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INVESTIGATION OF ONE-DIMENSIONAL PHOTONIC CRYSTAL STRUCTURES WITH TWO SUBLATTICES IN MICROWAVES

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The behavior of eigenfrequencies of an irregular microstrip resonator, being an integral part of the one-dimensional photonic crystal (PC) with two sublattices, is investigated as a function of its design parameters. The possibility of a significant increase of the second band gap of the PC with two sublattices is demonstrated that essentially improves the frequency selection properties of the bandpass filters designed on its basis. Good agreement between the calculated results and measurements with the prototype of the original filter built around the microstrip photonic crystal of new design comprising 19 layers is demonstrated.

Keywords: photonic crystal, resonator, dielectric permittivity, pass band.

It is well known that propagation of electromagnetic waves through waveguide structures comprising alternating sections of regular lines with different impedance values, but identical electric lengths is accompanied by the formation of transparency windows (equidistant frequency bands where the attenuation of the transmitted power is minimal) in the frequency response (FR) subdivided by regions of almost complete incident wave reflection (stop bands) [1, 2]. It is well known also that such irregular structures are widely used to develop various microwave devices. In particular, high-Q resonators, phase shifters, and various filters are built on their basis [3–5]. These structures are called photonic or electromagnetic crystals [6].

In the present work, the eigenfrequencies of structural cells of the one-dimensional photonic crystal (PC) comprising two sublattices [3, 5] are investigated together with the properties of such crystal and the possibility of designing on its basis of stop band and bandpass filters with improved characteristics. In this case, the microstrip design of PCs operating in the microwave range is considered. The design principle of the one-dimensional microstrip photonic crystals which are good analogs of optical multilayered structures made from dielectric materials with different refractive indices is based on a strong dependence of the wave impedance Z and effective dielectric permittivity ε_{eff} of the microstrip transmission line on the width w of the strip conductor and thickness h of the substrate. The propagation velocity and hence the electromagnetic wavelength of the line is determined by the ε_{eff} value that can be calculated from the known relative dielectric permittivity ε of the substrate and the main design parameters of the line itself [7]:

$$\varepsilon_{\text{eff}} = \frac{\varepsilon + 1}{2} + \frac{\varepsilon - 1}{2} P . \quad (1)$$

Here $P = (1 + 12h/w)^{-0.5}$ for $w \geq h$ and $P = (1 + 12h/w)^{-0.5} + 0.04\,(1 - h/w)^2$ for $w \leq h$. We note that formula (1) is valid for zero thickness of the strip conductor and only in the quasi-static frequency range when the transverse sizes w and h of the microstrip line are smaller than the wavelength of propagating electromagnetic radiation. In the quasi-static frequency range, wave impedances of regular microstrip line sections from which the photonic crystal is designed can

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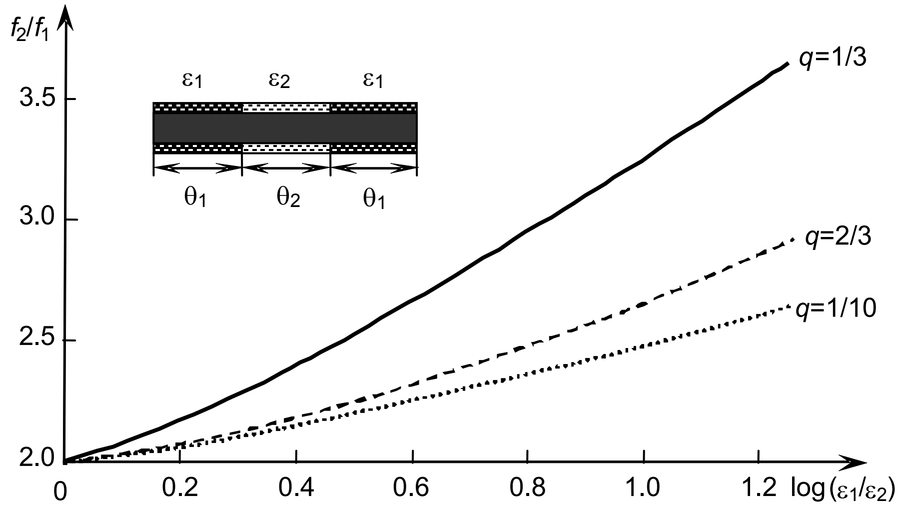

Fig. 1. Dependences of the relative frequency of the second resonance of the microstrip structure on the ratio of the dielectric permittivites of the end and central sections of its hybrid substrate.

be calculated from formulas [7]

$$Z = \begin{cases} \dfrac{120\pi/\sqrt{\varepsilon_{\text{eff}}}}{1.393 + w/h + 0.667\ln(w/h + 1.444)}, & \text{for } w \geq h, \\ \dfrac{60}{\sqrt{\varepsilon_{\text{eff}}}} \ln\left(\dfrac{8h}{w} + \dfrac{w}{4h}\right), & \text{for } w \leq h. \end{cases} \quad (2)$$

The regular line sections are resonators whose coupling is determined by impedance discontinuities [8]. Coupling of the external resonators with the external propagation path is determined by the difference between the wave impedance of the connected transmission lines Z_0= 50 Ω and the corresponding microstrip line sections that form the end sections of the external resonators. From formula (1) it can be seen that a decrease in w for the fixed thickness of the substrate leads to a monotonic decrease of the effective dielectric permittivity and *vice versa*.

We note that in the conventional design of the one-dimensional PC [1, 2], the central frequencies of all pass bands are equidistant, and the widths of the stop bands between them are less than one octave. This is due to the fact that the regular half-wave microstrip resonator (MSR), like any dielectric PC layer, has the equidistant eigenfrequency spectrum. However, filters with wide stop bands when the nearest parasitic pass band is spaced from the operating band at a distance greater than one octave are often required. Such problem can be solved by using irregular microstrip resonators possessing non-equidistant eigenfrequency spectrum.

Figure 1 illustrates the microstrip resonator design and the behavior of the ratio of its first two resonance frequencies f_2/f_1 depending on the discontinuity of the dielectric permittivity of the compound substrate for which $\varepsilon_1 = 16$ and ε_2 decreases from 16 to 1. Calculations were performed in the quasi-static approximation for the one-dimensional resonator model. Curves were drawn for several ratios of the electric length of the central section to the total electric length of the resonator $q = \theta_2/(2\theta_1 + \theta_2)$. Investigations demonstrated that the greater the difference between the dielectric permittivities of the sections, the larger the separation between the frequencies f_1 and f_2.

Figure 2 shows the dependences of the frequencies of the first and second MSR oscillation modes normalized by the frequency f_{01} of the first mode of the regular resonator on the relative electric length of the central section q drawn for three fixed ratios $\varepsilon_1/\varepsilon_2$ in two cases: when the dielectric permittivity of the central MSR section is greater than that of the end sections (for negative q values) and *vice versa* (for positive q values). It can be seen that the frequency of the first mode monotonically increases with q when the dielectric permittivity of the central section is less than that of

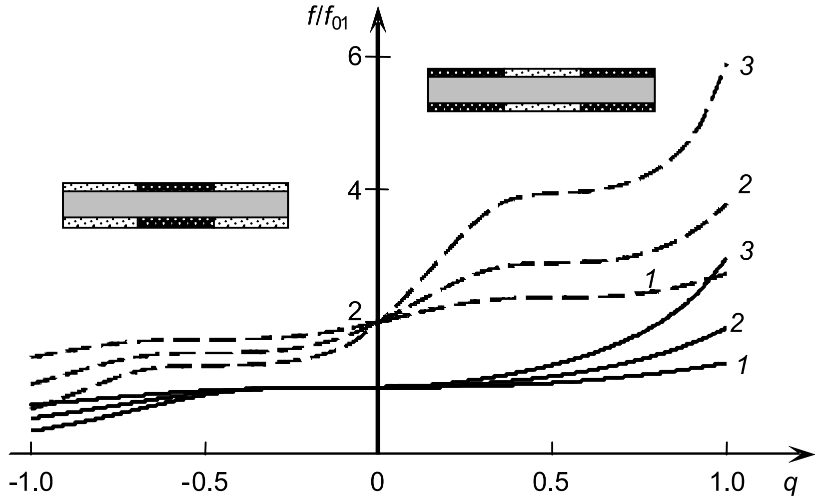

Fig. 2. Dependences of the eigenfrequencies of the first (solid curves) and second oscillation modes (dashed curves) of the irregular microstrip resonator on the relative electric length of the central section when the dielectric constants of the substrates differ by two (curve *1*), four (curve *2*), and ten times (curve *3*).

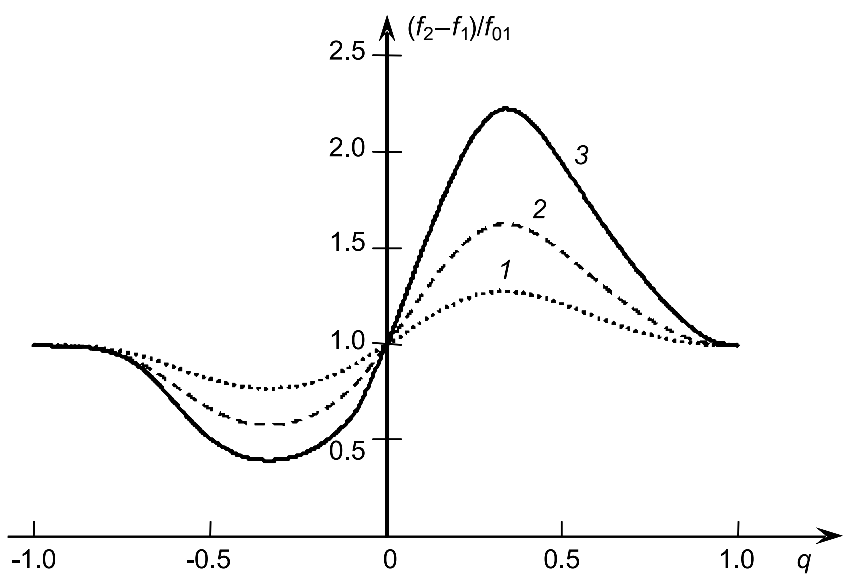

Fig. 3. Dependences of the relative difference of eigenfrequencies of the first and second oscillation modes of the microstrip resonator on the relative electric length of the central section when the dielectric constants of the substrates differed by two (curve *1*), four (curve *2*), and ten times (curve *3*).

the end sections, but when the dielectric permittivity of the central section is greater than that of the end sections, the frequency monotonically decreases with increasing q modulus. However, the frequencies of the second mode behave non-monotonically. Keeping for the dependences the general tendency to eigenfrequency increase or decrease by analogy with the first mode frequency, they have wider ranges where their frequencies remain almost constant.

Figure 3 shows the dependences of the relative difference of eigenfrequencies for the first and second oscillation modes of the microstrip resonator on the relative electric length of its central section drawn also for three fixed ratios $\varepsilon_1/\varepsilon_2$. It can be seen that irrespective of the ratio $\varepsilon_1/\varepsilon_2$, the maximum increase in the frequency of the second

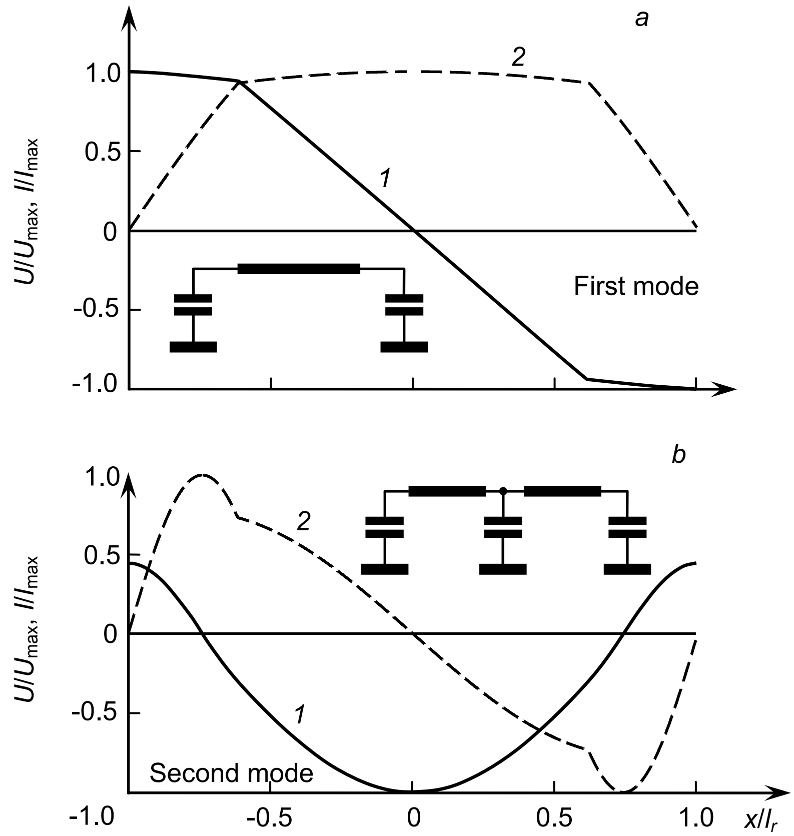

Fig. 4. Distributions of the high-frequency voltage (curve *1*) and current amplitudes (curve *2*) across the relative length of the MSR conductor x/l_r at the eigenfrequencies of the first two oscillation modes of the resonator with $q = 1/3$ and $\varepsilon_1/\varepsilon_2 = 10$. Equivalent circuits corresponding to these modes are shown in the inserts.

resonance compared to the first one as well as its greatest decrease is observed for equal electric lengths of all three MSR sections. Thus, for a fixed discontinuity in the wave impedance of the regular sections of microstrip lines comprising the irregular microstrip resonator, the frequencies of the first and second modes can be spaced almost by two octaves. Moreover, we note that in such irregular resonators, the resonant frequencies of almost any oscillation mode can be changed purposefully by changing the number, positions, and magnitudes of impedance discontinuities.

To explain the revealed behavior of the eigenfrequencies of the examined MSR, we now find the distributions of the normalized current and voltage amplitudes asross the length of the strip conductor of the irregular microstrip resonator for its first and second oscillation modes (Fig. 4). For the first oscillation mode (*a*), the high-frequency current antinode is located in the resonator center, and the high-frequency voltage antinodes are located at the resonator ends; therefore, the equivalent resonator circuit represents the oscillatory contour comprising one inductance and two capacitors. For the second oscillation mode (*b*), the equivalent circuit consists already of two inductances and three capacitors, according to the number of the high-frequency current and voltage antinodes. Considering that the inductivity is independent of the dielectric permittivity of the substrate and the capacitances are directly proportional to it, the dependences of the frequencies of the first and second oscillation modes of the irregular resonator on the relative electric length of the central section (Fig. 2) become understandable. Indeed, when the electric length of the central section increases given that the dielectric permittivity of its substrate is smaller than at the resonator ends, the MSR eigenfrequencies of the first and second oscillation modes will increase because of the decreased capacities of the capacitors. In this case, the frequency of the first mode in the initial section of the dependence changes weakly, because the MSR section with low dielectric permittivity is located in the current antinode that corresponds to the inductance.

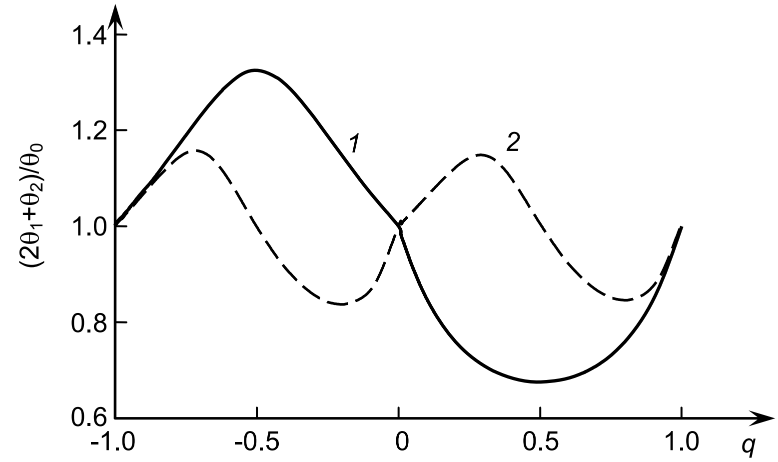

Fig. 5. Dependences of the total electric length of the irregular resonator sections normalized by the electric length of the regular resonator on the relative electric length of the central section for the first (curve *1*) and second oscillation modes (curve *2*) at $\varepsilon_1/\varepsilon_2 = 10$.

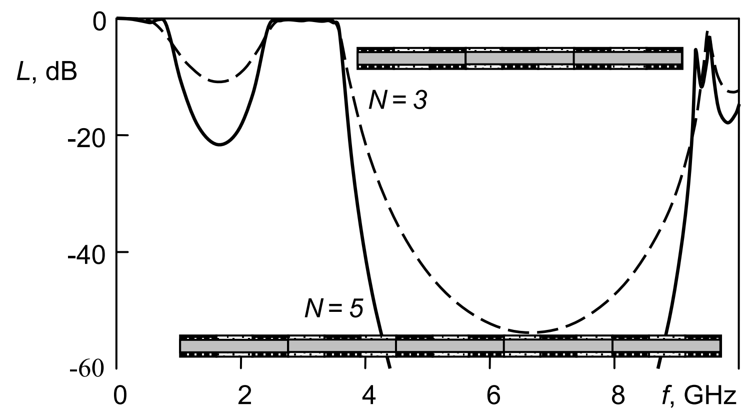

Fig. 6. Frequency response of the one-dimensional photonic crystals with two sublattices built around irregular microstrip resonators.

The frequency of the second mode, on the contrary, fast increases, because the high-frequency voltage antinode is located in the MSR center; therefore, the central part of the resonator plays the role of a capacitor.

Figure 5 shows the dependences of the sums of electric lengths of the examined irregular resonator sections normalized by the electric length of the regular MSR θ_0 on q. The curves were drawn for the resonant frequencies of the first oscillation mode (curve *1*; $\theta_0 = \pi$) and second oscillation mode (curve *2*; $\theta_0 = 2\pi$). It can be seen that the total electric length of the irregular resonator can be smaller or larger than the electric length of the regular MSR.

Based on the investigated irregular MSR, bandpass and stop band filters (mirrors) can be designed by simple series connection of resonators one after another. In this case, the MSR with high wave impedance of the end line sections ($q > 0$) and low wave impedance ($q < 0$) must be alternated. To adjust the filter design, it is necessary to satisfy simultaneously three conditions. First, according to the preset bandwidth of the bandpass filter and the level of return, it is required to provide optimal coupling of the external resonators with input and output transmission lines. Second, it is necessary to provide the balance of coupling of all resonators with each other and, finely, third, it is necessary to match the resonant frequencies of resonators to the central frequency f_0 of the bandpass filter.

Figure 6 shows the frequency response (FR) of two filters with the number of resonators N = 3 and 5. The filters had the relative bandwidths $\Delta f/f_0$ = 40% (f_0 = 3 GHz) measured on a level of –3 dB, and their hybrid substrates

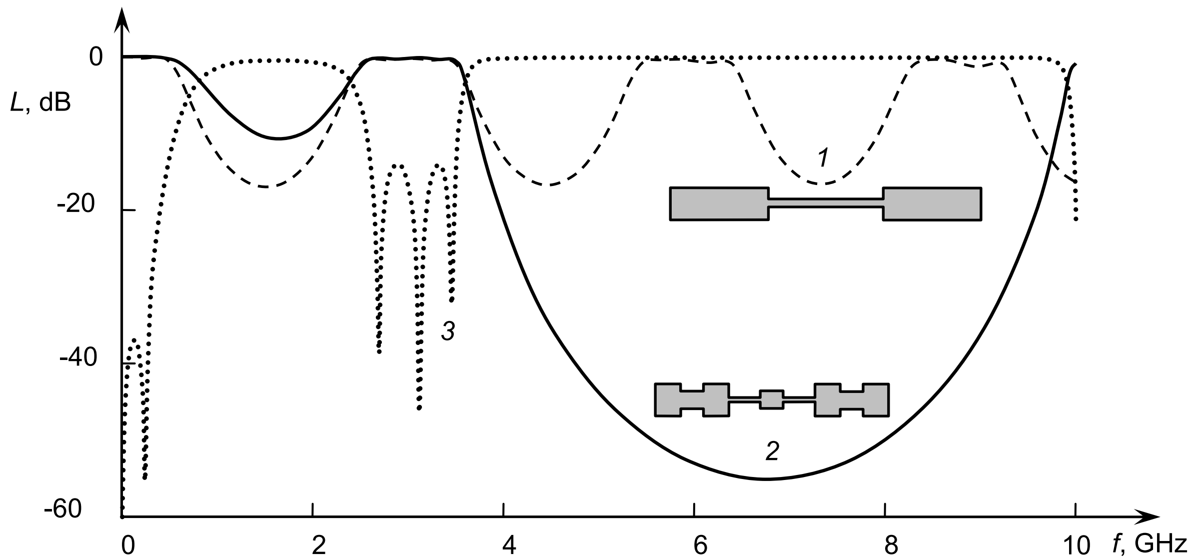

Fig. 7. Frequency response of microstrip photonic crystals with one (curve *1*) and two sublattices (curve *2*). Curve *3* shows the frequency dependence of the return losses.

comprised two materials with dielectric permittivity values $\varepsilon_1 = 16$ and $\varepsilon_2 = 1$. We note that addition of two resonators to the design is accompanied by an increase in the attenuation in the low-frequency stop band by more than 10 dB, and by more than 20 dB in the high-frequency stop band. It should be noted that the attenuation in the stop band essentially increases with decreasing bandwidth of the device; however, this is almost impossible for this design, because too large wave impedance discontinuities are required in lines forming the resonator for this purpose.

It is also impossible to obtain the relative bandwidth exceeding 40% using conventional microstrip photonic crystal filters with a monolithic substrate when the required wave impedance discontinuities are provided with a step change of the strip conductor widths. Figure 7 shows the FR of filters built around photonic crystals with three resonators and one (curve *1*) and two sublattices (curve *2*). The filters are arranged on the substrate 1 mm wide with $\varepsilon = 16$. For convenience of comparison, the band parameters of the designs were the same: $f_0 = 3$ GHz, $\Delta f/f_0 = 40\%$, and maxima of losses in the pass band were at a level of –14 dB. It can be seen that the PC with one sublattice has equidistant alternating transparency and stop bands, and the high-frequency stop band of the PC with two sublattices is much wider. This means that the examined rather simple design of the one-dimensional photonic crystal can serve simultaneously not only as a bandpass filter with increased stop band, but also as a good mirror with the reflection coefficient close to unity in a sufficiently wide frequency range. We note that the antinodes of the high-frequency electric field *E* are always localized at the ends of the external resonators, and the antinodes of the magnetic field *H* are always localized in the center. The central resonator, on the contrary, has antinodes of *H* localized at its ends and the antinode of *E* localized in the center.

To develop photonic crystal filters with a relative bandwidth of 40%, we suggested a new design of the PC with two sublattices (Fig. 8) in which the examined irregular half-wave resonators are also used to expand the high-frequency stop band (the second bandgap), but they are coupled through quarter wavelength line sections with high wave impedance. This allowed us not only to weaken considerably the interaction of the resonators to reduce the pass band of the filter, but also to increase the suppression of the power transmitted in the low-frequency stop band (the first bandgap).

The irregular microstrip resonators of the filter were made from TBNS ceramics ($\varepsilon = 97.5$) and arranged on dielectric substrates 1 mm thick, and quarter-wavelength regular line sections were made from polycor ($\varepsilon = 10$) and arranged on substrates also 1 mm thick. As a result of investigations of the filter design, it had five irregular half-wavelength microstrip resonators and four regular quarter-wavelength sections, a total of 19 sections of regular microstrip lines. The filter with the preset fractional bandwidth $\Delta f/f_0 = 10\%$ and central frequency of the pass band

TABLE 1. Lengths and Widths of Strip Conductors in Regular Sections of the Bandpass Filter Built around the Microstrip Photonic Crystal with Two Sublattices

Serial number of the line section	1	2	3	4	5	6	7	8	9	10
Width of the conductor	5.6	0.4	5.6	0.62	5.6	0.48	5.6	0.2	5.6	0.49
Length of the conductor	3.545	3.48	3.545	22.63	3.69	3.69	3.69	19.3	3.51	4

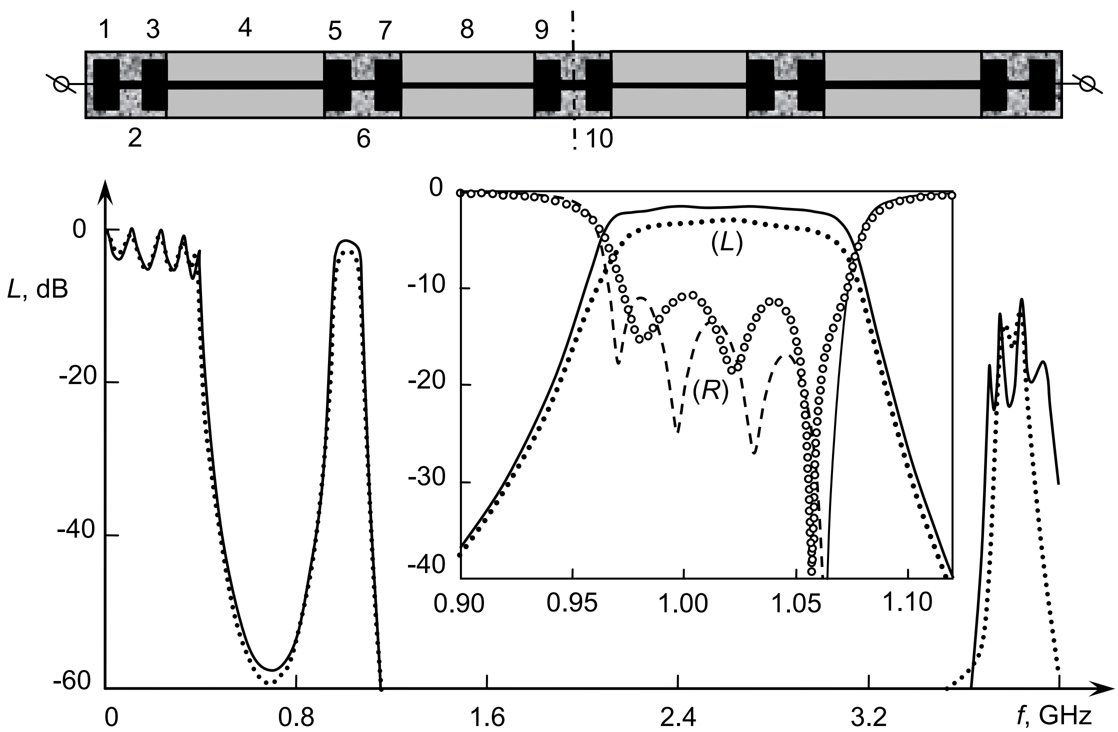

Fig. 8. Design and frequency response of the filter built around the microstrip PC with two sublattices having a fractional bandwidth of 10%. Here curves show results of calculations, and circles show experimental results. The photograph of the device is shown to the right of the figure.

f_0 = 1 GHz was synthesized by selection of lengths and width of strip conductors for all regular sections with the use of numerical analysis of microstrip structure. Results of parametrical synthesis are presented in Table 1 (all sizes are in millimeters).

Thus, the spectra of natural oscillations of irregular microstrip resonators have been investigated and the possibility of significant spacing of the frequencies of the first two resonances has been demonstrated that allows the bandpass filters with expanded high-frequency stop band to be designed. The new design of the bandpass filter built around the photonic crystal with two sublattices has been suggested whose half-wavelength irregular resonators are coupled with each other through quarter-wavelength sections of regular lines. This allowed coupling between the resonators to be significantly decreased and hence the fractional bandwidth of the pass band to be decreased. Good agreement of the results of quasi-static numerical analysis of microstrip models in such filters with experimental data allows the high-speed parametrical synthesis to be performed to obtain the design parameters of the device with the preset characteristics. The developed design, even with a rather small number of resonators (only 5), demonstrates fairly high selective characteristics. It simultaneously can play the role of good bandpass filter and mirror with high reflection coefficient in a wide frequency band.

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