ANALYSIS OF THE QUALITY FACTOR OF THE OPTICAL HALF-WAVELENGTH RESONATOR

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ABSTRACT: *An electrodynamic analysis of an optical resonator comprising a half-wavelength dielectric layer that is covered from both sides by quarter-wavelength layers with higher and lower refractive indices (RIs) is carried out. For such a resonator, the dependences of the external quality factor on number and RIs of the layers are obtained first. That allows one to determine minimum number of the layers in the structure to achieve the required quality factor at practically any contrast of the RIs. The results of this study provide an easy way to optimize the structures of 1D photonic crystal devices, particularly high-selectivity bandpass filters.* Microwave Opt Technol Lett 55:1613–1616, 2013; View this article online at wileyonlinelibrary.com. DOI 10.1002/mop.27662

Key words: *multilayers; resonance; coupled resonators; photonic crystals; filters*

1. INTRODUCTION

Multilayered structures composed of alternating dielectric layers with different refractive indices (RIs) which period is comparable to the electromagnetic wavelength of optical bands, so-called 1D photonic crystals (PCs), are used to design different optoelectronic devices: filters, polarizers, and mirrors [1–4]. In essence, the PCs are systems of interacting resonators, so they have periodically repeated transparency windows and rejection bands [5].

In narrowband filters and some other resonant devices, all the adjacent resonators must have weak couplings, whereas the input and output resonators must have high external quality factors [6]. But external Q factors of the input and output resonators in the PC as well as the coupling coefficients of the adjacent resonators depend on the difference of RIs of the contacting mediums—the larger the difference, the smaller the coupling coefficient and the higher the Q factor of the resonator [7, 8]. In view of the fact that RIs of materials in optics are not so high, the external Q factor of a half-wavelength (HWL) dielectric layer being in free space is not high too.

Thus, there are two related difficulties in design of PC devices. The first one is the difficulty of increasing the external Q factor of the input and output resonators, and the second one is the difficulty of weakening the coupling between the adjacent resonators. These difficulties are usually overcome with a no resonant quarter-wavelength (QWL) multilayer that separates the resonant HWL layer from free space or the neighbor resonant HWL layer [1].

In this article, we analyze the external Q factor of the HWL dielectric layer that is covered from both sides by the QWL multilayers and placed in free space.

2. COMPUTATION OF WAVE TRANSMISSION THROUGH THE MULTILAYERED STRUCTURE

To calculate the wave transmission through the multilayered dielectric structures, we use a 1D model where the strengths of the fields depend only on the coordinate z being perpendicular to the layers, and the layers themselves are considered as cascaded transmission line sections. The normalized transfer *ABCD* matrix [9]

$$\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} = \begin{pmatrix} \cos\theta_k & -iZ_k \sin\theta_k \\ -iZ_k^{-1}\sin\theta_k & \cos\theta_k \end{pmatrix} \tag{1}$$

is convenient to characterize electrodynamic behavior of the kth transmission line section. Here, Z_k is normalized impedance, and θ_k is electrical length of the kth section. It is supposed that the oscillations of the field components occur as exp($-\omega t$). In this case,

$$Z_k = 1/n_k, \quad \theta_k = \frac{\omega}{c} n_k T_k \tag{2}$$

where n_k is RI, T_k is the thickness of the kth dielectric layer.

The transfer matrix (1) together with the formula

$$\begin{pmatrix} E_1 \\ Z_0 H_1 \end{pmatrix} = \begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} \begin{pmatrix} E_2 \\ Z_0 H_2 \end{pmatrix} \tag{3}$$

enables to calculate the strengths of the electric and magnetic fields on the first surface of the dielectric layer at the given field strengths on the second surface. Here, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the characteristic impedance of free space.

When the line sections are cascaded their transfer matrices are multiplied in the same order. Therefore, the *ABCD* matrix of the n-layered structure can be calculated by the formula

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \prod_{k=1}^{n} \begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix}. \tag{4}$$

Power transmission through multilayered structure we characterize by $|S_{21}|^2$ measured in dB where S_{21} is the matrix element of the scattering matrix. This element is related to the *ABCD* matrix by the formula [9]

$$S_{21} = \frac{2\sqrt{Z_2 Z_1}}{AZ_2 + B + CZ_2 Z_1 + DZ_1} \tag{5}$$

where Z_1 and Z_2 are the characteristic impedances of mediums before and after the structure.

Having the transmission spectrum and neglecting dielectric loss, one can estimate the external Q factor of the resonance by the formula

$$Q = f_1/\Delta f, \tag{6}$$

where Δf is the 3-dB bandwidth of the passband and f_1 is the resonant frequency.

3. DERIVATION OF FORMULAS FOR THE EXTERNAL Q FACTOR

Now, we shall derive the formulas for the external Q factor of the HWL dielectric layer placed in the middle of the layered structure composed of the QWL dielectric layers. We suppose that the outer layers have the RI n_H and the RIs of all inner layers take by turns values n_L and n_H where $n_\mathrm{L} < n_\mathrm{H}$.

The external Q factor of any oscillating system in the absence of the internal loss is related to the complex frequency of its free oscillation by formula [10]

$$Q_e = -\mathrm{Re}\omega/(2\mathrm{Im}\omega). \tag{7}$$

As the electrical thickness of the QWL dielectric layers θ is proportional to the frequency ω, one can rewrite the formula (7) in the following way

$$Q_e = -\mathrm{Re}\theta/(2\mathrm{Im}\theta). \tag{8}$$

At the resonant frequency, we have $\mathrm{Re}\theta = \pi/2$.

To simplify the calculation, we take into account the symmetry plane of the layered structure. We put zero of the coordinate z into the center of the resonant HWL layer. Then, it is enough to take into account the strength distributions of the field only for $z > 0$. The HWL layer is equivalent to the transmission line section short-circuited at both ends when its RI (n_R) is much more less than RI of the surrounding layers (n_H) and is equivalent to the open-circuited transmission line section in opposite case ($n_R >> n_L$). Nevertheless, the electric field strength $E(z)$ in the HWL layer has a cosine distribution law if $n_R = n_L$ and a sinusoidal distribution if $n_R = n_H$ at any contrast degree of the RIs.

Thus, the distribution functions for the three-layered structure ($n_R = n_L$) have the form

$$E_x(z) = \begin{cases} A_1 \cos(k_1 z) & \text{for } 0 < z < z_1, \\ A_2 e^{ik_2(z-z_1)} + A_3 e^{-ik_2(z-z_1)} & \text{for } z_1 < z < z_2, \\ A_4 e^{ik_0(z-z_2)} & \text{for } z_2 < z < \infty, \end{cases}$$

$$Z_0 H_y(z) = \begin{cases} in_L A_1 \sin(k_1 z) & \text{for } 0 < z < z_1, \\ n_H A_2 e^{ik_2(z-z_1)} - n_H A_3 e^{-ik_2(z-z_1)} & \text{for } z_1 < z < z_2, \\ A_4 e^{ik_0(z-z_2)} & \text{for } z_2 < z < \infty \end{cases} \tag{9}$$

where z_1 and z_2 are the layer boundary coordinates, A_m ($m = 1$, 2, 3, and 4) are unknown wave amplitudes, and $k_1 = \omega n_L/c$, $k_2 = \omega n_H/c$, $k_0 = \omega/c$.

In accordance with the electrodynamic boundary conditions, the functions (9) must be continuous at z_1 and z_2. Therefore, we get the system of equations

$$\begin{aligned} \cos\theta A_1 - A_2 - A_3 &= 0, \\ in_L \sin\theta A_1 - n_H A_2 + n_H A_3 &= 0, \\ e^{i\theta} A_2 + e^{-i\theta} A_3 - A_4 &= 0, \\ n_H e^{i\theta} A_2 - n_H e^{-i\theta} A_3 - A_4 &= 0. \end{aligned} \tag{10}$$

Homogeneous system has nontrivial solution when its determinant is equal to zero. Hence, we get the equation for obtaining the frequency of free oscillation

$$n_L \tan^2\theta + in_H(n_H + n_L)\tan\theta - n_H = 0. \tag{11}$$

It has the solution

$$\theta = \frac{\pi}{2} - i\,\mathrm{arcoth}\left(\frac{n_H + n_L}{2n_L} n_H + \sqrt{\left(\frac{n_H + n_L}{2n_L} n_H\right)^2 - \frac{n_H}{n_L}}\right). \tag{12}$$

After substitution of (12) into (8), we obtain the external Q factor of the HWL resonator in three-layered structure

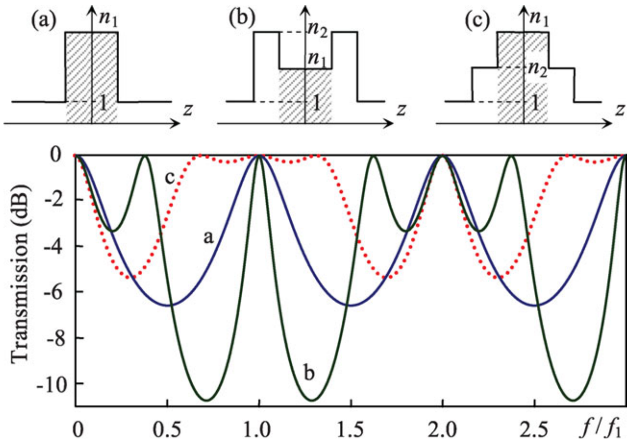

Figure 1 Transmission spectrum through the HWL layer when (a) it is free, (b) it is covered by the QWL layers with high RI, and (c) it is covered by the QWL layers with low RI (Insets show the profiles of RIs. The HWL layer is shaded). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

$$Q_{e3} = \frac{\pi}{4\mathrm{arcoth}\left(\frac{n_H + n_L}{2n_L} n_H + \sqrt{\left(\frac{n_H + n_L}{2n_L} n_H\right)^2 - \frac{n_H}{n_L}}\right)} \tag{13}$$

Adding one more QWL layer on each side of the HWL resonator results in substantial increase of the Q factor. In this case, the RI of the HWL layer must be greater than the index of adjacent QWL layer.

The quality factor of the HWL resonator in the five-layered structure can be calculated by the formula

$$Q_{e5} = \frac{\pi}{4\,\mathrm{arcoth}\,x} \tag{14}$$

where x is the real root of the cubic equation

$$\begin{aligned} n_L^2 x^3 - n_H\left(n_H^2 + n_L n_H + n_L^2\right)x^2 \\ + n_H(n_L + n_H + n_L)x - n_H^2 n_L = 0. \end{aligned} \tag{15}$$

We have obtained these formulas the same way as formula (13).

4. COMPUTATIONAL RESULTS

Figure 1(a) shows the calculated frequency dependence of the wave transmission through a single HWL layer with the RI $n_1 = 4$. As shown elsewhere [11], the mentioned value of n_1 is close to maximum for materials in optical band. Such dielectric layer being in air ($n = 1$) has relatively low external quality factor $Q \approx 2.8$ at frequency f_1 of the first resonance even for such high difference between the RIs of the mediums. We note that Q factors of the high-order resonances grow proportionally to the mode number.

For comparison, Figures 1(b) and 1(c) show the frequency dependences of the wave transmission through the three-layered structures with different RIs: (b) when the HWL layer with $n_1 = 2$ is surrounded by the QWL layers with $n_2 = 4$ and (c) when the HWL layer with $n_1 = 4$ is surrounded by the ordinary antireflective QWL layers with $n_2 = 2$. One can see that in case (b) the Q factor increases nearly three times up to $Q \approx 8.3$.

Figure 2 shows the Q factor dependences of the HWL resonator in the three-layered structure on the RI of the QWL layers.

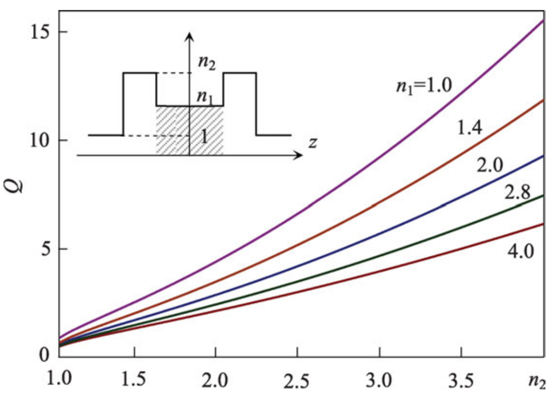

Figure 2 The Q factor dependences of the three-layered resonator on the RI of QWL layers for some values of the HWL layer RIs. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

The curves were built using (13) for some values of the RI of the HWL layer. One can see that the Q factor rapidly increases with increase of n_2 and also increases with decrease of n_1. In comparison with single-layer HWL resonator, the Q factor of the three-layered structure increases more than five times.

Figure 3 shows the Q factor dependences of the HWL resonator in the five-layered structure on the RI of the internal QWL layers. The curves were built using (14) for some values of the RI of the HWL resonant layer. One can see that the Q factor in this case, contrariwise, rapidly decrease when the index n_2 increases or the index n_1 decreases. Compared to single-layer HWL resonator with $n_1 = 4$, the Q factor in the five-layered structure increases more than 20 times.

The presented approach [formulas (1)–(6)] allows one to compute the Q factor of the HWL layer at any number N of the surrounding QWL layers with alternating high and low optical densities. Such dependencies for some ratios of the RIs are shown in Figure 4. These dependencies demonstrate practically exponential increase in the Q factor with increase in number of the surrounding QWL layers.

5. CONCLUSION

Thus, we carried out an electrodynamic analysis of the optical resonator representing a HWL dielectric layer with low or high RI that is covered on either side with the QWL layers with alternating high and low RIs. We first obtained the formulas for calculating the Q factor of such resonator in free space. We investigated the Q factor dependences on both number of the dielectric layers in the structure and their optical properties. The proposed approach enables one to determine necessary number of the QWL dielectric layers covering the HWL dielectric resonator to get the required Q factor. Results of this research enable to optimize the structures of 1D PC devices, particularly the bandpass filters having high frequency selectivity and low loss.

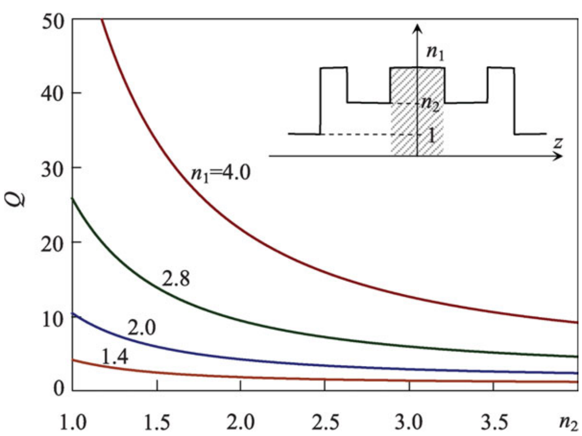

Figure 3 The Q factor dependences of the five-layered resonator on the RI of QWL layers for some values of the HWL layer RIs. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

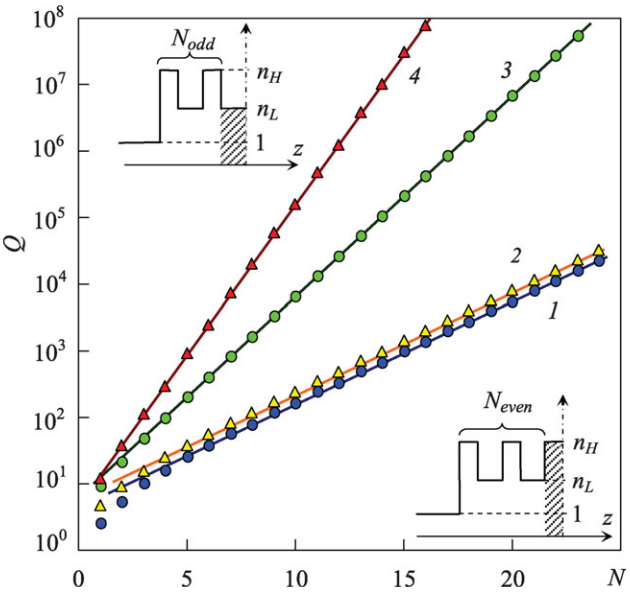

Figure 4 The Q factor dependences of the HWL resonator on the number of the QWL layers on each side: 1 − $n_H = 4$, $n_L = 2$; 2 − $n_H = 8$, $n_L = 4$; 3 − $n_H = 16$, $n_L = 4$; and 4 − $n_H = 16$, $n_L = 2$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

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RADIO-FREQUENCY MICROFLUIDIC INTERFEROMETER IN PRINTED CIRCUIT BOARD PROCESS

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ABSTRACT: *We report a process to integrate microfluidic channels on standard radio-frequency (RF) printed circuit board (PCB) substrate. A 10.56-GHz RF microstrip interferometer was fabricated using the proposed technique and used to compare electrical responses for liquids with various dielectric properties and thereby demonstrate the capability of RF PCB-integrated microfluidic devices for biological sensing. Microchannels were milled into a substrate and prepreg outlines for microchannels were cut using a laser system. Substrates were adhered and bonded by clamping and baking. The RF interferometer responded to changes in liquid permittivity with high sensitivity relative to many other types of dielectric sensors.* © 2013 Wiley Periodicals, Inc. Microwave Opt Technol Lett 55:1616–1618, 2013; View this article online at wileyonlinelibrary.com. DOI 10.1002/mop.2766

Key words: *microfluidic waveguide; microstrip interferometer; printed circuit board; radio frequency; liquid permittivity*

1. INTRODUCTION

Electrical interrogation in microfluidics has been explored extensively because of the advantages of using integrated, compact microelectronics to perform liquid sensing and control. The traditional fabrication route is to create an electrical network on one substrate and then use an elastomer or other thermoplastic to form fluid channels that are then adhered to the electrical network [1, 2]. Alternatively, liquid channels and the electrical network are formed by lithography, etching, and deposition processes performed in a cleanroom [3, 4]. These processes generally require materials and adhesives that are not found in traditional printed circuit board (PCB) fabrication processes. This is particularly true for radio-frequency (RF) circuits that require substrates and fabrication techniques with carefully controlled properties. It would be advantageous to use tools and techniques that are already found in industry to fabricate microfluidics that can be incorporated simply with standard multilaminate RF PCB processes for reduction in fabrication complexity, time, and cost. This would allow microfluidic devices to be fabricated by researchers without cleanrooms and simplify the integration of commercial devices with RF interrogation electronics.

The RF broadband dielectric properties of liquids can indicate the physical characteristics and state of liquids as well as biological processes occurring in them [5]. Thus, measuring liquid dielectric properties and dielectric changes are important in both fundamental and applied studies [6]. Broadband RF measurements are often used to show dielectric relaxation characteristics of cells and proteins in solutions [7]. Broadband measurement techniques can vary widely in construction and measurement and use solution volumes from milliliters to nanoliters [8]. At a single frequency, interferometers can be used for exquisitely sensitive measurements. Dual-band technology has been utilized for increased stability and robustness [9]. Recently, a microfabricated RF interferometer for liquids demonstrated increased sensitivity to methanol–water mixture changes [10].

A modest amount of work on PCB integrated microfluidic devices has already been reported. Microfluidic integration in FR4 substrates (traditional substrate for most low-frequency electronics) has been developed [11]. A wide range of micromechanical devices such as RF switches have been demonstrated by direct fabrication in PCBs [12]. In this article, we demonstrate a process to form microfluidic channels in a standard RF board fabrication process to create a liquid RF interferometer.

2. INTERFEROMETER FABRICATION

The RF interferometer was fabricated using traditional PCB processes as shown in Figure 1. The substrates were RO4003C board (Rogers) that has copper layers on both sides. The dimensions of the board were cut to 1×2 in.^{2}. The top board that contains the RF waveguides was patterned using a FP-21T PCB Prototyping Machine (MITS Electronics). The copper was peeled from the underside of the board, removing the original ground plane. Access holes in the top substrate were drilled.

The copper layer was peeled from a second board of equal dimensions and then milled at a depth of 10 mil to form microchannels. RO4450F prepreg (Rogers) was then laser cut to the same size as the top and bottom boards and to remove areas corresponding to the microchannels using a Rayjet 50 C30 laser engraver (Trotec).

The two boards were then aligned with the microchannels on the inside and sandwiched together with the prepreg layer in between. The boards were clamped together between aluminum blocks and then baked at 210°C for an hour in an oven (J. C. Penney, Model 4780/1) [13]. The prepreg sealed the boards and the microchannels. A photograph of the completed device is shown in Figure 2. Flexible plastic tubing was attached with Loctite® Epoxy Marine (Henkel) for liquid introduction into the microchannels.

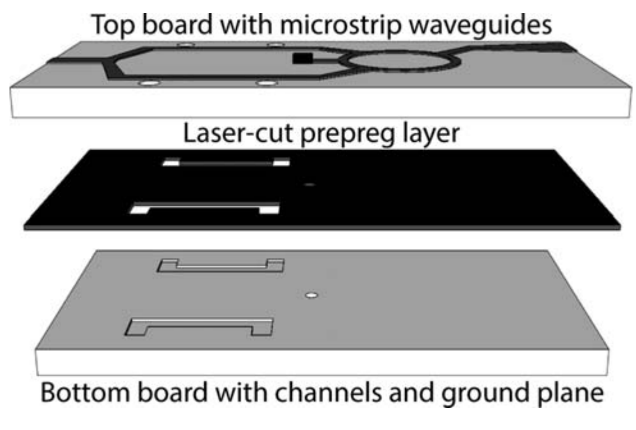

Figure 1 Fabrication stack for integrated microchannels and RF waveguides (not to scale). The top layer contains the microstrip waveguides and resistor. The middle layer is the laser-cut prepreg adhesive layer. The bottom layer contains the milled channels and bottom ground plane