SB in coupled nonlinear microcavities by adding pulses to the side with lower output power. We employ that approach to demonstrate full light switching of outputs from the left to the right. Applying two ultrashort light impulses of length over a decades of periods of light oscillations enough separated in time, we manage to switch the light outputs as shown from Fig. 2(d).

Figure 3 illustrates SB. The optical streamlines computed by use of the stream function [17] show vortical structure for the SB solution as was found in [13].

Second, let us consider the light outputs as dependent on the phase difference θ of the inputs. For the linear case, we would have obviously found that the transmissions follow the $\sin\theta$ as, indeed, Figs. 4(a) and 4(b) demonstrate by the dashed lines. For the nonlinear case, we reveal a wealth of new solutions with periodical phase behavior crucially different from the linear case as shown in Fig. 4. The first type of solutions shown by the dotted—dashed lines in Fig. 4(b) are located near $\theta = \pm\pi$. The second type of solution shown in Fig. 4(c)

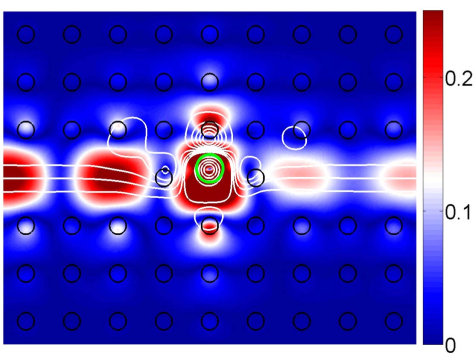

Fig. 3. (Color online) Absolute value of light amplitude (electric field) and optical streamlines (white lines) in the PhC waveguide with single nonlinear defect shown by gray open circle for $\omega a/2\pi c = 0.36$, $P = 10$ W$/a$, $\theta = 0$.

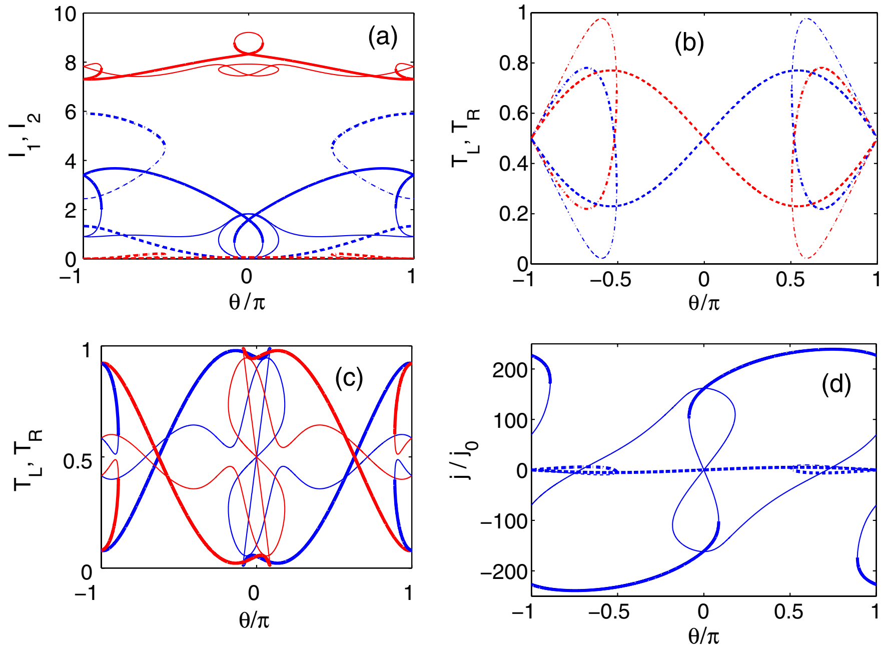

Fig. 4. (Color online) (a) Intensities of dipole modes (red for I_1 and blue for I_2), (b) and (c) transmissions to the left (blue lines) and to the right (red lines), and (d) ratio of maximal value of the Poynting vector power current in the interior of the defect rod to the input Poynting vector for $E_{\text{in}} = 0.064$, $\omega a/2\pi c = 0.361$ as a function of the phase difference of the light inputs. In (a), blue lines show the intensity of even dipole mode, red lines show the intensity of odd dipole mode. Thicker lines mark the stable domains of the solutions.

has the symmetrical points $\theta = 0, \pm\pi$ where the SB solutions substitute each other $T_L \leftrightarrow T_R$. More generally, there is the following symmetry: for $\theta \to -\theta$, $I_1(\theta) = I_2(-\theta)$, $T_L(\theta) = T_R(-\theta)$. These stable solutions are remarkable because they can reach almost unity or zero to pave the path for all-optical switching in some vicinity of symmetrical points. Beyond this vicinity, we lose this possibility. Finally, in Fig. 4 we show the evolution of the maximal value of the Poynting vector power current in the interior of the dipole defect with the phase for all three types of the solution. One can see a drastic difference between the symmetry-preserving and SB solutions.

4. CONCLUSIONS

The eigenstates of two defects might be classified as even and odd (symmetric and antisymmetric) states [6] with respect to inversion of the waveguide axis. Therefore, in view of the symmetries of the resonant states, the system of two defects is similar to the present system with the single dipole defect in the directional waveguide. However, in the former system of two defects aligned along the waveguide, the symmetry is breaking irrespectively to the position of the defects relative to the center line of waveguide [4,5], while in the present system, the symmetry is breaking only for the shifted position of the dipole defect. Dipole modes are located prevailingly in the vicinity of the defect. For the SB solution, both modes are excited with different phases. That results in a giant optical vortex of the Poynting vector of the power current [13] around the defect as seen in Fig. 3. The dynamical behavior of the dipole modes is extremely sensitive to the phases of ingoing waves even for symmetrical injecting conditions. Respectively, we obtained a mass of new solutions shown in Fig. 4 dependent on the phase difference. That opens a wide spectrum of possibilities to manipulate light propagation in the PhC waveguides. It is remarkable that this variety of phase features might be observed in the wave outputs as shown in Figs. 4(b) and 4(c).

ACKNOWLEDGMENTS

This work is partially supported by the Integration Project of the Siberian Branch of the Russian Academy of Sciences (RAS) (project no. 29) and the Russian Fund of Basic Research (RFBR) grant 12-02-000094. We thank D. N. Maksimov for assistance in the manuscript preparation.

REFERENCES

1. K. Otsuka and K. Ikeda, “Hierarchical multistability and cooperative flip-flop operation in a bistable optical system with distributed nonlinear elements,” Opt. Lett. **12**, 539–601 (1987).
2. M. Haelterman and P. Mandel, “Pitchfork bifurcation using a two-beam nonlinear Fabry–Perot interferometer: an analytical study,” Opt. Lett. **15**, 1412–1414 (1990).
3. I. V. Babushkin, Yu. A. Logvin, and N. A. Loiko, “Symmetry breaking in optical dynamics of two bistable thin films,” Quantum Electron. **28**, 104–107 (1998).
4. B. Maes, M. Soljačić, J. D. Joannopoulos, P. Bienstman, R. Baets, S.-P. Gorza, and M. Haelterman, “Switching through symmetry breaking in coupled nonlinear micro-cavities,” Opt. Express **14**, 10678–10683 (2006).
5. B. Maes, P. Bienstman, and R. Baets, “Symmetry breaking with coupled Fano resonances,” Opt. Express **16**, 3069–3076 (2008).
6. E. Bulgakov, K. Pichugin, and A. Sadreev, “Symmetry breaking for transmission in a photonic waveguide coupled with two off-channel nonlinear defects,” Phys. Rev. B **83**, 045109 (2011).
7. E. Bulgakov and A. Sadreev, “Switching through symmetry breaking for transmission in a T-shaped photonic waveguide coupled with two identical nonlinear micro-cavities,” J. Phys. Condens. Matter **23**, 315303 (2011).

8. N. Akhmediev and A. Ankiewicz, "Novel soliton states and bifurcation phenomena in nonlinear fiber couplers," Phys. Rev. Lett. **70**, 2395–2398 (1993).
9. N. Dror and B. A. Malomed, "Spontaneous symmetry breaking in coupled parametrically driven waveguides," Phys. Rev. E **79**, 016605 (2009).
10. V. A. Brazhnyi and B. A. Malomed, "Spontaneous symmetry breaking in Schrödinger lattices with two nonlinear sites," Phys. Rev. A **83**, 053844 (2011).
11. S. G. Johnson, C. Manolatou, S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and H. A. Haus, "Elimination of cross talk in waveguide intersections," Opt. Lett. **23**, 1855–1857 (1998).
12. M. F. Yanik, S. Fan, M. Soljačić, and J. D. Joannopoulos, "All-optical transistor action with bistable switching in a photonic crystal cross-waveguide geometry," Opt. Lett. **28**, 2506–2508 (2003).
13. E. N. Bulgakov and A. F. Sadreev, "Giant optical vortex in photonic crystal waveguide with nonlinear optical cavity," Phys. Rev. B **85**, 165305–165306 (2012).
14. J. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals: Molding the Flow of Light* (Princeton University, 2008).
15. W. Suh, Z. Wang, and S. Fan, "Temporal coupled-mode theory and the presence of non-orthogonal modes in lossless multimode cavities," IEEE J. Quantum Electron. **40**, 1511–1518 (2004).
16. J. Bravo-Abad, S. A. Rodriguez, P. Bermel, S. G. Johnson, J. D. Joannopoulos, and M. Soljaĉić, "Enhanced nonlinear optics in photonic-crystal microcavities," Opt. Express **15**, 16161–16176 (2007).
17. C. F. Chien and R. V. Waterhouse, "Singular points of intensity streamlines in two-dimensional sound fields," J. Acoust. Soc. Am. **101**, 705–712 (1997).

84.237.90.20 {ts 2020-12-07 05:36:58}