

GENERALIZED REFRACTIVE MIXING DIELECTRIC MODEL OF MOIST SOILS CONSIDERING IONIC RELAXATION OF SOIL WATER

V. L. Mironov,^{1,3} P. P. Bobrov,² S. V. Fomin,¹ and A. Yu. Karavaiskii^{1,3}

UDC 621.371.3/205.2

A generalized multi-relaxation refractive mixing dielectric model of moist soils is suggested for frequencies in the range from 0.2 to 14.8 GHz. The model is based on dielectric measurements in this frequency range at a temperature of 20°C for relative moisture content (by weight) changing from 2 to 43%. The model expands the range of applicability of the single-relaxation model suggested previously to the megahertz frequency range with allowance for the Maxwell–Wagner ionic relaxation which is clearly manifested in this frequency range for bound soil water. It is demonstrated that the error in calculating the complex dielectric permittivity spectra of the moist soil has the same order of magnitude as the error of experimental data used for model construction.

Keywords: dielectric permittivity, moist soils, spectroscopic parameters.

INTRODUCTION

Nowadays the generalized refractive mixing dielectric model (GRMDM) of frequency spectra for moist soils suggested in [1] is an efficient tool for calculation of dielectric spectra of moist soils in the microwave range. This model considers only the dipole relaxation of water molecules in gigahertz frequency range and can be defined as the single-relaxation GRMDM. Errors in calculating the complex dielectric permittivity (CDP) for models developed on the basis of the single-relaxation GRMDM [2–5] appear much less than those of the widely used semi-empirical dielectric model suggested in [6]. Meanwhile, errors of the dielectric mixing models [2–5] considerably increase as the frequency decreases below 1.0 GHz. This is due to the fact that the single-relaxation GRMDM does not take into account considerable increase in the real and imaginary CDP components of moist clay soils observed experimentally in [7–11] with frequency decrease when going to the megahertz frequency range. In [11] it was demonstrated that this increase could be caused by the ionic Maxwell–Wagner relaxation [12] of the bound soil water.

In the present work, a multi-relaxation GRMDM with allowance for the dipole and ionic relaxations of soil water molecules at frequencies in the range from 0.2 to 14.8 GHz is developed. Procedures of evaluation of the parameters of dielectric mixture spectra for the multi-relaxation GRMDM are developed on an example of the black clay soil comprising ~50–60% quartz, ~15–20% plagioclase, ~10% dioctahedral mica, ~10% potassium feldspar, ~5–7% chloride, ~2–3% smectite, and ~2% organic components. The error of multi-relaxation GRMDM is estimated by the Pearson coefficient and standard deviation between the measured and calculated real and imaginary components of the complex dielectric permittivity for the model.

¹ L. V. Kirenskii Institute of Physics of the Siberian Branch of the Russian Academy of Sciences, Krasnoyarsk, Russia; ²Omsk State Pedagogical University, Omsk, Russia; ³Academician M. F. Reshetnev Siberian State Aerospace University, Krasnoyarsk, Russia, e-mail: rsd@ksc.krasn.ru. Translated from *Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika*, No. 3, pp. 75–79, March, 2013. Original article submitted June 15, 2012.

CONCEPT OF THE MULTI-RELAXATION GRMDM

Let us express the real, n_s , and imaginary, κ_s , components of the complex refractive index (CRI) of the moist soil as a function of moisture M by weight normalized by the dry sample weight for the refractive mixing dielectric model [1, 3, 5]:

$$\frac{n_s(M, f) - 1}{\rho_d} = \begin{cases} \frac{n_m - 1}{\rho_m} + (n_b(f) - 1)M, & 0 \leq M \leq M_{t1}, \\ \frac{n_s(M_{t1}, f) - 1}{\rho_d} + (n_t(f) - 1)(M - M_{t1}), & M_{t1} \leq M \leq M_{t2}, \\ \frac{n_s(M_{t2}, f) - 1}{\rho_d} + (n_u(f) - 1)(M - M_{t2}), & M \geq M_{t2}, \end{cases} \quad (1)$$

$$\frac{\kappa_s(M, f) - 1}{\rho_d} = \begin{cases} \frac{\kappa_m}{\rho_m} + \kappa_b(f)M, & 0 \leq M \leq M_{t1}, \\ \frac{\kappa_s(M_{t1}, f)}{\rho_d} + \kappa_t(f)(M - M_{t1}), & M_{t1} \leq M \leq M_{t2}, \\ \frac{\kappa_s(M_{t2}, f)}{\rho_d} + \kappa_u(f)(M - M_{t2}), & M \geq M_{t2}, \end{cases} \quad (2)$$

where n_s , n_m , n_b , n_t , and n_u and κ_s , κ_m , κ_b , κ_t , and κ_u are the real and imaginary CRI components, f designates the electromagnetic field frequency, and ρ_d is the dry sample density. Here the subscripts s , m , b , t , and u in Eqs. (1) and (2) and in the subsequent expressions below designate the moist soil, organo-mineral soil component, and bound (adsorbed), tightly bound (film), and unbound (capillary) soil waters, respectively. In turn, M_{t1} and M_{t2} designate the maximum bound water fraction and the maximum total bound and tightly bound water fraction in a concrete soil. The real, n_p , and imaginary, κ_p , CRI components are expressed through the real, ε_p' , and imaginary, ε_p'' , CDP components:

$$n_p \sqrt{2} = \sqrt{\sqrt{(\varepsilon_p')^2 + (\varepsilon_p'')^2} + \varepsilon_p'}, \quad \kappa_p \sqrt{2} = \sqrt{\sqrt{(\varepsilon_p')^2 + (\varepsilon_p'')^2} - \varepsilon_p'}, \quad (3)$$

where the subscript p takes values $p = s, b, t, \text{ and } u$ for the moist soil and bound, tightly bound, and unbound waters, respectively.

Let us express the real and imaginary CDP components of the soil water in Eqs. (3) through the equation for the double Debye relaxation [13] of non-conductive liquids which considers only bias currents:

$$\varepsilon_p' = \frac{\varepsilon_{0pL} - \varepsilon_{0pH}}{1 + (2\pi f \tau_{pL})^2} + \frac{\varepsilon_{0pH} - \varepsilon_{\infty pH}}{1 + (2\pi f \tau_{pH})^2} + \varepsilon_{\infty pH}, \quad \varepsilon_p'' = \frac{\varepsilon_{0pL} - \varepsilon_{0pH}}{1 + (2\pi f \tau_{pL})^2} 2\pi f \tau_{pL} + \frac{\varepsilon_{0pH} - \varepsilon_{\infty pH}}{1 + (2\pi f \tau_{pH})^2} 2\pi f \tau_{pH}. \quad (4)$$

Here ε_{0pL} and ε_{0pH} or ε_{0pH} and $\varepsilon_{\infty pH}$ are the low- or high-frequency maximum of the real CDP component and τ_{pL} or τ_{pH} are the relaxation time for the ionic or dipole relaxation, respectively; all these spectroscopic parameters should be taken for the bound ($p = b$), tightly bound ($p = t$), and unbound ($p = u$) soil waters; and $\varepsilon_r = 8.854 \cdot 10^{-12}$ F/m is the dielectric permittivity of vacuum. To determine the CDP of the unbound soil water, we take advantage of the single Debye relaxation equation that follows from Eq. (5) for $\varepsilon_{0uL} = \varepsilon_{0uH}$.

Since formulas (4) consider only the bias currents, the CRI values of the moist soil calculated from Eqs. (1)–(4) will differ from their values measured for moist soil samples possessing the direct current conductivity. However, formulas (1)–(4) can be used to calculate the real CDP component of the moist soil sample

$$\varepsilon'_s = n_s^2 - \kappa_s^2, \quad (5)$$

because this quantity is determined only by the bias currents. At the same time, the imaginary CDP component of the moist soil can be represented as the sum of terms caused first, by bias currents $2n_s\kappa_s$ and second, by direct current running through the moist sample $\sigma_s/2\pi f\varepsilon_r$, where σ_s is the specific conductivity of the moist sample and ε_r is the dielectric permittivity of the free space. We now represent the specific electrical conductivity of the moist sample as the sum of the specific conductivities of all water types presented in the soil for the given sample moisture taking into account their relative fractions $\sigma_s = W_b\sigma_b + W_t\sigma_t + W_u\sigma_u$, where $W_p = M_p\rho_d$ are fractions of water of the above-indicated types in the soil ($p = b, t, \text{ and } u$). As a result, the expression for the imaginary CDP component of the moist sample is written in the following form:

$$\varepsilon_s'' = \begin{cases} 2n_s\kappa_s + \rho_d(M)M\sigma_b(M) / 2\pi f\varepsilon_r, & 0 \leq M \leq M_{t1}, \\ 2n_s\kappa_s + \rho_d(M)[M_{t1}\sigma_b(M_{t1}) + (M - M_{t1})\sigma_t(M)] / 2\pi f\varepsilon_r, & M_{t1} \leq M \leq M_{t2}, \\ 2n_s\kappa_s + \rho_d(M)[M_{t1}\sigma_b(M_{t1}) + (M_{t2} - M_{t1})\sigma_t(M_{t2}) + (M - M_{t2})\sigma_u(M)] / 2\pi f\varepsilon_r, & M \geq M_{t2}. \end{cases} \quad (6)$$

In formula (6), the specific electrical conductivities of the bound, tightly bound, and unbound soil water depend generally on the sample moisture, because the soil solution can be diluted when water is added to the sample. However, as demonstrated below, these dependences are weak for the examined soil, and the conductivity of water of each type can be considered constant in the entire range of concentrations of the corresponding soil water type.

From Eqs. (1)–(6) it can be seen that for the suggested GRMDM, the CDP spectrum of a concrete soil with the moisture M and electromagnetic field frequency f can be calculated using the following set of the parameters:

- 1) real and imaginary parts of the CRI for the organo-mineral component: $(n_m - 1)/\rho_m$ and κ_m/ρ_m ,
- 2) dry soil density ρ_d ,
- 2) maximum bound (M_{t1}) and tightly bound ($M_{t2} - M_{t1}$) waters in the soil of the given type,
- 3) maximum low- (ε_{0pL} and ε_{0pH}) and high-frequency ($\varepsilon_{\infty pH}$) real CDP components for the ionic and dipole relaxations of the bound ($p = b$), tightly bound ($p = t$), and unbound ($p = u$) soil waters,
- 4) times of ionic (τ_{pL}) and dipole (τ_{pH}) relaxations of the bound ($p = b$), tightly bound ($p = t$), and unbound ($p = u$) soil waters,
- 5) specific electrical conductivity of the bound σ_b , tightly bound σ_t , and unbound σ_u soil waters.

By analogy with [14], all these parameters can be calculated for a concrete soil from conventional dielectric measurements of the moist soil for the single relaxation GRMDM. The procedure of determining the parameters of the spectroscopic model is based on the multidimensional regression analysis of the measured CDP spectra of the moist soil using Eqs. (1)–(6) as a theoretical regression model. Results of the regression analysis are presented below.

CALCULATION OF THE PARAMETERS OF THE MULTI-RELAXATION GRMDM

To determine the parameters of multi-relaxation models, laboratory measurements of the CDP spectra were performed at a temperature of 20°C for the moist black clay soil whose mineral composition is presented above. The setup and the measurement procedure were described in [5]. Measurements were performed at frequencies from 200 MHz to 14.8 GHz. The weight moisture of samples ranged from 0 to 43%. On the whole, the data were obtained for 13 moist samples. In addition, the dry sample density $\rho_d(M)$ was determined for each moisture. From the measured CRI dependences on the moisture, Eqs. (1) and (2), and regression analysis technique suggested in [3] we first determined the normalized real ($(n_m - 1)/\rho_m = 0.45$) and imaginary ($\kappa_m/\rho_m = 0.001$) CRI components of the organo-mineral soil constituent, maximum bound water fraction ($M_{t1} = 0.176$), and total maximum bound and tightly bound water fraction ($M_{t2} = 0.31$).

Then the measured spectra shown by symbols in Fig. 1 that corresponded to the presence in the sample of only bound water (curves 1 and 2), bound and tightly bound waters (curves 3, 4, and 5), and bound, tightly bound, and unbound waters (curve 6) were used. Applying the regression analysis technique suggested in [14] and formulas (1)–

TABLE 1. Spectroscopic Parameters and Errors of the Multi-relaxation Dielectric Model

Physical characteristics		Spectroscopic parameters						Error estimates	
M	ρ_d	Bound water		Tightly bound water		Unbound water		Linear regression ε'	$\varepsilon'_m = 0.009 + 0.969 \cdot \varepsilon'_p$
0.021	1.413	ε_{0bL}	750	ε_{0tL}	150	–	–	ρ	0.998
0.168	1.123	ε_{0bH}	40	ε_{0tH}	140	ε_{0uH}	100	σ	0.454
0.190	1.389	τ_{bL} (ns)	5.5	τ_{tL} (ns)	0.06	–	–	Linear regression ε''	$\varepsilon''_m = -0.125 + 0.994 \cdot \varepsilon''_p$
0.248	1.499	τ_{bH} (ps)	12.5	τ_{tH} (ps)	8	τ_{uH} (ps)	10.6	ρ	0.995
0.292	1.555	σ_b (S/m)	0.001	σ_t (S/m)	0.18	σ_u (S/m)	0.4	σ	0.361
0.335	1.408								

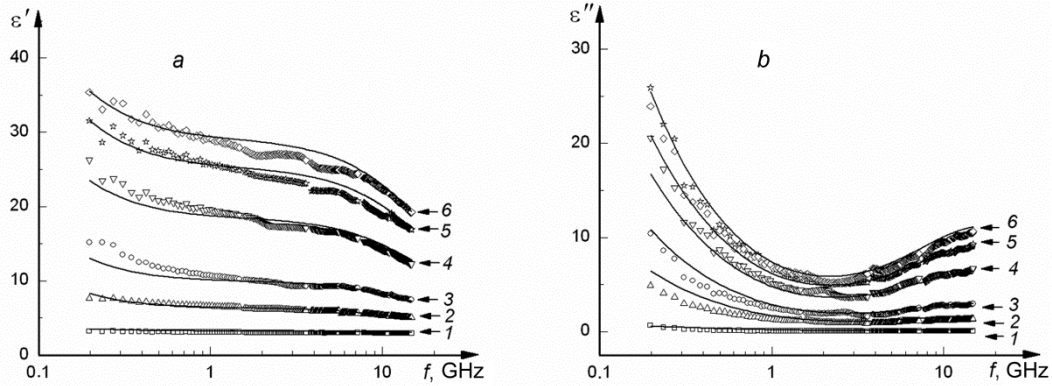


Fig. 1. Experimental spectra of the real (ε') and imaginary (ε'') CDP components (symbols) of the moist soil and results of their calculations from formulas (1)–(6) and tabulated data (solid curves). The weight moisture M (g/g) was 0.021 (curve 1), 0.168 (curve 2), 0.190 (curve 3), 0.248 (curve 4), 0.292 (curve 5), and 0.335 (curve 6).

(6), we calculated the spectroscopic parameters of the bound, tightly bound, and unbound soil waters from the measured dielectric spectra shown in Fig. 1. Values of the parameters of the multi-relaxation models calculated by this method are given in Table 1 from which it follows that the difference $\varepsilon_{0tL} - \varepsilon_{0tH}$ is nonzero. This indicates the presence of the ionic relaxation in the tightly bound water. We previously observed the presence of this relaxation only in the bound water [11]. It should also be noted that the table gives the average values of the conductivities of bound, tightly bound, and unbound waters. In addition to the tabulated data, the well-known value $\varepsilon_{\infty pH} = 4.9$ [1] was used. The dielectric spectra calculated for the determined parameters of the multi-relaxation GRMDM and Eqs. (1)–(6) are illustrated by solid curves in Fig. 1. A comparison of the calculated spectra with the measured ones demonstrates their good quantitative agreement. To determine the error in calculating the spectra for the multi-relaxation GRMDM, we performed correlation analysis whose results are presented below.

ERROR OF THE MULTI-RELAXATION GRMDM

Figure 2 shows the dependences of the measured CDP on its calculated value. If the calculated value did not deviate from the measured ones, the dependences shown in Fig. 2 by symbols would coincide with the bisectors shown by the dashed straight lines. We now estimate the statistical error of CDP calculation using the Pearson coefficient ρ and the standard deviation σ corresponding to the linear regression for the data shown in Fig. 2. The linear regression is shown in Fig. 2 by the solid straight lines. Values of the Pearson coefficient and standard deviations σ are given in

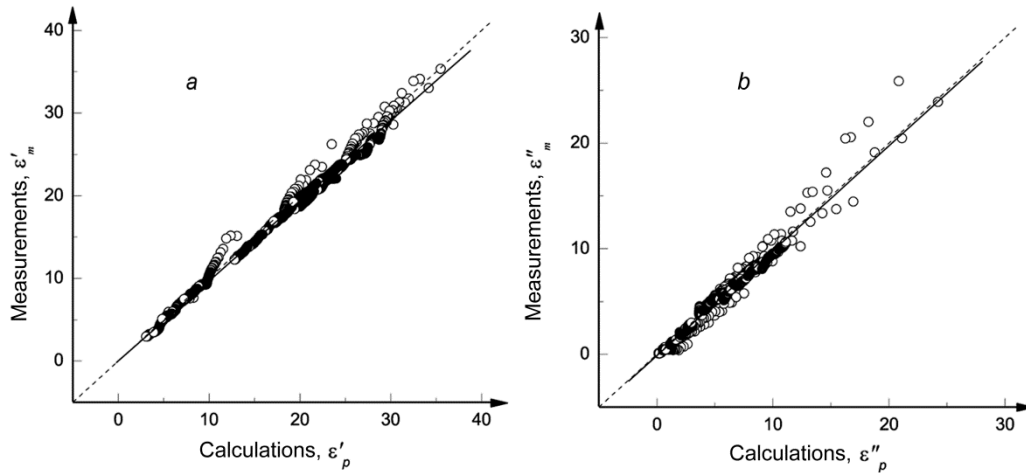


Fig. 2. Correlation of the calculated (ε'_p and ε''_p) and measured (ε'_m and ε''_m) real and imaginary CDP components (shown by symbols). The dashed straight line is the bisector. The solid curve shows the linear regression.

Table 1. Here the equations of linear regression which allow the systematic error (deviation of the regression straight line from the bisector) to be estimated are also presented. The errors of the multi-relaxation model presented in Table 1 in the form of standard deviations equal to 0.454 and 0.361 for the real and imaginary CDP components are comparable with the errors of dielectric measurements [15].

CONCLUSIONS

In the given work, the procedure of constructing the multi-relaxation GRMDM has been developed for calculation of the CDP spectra at frequencies in the range from 0.2 to 14.5 GHz for the moist soil of a concrete type at a preset temperature. The developed procedure is based on the data of dielectric measurements of the moist soil. In the suggested dielectric model, double relaxation spectra of the complex dielectric permittivity of the bound and tightly bound waters in soil were used. Errors of CDP calculation for the multi-relaxation GRMDM were comparable with errors of dielectric soil measurements. In this work, it was demonstrated for the first time that the tightly bound water in the soil possesses the ionic relaxation. The multi-relaxation GRMDM will further be used to construct dielectric models considering also dependences on the soil particle size distribution and temperature by analogy with [2–5]. Such dielectric models are necessary for radar and radiometric remote sensing of the dry land surface in gigahertz and megahertz frequency ranges.

REFERENCES

1. V. L. Mironov, M. C. Dobson, V. H. Kaupp, *et al.*, IEEE Trans. Geosci. Remote Sens., **42**, No. 4, 773–785 (2004).
2. V. L. Mironov, L. G. Kosolapova, and S. V. Fomin, IEEE Trans. Geosci. Remote Sens., **47**, No. 7, 2059–2070 (2009).
3. V. L. Mironov, R. D. De Roo, and I. V. Savin, IEEE Trans. Geosci. Remote Sens., **48**, No. 6, 2544–2556 (2010).
4. V. L. Mironov and S. V. Fomin, Progr. in Electromagn. Res. Online, **5**, No. 5, 411–415 (2009).
5. V. L. Mironov and Yu. I. Lukin, Russ. Phys. J., **53**, No. 9, 956–963 (2011).

6. M. C. Dobson, F. T. Ulaby, M. T. Hallikainen, and M. A. El-Rayes, *IEEE Trans. Geosci. Remote Sens.*, **23**, No. 1, 35–46 (1985).
7. J. O. Curtis, C. A. Weiss, Jr., and J. B. Everett, Technical Report EL-95-34, U.S. Army Corps Eng. Waterways Exp. Station, Vicksburg MS (Dec. 1995).
8. T. J. Kelleners, D. A. Robinson, P. J. Shouse, *et al.*, *Soil Sci. Soc. Am. J.*, **69**, No. 1, 67–76 (2005).
9. Yu. I. Lukin and S. A. Komarov, in: *Proc. Int. Geosci. and Remote Sensing Symp.* (2007), pp. 735–737.
10. N. Wagner, K. Emmerich, F. Bonitz, and K. Kupfer, *IEEE Trans. Geosci. Remote Sens.*, **49**, No. 7, 2518–2530 (2011).
11. P. P. Bobrov, V. L. Mironov, O. V. Kondrat'eva, and A. V. Repin, in: *Proc. XIIth Int. Conf. "Physics of Dielectrics"* (Dielectrics-2011), Vol. 1, Sankt Petersburg (2011), pp. 207–209.
12. F. Kremer, A. Schonhals, and W. Luck, *Broadband Dielectric Spectroscopy*, Springer Verlag (2002).
13. Ya. Yu. Akhadov, *Dielectric Property of Pure Liquids* [in Russian], Publishing House of Standards, Moscow (1972).
14. V. L. Mironov, P. P. Bobrov, L. G. Kosolapova, *et al.*, in: *Proc. Int. Geosci. and Remote Sens. Symp. IGARSS'06*, Vol. 6 (2006), pp. 2957–2961.
15. V. L. Mironov, S. A. Komarov, Yu. I. Lukin, and D. S. Shatov, *Radiotekh. Elektron.*, **55**, No. 12, 1465–1470 (2010).